FUZZY α -WEAKLY (r, s) -CONTINUOUS MAPPINGS

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Abstract. In this paper, we introduce the concept of fuzzy α -weakly (r, s) -continuous mappings on an fuzzy topological space and investigate some properties of such mappings.

1. Introduction

The concept of fuzzy set was introduced by Zadeh [17]. As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Chang [2] defined fuzzy topological spaces using fuzzy sets. In [3], Chattopadhyay, Hazra and Samanta introduced the concept of smooth fuzzy topological spaces which are a generalization of fuzzy topological spaces.

Çoker and his colleagues [4,5,6,7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. In [5], Çoker and Demirci introduced intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth fuzzy topological spaces and intuitionistic fuzzy topological spaces. The author introduced the concept of fuzzy S -weakly (r, s) -continuous mappings [15] as a generalization of fuzzy (r, s) -continuous mappings and fuzzy weakly (r, s) -continuous mappings. In this paper, we introduce fuzzy α -weakly (r, s) -continuous mappings as a generalization of fuzzy strongly (r, s) -semicontinuous mappings and fuzzy weakly (r, s) -continuous mappings and investigate some properties. In particular, we investigate the relationships among fuzzy weakly (r, s) -continuous mappings, fuzzy S -weakly (r, s) -continuous and fuzzy α -weakly (r, s) -continuous mappings.

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2. Preliminaries

Let I be the unit interval $[0, 1]$ of the real line. A member μ of I^X is called a fuzzy set of X . By $\tilde{0}$ and $\tilde{1}$ we denote constant maps on X with value 0 and 1, respectively. For any $\mu \in I^X$, μ^c denotes the complement $\tilde{1} - \mu$. All other notations are standard notations of fuzzy set theory.

Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair

$$A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X \} \text{ (simply, } A = (\mu_A, \gamma_A) \text{)}$$

where the functions $\mu_A : X \rightarrow I$ and $\gamma_A : X \rightarrow I$ denote the degree of membership and the degree of nonmembership, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for $x \in X$. We will denote the set of all intuitionistic fuzzy sets in X by $IF(X)$.

An *intuitionistic fuzzy point* $x_{(\alpha, \beta)}$ in X is an intuitionistic fuzzy set

$$x_{(\alpha, \beta)} = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \rightarrow I$ and $\gamma_A : X \rightarrow I$ are defined as follows:

$$(\mu_A(y), \gamma_A(y)) = \begin{cases} (\alpha, \beta), & \text{if } y = x, \\ (0, 1), & \text{if } y \neq x; \end{cases}$$

and $0 \leq \alpha + \beta \leq 1$.

An intuitionistic fuzzy point $x_{(\alpha, \beta)}$ is said to belong to an intuitionistic fuzzy set $A = (\mu_A, \gamma_A)$ in X , denoted by $x_{(\alpha, \beta)} \in A$, if $\mu_A(x) \geq \alpha$ and $\gamma_A(x) \leq \beta$ for $x \in X$.

An intuitionistic fuzzy set A in X is the union of all intuitionistic fuzzy points which belong to A .

Definition 2.1. ([1]) Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy sets on X . Then

- (1) $A \subseteq B$ iff $\mu_A \leq \mu_B$ and $\gamma_A \geq \gamma_B$.
- (2) $A = B$ iff $A \subseteq B$ and $B \subseteq A$.
- (3) $A^c = (\gamma_A, \mu_A)$.
- (4) $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$.
- (5) $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$.
- (6) $0_\sim = (\tilde{0}, \tilde{1})$ and $1_\sim = (\tilde{1}, \tilde{0})$.

Let f be a map from a set X to a set Y . Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy set of X and $B = (\mu_B, \gamma_B)$ an intuitionistic fuzzy set of Y . Then:

(1) The image of A under f , denoted by $f(A)$ is an intuitionistic fuzzy set in Y defined by

$$f(A) = (f(\mu_A), \tilde{1} - f(\tilde{1} - \gamma_A)).$$

(2) The inverse image of B under f , denoted by $f^{-1}(B)$ is an intuitionistic fuzzy set in X defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)).$$

A *smooth fuzzy topology* [16] on X is a map $T : I^X \rightarrow I$ which satisfies the following properties:

- (1) $T(\tilde{0}) = T(\tilde{1}) = 1$.
- (2) $T(\mu_1 \wedge \mu_2) \geq T(\mu_1) \wedge T(\mu_2)$ for $\mu_1, \mu_2 \in I^X$.
- (3) $T(\bigvee \mu_i) \geq \bigwedge T(\mu_i)$ for $\mu_i \in I^X$.

The pair (X, T) is called a *smooth fuzzy topological space*.

An *intuitionistic fuzzy topology* on X is a family T of intuitionistic fuzzy sets in X which satisfies the following properties:

- (1) $0_\sim, 1_\sim \in T$.
- (2) If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$.
- (3) If $A_i \in T$ for all $i \in J$, then $\bigcup A_i \in T$.

The pair (X, T) is called an *intuitionistic fuzzy topological space*.

Let $IF(X)$ be a family of all intuitionistic fuzzy sets of X and let $I \otimes I$ be the set of the pair (r, s) such that $r, s \in I$ and $0 \leq r + s \leq 1$.

Definition 2.2. ([5]) Let X be a nonempty set. An *intuitionistic fuzzy topology in Šostak's sense* (SoIFT for short) $\mathcal{T} = (T_1, T_2)$ on X is a map $\mathcal{T} : IF(X) \rightarrow I \otimes I$ which satisfies the following properties:

- (1) $T_1(0_\sim) = T_1(1_\sim) = 1$ and $T_2(0_\sim) = T_2(1_\sim) = 0$.
- (2) $T_1(A \cap B) \geq T_1(A) \wedge T_1(B)$ and $T_2(A \cap B) \leq T_2(A) \vee T_2(B)$.
- (3) $T_1(\bigcup A_i) \geq \bigwedge T_1(A_i)$ and $T_2(\bigcup A_i) \leq \bigvee T_2(A_i)$.

The $(X, \mathcal{T}) = (X, T_1, T_2)$ is said to be an *intuitionistic fuzzy topological space in Šostak's sense* (SoIFTS for short). Also, we call $T_1(A)$ a *gradation of openness* of A and $T_2(A)$ a *gradation of nonopenness* of A . The *fuzzy (r, s) -closure* and the *fuzzy (r, s) -interior* of A , denoted by $cl(A, r, s)$ and $int(A, r, s)$, respectively, are defined as

$$cl(A, r, s) = \bigcap \{B \in IF(X) : A \subseteq B \text{ and } B \text{ is fuzzy } (r, s)\text{-closed}\},$$

$$int(A, r, s) = \bigcup \{B \in IF(X) : B \subseteq A \text{ and } B \text{ is fuzzy } (r, s)\text{-open}\}.$$

Definition 2.3. Let A be an intuitionistic fuzzy set in an SoIFTS (X, T_1, T_2) and $(r, s) \in I \otimes I$. Then A is said to be

- (1) *fuzzy (r, s) -semiopen* [8] if there is a fuzzy (r, s) -open set B in X such that $B \subseteq A \subseteq cl(B, r, s)$,
- (2) *fuzzy (r, s) -preopen* [10] if $A \subseteq int(cl(A, r, s), r, s)$,
- (3) *fuzzy strongly (r, s) -semiopen* [11] if $A \subseteq int(clint((A, r, s), r, s))$.

Let A be an intuitionistic fuzzy set in an SoIFTS (X, T_1, T_2) and $(r, s) \in I \otimes I$. The *fuzzy strongly (r, s) -semi-closure* and the *fuzzy strongly (r, s) -semi-interior* of A , denoted by $sscl(A, r, s)$ and $ssint(A, r, s)$, respectively, are defined as

$$\begin{aligned} sscl(A, r, s) &= \cap \{B \in IF(X) : A \subseteq B \text{ and} \\ &\quad B \text{ is fuzzy strongly } (r, s)\text{-semiclosed}\}, \\ ssint(A, r, s) &= \cup \{B \in IF(X) : B \subseteq A \text{ and} \\ &\quad B \text{ is fuzzy strongly } (r, s)\text{-semiopen}\}. \end{aligned}$$

Definition 2.4. Let $f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2)$ be a mapping from an SoIFTS X to another SoIFTS Y and $(r, s) \in I \otimes I$. Then f is said to be

- (1) *fuzzy (r, s) -continuous* [8] if for each fuzzy (r, s) -open set B of Y , $f^{-1}(B)$ is fuzzy (r, s) -open in X ;
- (2) *fuzzy strongly (r, s) -continuous* [9] if for each fuzzy (r, s) -open set B of Y , $f^{-1}(B)$ is fuzzy strongly (r, s) -semiopen in X ;
- (3) *fuzzy weakly (r, s) -continuous* [8] if for each fuzzy (r, s) -open set B of Y , $f^{-1}(B) \subseteq int(f^{-1}(cl(B, r, s)), r, s)$ in X ;
- (4) f is said to be *fuzzy S -weakly (r, s) -continuous* [15] if for each fuzzy (r, s) -open set B of Y , $f^{-1}(B) \subseteq sint(f^{-1}(cl(B, r, s)), r, s)$ in X .

3. Main Results

Definition 3.1. Let $f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2)$ be a mapping from an SoIFTS X to another SoIFTS Y and $(r, s) \in I \otimes I$. Then f is said to be *fuzzy α -weakly (r, s) -continuous* if for each fuzzy (r, s) -open set μ of Y , $f^{-1}(\mu) \subseteq ssint(f^{-1}(cl(\mu, r, s)), r, s)$.

Remark 3.2. Every fuzzy strongly (r, s) -semicontinuous mapping is fuzzy α -weakly (r, s) -continuous but the converse is not always true.

fuzzy (r, s) -continuous $\Rightarrow$ fuzzy strongly (r, s) -semicontinuous $\Rightarrow$ fuzzy α -weakly (r, s) -continuous

Example 3.3. Let $X = I$ and let A and B be intuitionistics fuzzy sets of X defined as

$$A(x) = (-\frac{1}{3}x + \frac{2}{3}, \frac{1}{3}x + \frac{1}{3}), \text{ for } x \in I,$$

$$B(x) = (\frac{1}{2}x, \frac{1}{2}x), \text{ for } x \in I.$$

Define an SoIFT $\mathcal{T}_1 : IF(X) \rightarrow I \otimes I$ by

$$\mathcal{T}_1(\sigma) = (T_1(\sigma), T_2(\sigma)) = \begin{cases} (1, 0), & \text{if } \sigma = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{3}), & \text{if } \sigma = A, \\ (0, 1), & \text{otherwise,} \end{cases}$$

and define an SoIFT $\mathcal{T}_2 : IF(X) \rightarrow I \otimes I$ by

$$\mathcal{T}_2(\sigma) = (U_1(\sigma), U_2(\sigma)) = \begin{cases} (1, 0), & \text{if } \sigma = 0_\sim, 1_\sim, \\ (\frac{2}{3}, \frac{1}{3}), & \text{if } \sigma = B, \\ (0, 1), & \text{otherwise.} \end{cases}$$

Consider a mapping $f : (X, \mathcal{T}_1) \rightarrow (X, \mathcal{T}_2)$ defined as follows $f(x) = x$ for all $x \in X$. Then f is a fuzzy α -weakly $(\frac{1}{2}, \frac{1}{3})$ -continuous mapping but it is not fuzzy strongly $(\frac{1}{2}, \frac{1}{3})$ -semicontinuous.

Theorem 3.4. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (Y, U_1, U_2)$ be a mapping from an SoIFTS X to another SoIFTS Y and $(\alpha, \beta), (r, s) \in I \otimes I$. Then f is a fuzzy α -weakly (r, s) -continuous mapping if and only if for an intuitionistic fuzzy point $x_{(\alpha, \beta)}$ and each fuzzy (r, s) -open set V containing $f(x_{(\alpha, \beta)})$, there exists a fuzzy strongly (r, s) -semiopen set U containing $x_{(\alpha, \beta)}$ such that $f(U) \subseteq cl(V, r, s)$.

Proof. Suppose f is a fuzzy α -weakly (r, s) -continuous mapping. Let $x_{(\alpha, \beta)}$ be an intuitionistic fuzzy point in X and V a fuzzy strongly (r, s) -semiopen set containing $f(x_{(\alpha, \beta)})$. Then there exists a fuzzy (r, s) -open set B such that $f(x_{(\alpha, \beta)}) \in B \subseteq V$. Since f is a fuzzy α -weakly (r, s) -continuous mapping,

$$f^{-1}(B) \subseteq ssint(f^{-1}(cl(B, r, s)), r, s) \subseteq ssint(f^{-1}(cl(V, r, s)), r, s).$$

Set $U = ssint(f^{-1}(cl(V, r, s)), r, s)$. Then U is a fuzzy strongly (r, s) -semiopen set such that $f^{-1}(B) \subseteq U$. So $f(U) \subseteq cl(V, r, s)$.

For the converse, let V be a fuzzy (r, s) -open set in Y . For each $x_{(\alpha, \beta)} \in f^{-1}(V)$, there exists a fuzzy strongly (r, s) -semiopen set $U_{x_{(\alpha, \beta)}}$ containing $x_{(\alpha, \beta)}$ such that $f(U_{x_{(\alpha, \beta)}}) \subseteq cl(V, r, s)$. i.e., $x_{(\alpha, \beta)} \in U_{x_{(\alpha, \beta)}} \subseteq f^{-1}(cl(V, r, s))$ for each $x_{(\alpha, \beta)} \in f^{-1}(V)$.

Thus $\cup\{U_{x_{(\alpha,\beta)}} : x_{(\alpha,\beta)} \in f^{-1}(V)\} \subseteq f^{-1}(cl(V, r, s))$. Since $\cup\{U_{x_{(\alpha,\beta)}} : x_{(\alpha,\beta)} \in f^{-1}(V)\}$ is fuzzy strongly (r, s) -semiopen, we have $f^{-1}(V) \subseteq ssint(f^{-1}(cl(V, r, s)), r, s)$. $\square$

Theorem 3.5. Let $f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2)$ be a mapping from an SoIFTS X to another SoIFTS Y and $(r, s) \in I \otimes I$. Then the following statements are equivalent:

- (1) f is fuzzy α -weakly (r, s) -continuous.
- (2) $sscl(f^{-1}(int(F, r, s)), r, s) \subseteq f^{-1}(F)$ for each fuzzy (r, s) -closed set F in Y .
- (3) $sscl(f^{-1}(int(cl(B, r, s), r, s)), r, s) \subseteq f^{-1}(cl(B, r, s))$ for $B \in IF(Y)$.
- (4) $f^{-1}(int(B, r, s)) \subseteq ssint(f^{-1}(cl(int(B, r, s), r, s)), r, s)$ for $B \in IF(Y)$.
- (5) $sscl(f^{-1}(V), r, s) \subseteq f^{-1}(cl(V, r, s))$ for a fuzzy (r, s) -open set V in Y .

Proof. (1) $\Rightarrow$ (2) Let F be any fuzzy (r, s) -closed set of Y . Then from (1), it follows

$$\begin{aligned} f^{-1}(1_{\sim} - F) &\subseteq ssint(f^{-1}(cl(1_{\sim} - F, r, s)), r, s) \\ &= ssint(f^{-1}(1_{\sim} - int(F, r, s)), r, s) \\ &= ssint(1_{\sim} - f^{-1}(int(F, r, s)), r, s) \\ &= 1_{\sim} - sscl(f^{-1}(int(F, r, s)), r, s). \end{aligned}$$

Hence we have $sscl(f^{-1}(int(F, r, s)), r, s) \subseteq f^{-1}(F)$.

(2) $\Rightarrow$ (3) Since $cl(B, r, s)$ is a fuzzy (r, s) -closed set in Y , by (2), it is obvious.

(3) $\Rightarrow$ (4) For $B \in I^Y$, from (3),

$$\begin{aligned} f^{-1}(int(B, r, s)) &= 1_{\sim} - (f^{-1}(cl(1_{\sim} - B, r, s))) \\ &\subseteq 1_{\sim} - sscl(f^{-1}(int(cl(1_{\sim} - B, r, s), r, s)), r, s) \\ &= ssint(f^{-1}(cl(int(B, r, s), r, s)), r, s). \end{aligned}$$

Hence $f^{-1}(int(B, r, s)) \subseteq ssint(f^{-1}(cl(int(B, r, s), r, s)), r, s)$.

(4) $\Rightarrow$ (5) Let V be any fuzzy (r, s) -open set of Y . Then from (4) and $(V, r, s) \subseteq \text{int}(cl(V, r, s), r, s)$,

$$\begin{aligned} 1_{\sim} - f^{-1}(cl(V, r, s)) &= f^{-1}(\text{int}(1_{\sim} - V, r, s)) \\ &\subseteq \text{ssint}(f^{-1}(cl(\text{int}(1_{\sim} - V, r, s), r, s)), r, s) \\ &= \text{ssint}(1_{\sim} - (f^{-1}(\text{int}(cl(V, r, s), r, s))), r, s) \\ &= 1_{\sim} - \text{sscl}(f^{-1}(\text{int}(cl(V, r, s), r, s)), r, s) \\ &\subseteq 1_{\sim} - \text{sscl}(f^{-1}(V), r, s). \end{aligned}$$

Hence

$$\text{sscl}(f^{-1}(V), r, s) \subseteq f^{-1}(cl(V, r, s)).$$

(5) $\Rightarrow$ (1) Let V be a fuzzy (r, s) -open set in Y . From $V \subseteq \text{int}(cl(V, r, s), r, s)$ and (5),

$$\begin{aligned} f^{-1}(V) &\subseteq f^{-1}(\text{int}(cl(V, r, s), r, s)) \\ &= 1_{\sim} - f^{-1}(cl(1_{\sim} - cl(V, r, s), r, s)) \\ &\subseteq 1_{\sim} - \text{sscl}(f^{-1}(1_{\sim} - cl(V, r, s)), r, s) \\ &= \text{ssint}(f^{-1}(cl(V, r, s)), r, s). \end{aligned}$$

Hence f is a fuzzy α -weakly (r, s) -continuous mapping. □

Theorem 3.6. Let $f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2)$ be a mapping from an SoIFTS X to another SoIFTS Y and $(r, s) \in I \otimes I$. Then f is fuzzy α -weakly (r, s) -continuous if and only if $\text{sscl}(f^{-1}(\text{int}(cl(V, r, s), r, s)), r, s) \subseteq f^{-1}(cl(V, r, s))$ for each fuzzy (r, s) -preopen set V in Y .

Proof. Assume f is fuzzy α -weakly (r, s) -continuous. Let V be a fuzzy (r, s) -preopen set of Y and $A = \text{int}(cl(V, r, s), r, s)$. Since A is a fuzzy (r, s) -open set, by Theorem 3.5 (5), we have

$$\text{sscl}(f^{-1}(\text{int}(cl(A, r, s), r, s)), r, s) \subseteq f^{-1}(cl(A, r, s)).$$

From $cl(A, r, s) = cl(V, r, s)$, it follows

$$\text{sscl}(f^{-1}(\text{int}(cl(V, r, s), r, s)), r, s) \subseteq f^{-1}(cl(V, r, s)).$$

For the converse, let G be a fuzzy (r, s) -open set of Y . Then since G is a fuzzy (r, s) -preopen set, we have

$$\text{sscl}(f^{-1}(G), r, s) \subseteq \text{sscl}(f^{-1}(\text{int}(cl(G, r, s), r, s)), r, s) \subseteq f^{-1}(cl(G, r, s)).$$

Hence, by Theorem 3.5 (5), f is a fuzzy α -weakly (r, s) -continuous mapping. □

Theorem 3.7. Let $f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2)$ be a mapping from an SoIFTS X to another SoIFTS Y and $(r, s) \in I \otimes I$. Then f is fuzzy α -weakly (r, s) -continuous if and only if $sscl(f^{-1}(int(cl(G, r, s), r, s)), r, s) \subseteq f^{-1}(cl(G, r, s))$ for each fuzzy (r, s) -semiopen set G in Y .

Proof. Assume f is fuzzy α -weakly (r, s) -continuous. Let V be a fuzzy (r, s) -semiopen set in Y and $F = cl(int(V, r, s), r, s)$. Since F is a fuzzy (r, s) -closed set, by Theorem 3.5 (2), we have

$$sscl(f^{-1}(int(F, r, s)), r, s) \subseteq f^{-1}(F).$$

From $cl(V, r, s) = cl(int(V, r, s), r, s) = F$, it follows

$$sscl(f^{-1}(int(cl(V, r, s), r, s)), r, s) \subseteq f^{-1}(cl(V, r, s)).$$

For the converse, let G be a fuzzy (r, s) -open set of Y . Then since G is a fuzzy (r, s) -semiopen set, by Theorem 3.5 (5), f is a fuzzy α -weakly (r, s) -continuous mapping. $\square$

Theorem 3.8 ([9]). Let $f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2)$ be a mapping from an SoIFTS X to another SoIFTS Y and $(r, s) \in I \otimes I$. Then f is fuzzy strongly (r, s) -continuous if and only if $sscl(f^{-1}(cl(G, r, s)), r, s) \subseteq f^{-1}(cl(G, r, s))$ for each intuitionistics fuzzy set G in Y .

Theorem 3.9. Let $f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2)$ be a mapping from an SoIFTS X to another SoIFTS Y and $(r, s) \in I \otimes I$. Then if f is fuzzy strongly (r, s) -semicontinuous, then it is fuzzy weakly (r, s) -continuous.

Proof. Let B be fuzzy (r, s) -open in Y . Since f is fuzzy strongly (r, s) -continuous, $f^{-1}(B)$ is fuzzy strongly (r, s) -semiopen and $f^{-1}(cl(B, r, s))$ is fuzzy strongly (r, s) -semiclosed in X . Thus from Theorem 3.8, it follows

$$\begin{aligned} f^{-1}(B) &\subseteq int(cl(int(f^{-1}(B), r, s), r, s), r, s) \\ &\subseteq cl(int(cl(f^{-1}(B), r, s), r, s), r, s) \\ &\subseteq sscl(f^{-1}(cl(B, r, s)), r, s) \\ &\subseteq f^{-1}(cl(B, r, s)). \end{aligned}$$

This implies $f^{-1}(B) \subseteq int(f^{-1}(cl(B, r, s)), r, s)$. Hence f is fuzzy weakly (r, s) -continuous. $\square$

Remark 3.10. From Definition 2.4 and Definition 3.1, we know every fuzzy α -weakly (r, s) -continuous mapping is fuzzy S -weakly (r, s) -continuous. But the converse is not always true as shown in the next example.

Example 3.11. Let $X = \{x, y\}$ and A_1, A_2, μ and σ be intuitionistic fuzzy sets in X defined as

$$\begin{aligned} A_1(x) &= (0.2, 0.8), & A_1(y) &= (0.5, 0.5); \\ A_2(x) &= (0.5, 0.2), & A_2(y) &= (0.5, 0.5); \\ \mu(x) &= (0.6, 0.2), & \mu(y) &= (0.5, 0.5); \\ \sigma(x) &= (0.2, 0.7), & \sigma(y) &= (0.5, 0.5); \end{aligned}$$

Define an SoIFT $\mathcal{T}_1 : IF(X) \rightarrow I \otimes I$ by

$$\mathcal{T}_1(A) = (T_1(A), T_2(A)) = \begin{cases} (1, 0), & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{3}), & \text{if } A = A_1, A_2, \\ (0, 1), & \text{otherwise,} \end{cases}$$

and define an SoIFT $\mathcal{T}_2 : IF(X) \rightarrow I \otimes I$ by

$$\mathcal{T}_2(A) = (U_1(A), U_2(A)) = \begin{cases} (1, 0), & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{3}), & \text{if } A = \mu, \sigma, \\ (0, 1), & \text{otherwise.} \end{cases}$$

Consider the identity mapping $f : (X, \mathcal{T}_1) \rightarrow (X, \mathcal{T}_2)$. Then clearly f is a fuzzy S -weakly $(\frac{1}{2}, \frac{1}{3})$ -continuous mapping but it is not fuzzy α -weakly $(\frac{1}{2}, \frac{1}{3})$ -continuous.

Lemma 3.12. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from an SoIFTS X to another SoIFTS Y and $(r, s) \in I \otimes I$. Then if f is a fuzzy weakly (r, s) -continuous mapping, then it is fuzzy α -weakly (r, s) -continuous.

Proof. Obvious. □

Example 3.13. Let $X = I$ and A_1, A_2, B_1 and B_2 be intuitionistic fuzzy sets of X defined as

$$\begin{aligned} A_1(x) &= (1/4, 3/4); \\ A_2(x) &= (\frac{5}{8}, \frac{3}{8}). \\ B_1(x) &= (\frac{1}{2}, \frac{3}{4}); \\ B_2(x) &= (\frac{7}{9}, \frac{1}{9}). \end{aligned}$$

Define an SoIFT $\mathcal{T}_1 : IF(X) \rightarrow I \otimes I$ by

$$\mathcal{T}_1(A) = (T_1(A), T_2(A)) = \begin{cases} (1, 0), & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{4}), & \text{if } A = A_1, A_2, \\ (0, 1), & \text{otherwise,} \end{cases}$$

and define an SoIFT $\mathcal{T}_2 : IF(X) \rightarrow I \otimes I$ by

$$\mathcal{T}_2(A) = (U_1(A), U_2(A)) = \begin{cases} (1, 0), & \text{if } A = 0_{\sim}, 1_{\sim}, \\ (\frac{1}{2}, \frac{1}{3}), & \text{if } A = B_1, B_2, \\ (0, 1), & \text{otherwise.} \end{cases}$$

Consider a mapping $f : (X, \mathcal{T}_1) \rightarrow (X, \mathcal{T}_2)$ defined as follows $f(x) = x$ for all $x \in X$. Then f is a fuzzy α -weakly $(\frac{1}{2}, \frac{1}{3})$ -continuous mapping but it is not fuzzy weakly $(\frac{1}{2}, \frac{1}{3})$ -continuous.

Finally we get the following implications:

fuzzy (r, s) -continuous $\Rightarrow$ fuzzy strongly (r, s) -semicontinuous $\Rightarrow$ fuzzy weakly (r, s) -continuous $\Rightarrow$ fuzzy α -weakly (r, s) -continuous $\Rightarrow$ fuzzy S -weakly (r, s) -continuous

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