ON THE PHYSICAL BASIS OF THE TULLY-FISHER RELATION

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ABSTRACT

We analyse the results of mass models derived from the HI rotation curves of spiral galaxies and find that the slope of the luminous mass–circular velocity relation is close to 4. The luminous mass–circular velocity relation with a slope of about 4 can be explained by an anti-correlation between the mass surface density of luminous matter and the mass ratio of the dark and luminous components. We also argue that the *conspiracy* between luminous and dark matter exists in a *local* sense (producing a flat or smooth rotation curve) and in a *global* sense (affecting the mass ratio of the dark and luminous matter), maintaining the luminous mass–circular velocity relation with a slope of about 4. We therefore propose that the physical basis of the Tully-Fisher relation lies in the luminous mass–circular velocity relation. While the slope of the luminous mass–circular velocity relation is fairly well defined regardless of the dark matter contribution, the zero-point of the relation is still to be determined. The determination of the slope of the Tully-Fisher relation needs one more step: the mean trend of the luminosity–luminous mass relation determines the overall shape (slope) of the Tully-Fisher relation. The key parameter needed to determine the zero-point of the luminous mass–circular velocity relation and the slope of the Tully-Fisher relation obviously is the luminous mass-to-light ratio.

Key words : galaxies: distances and redshift – galaxies: fundamental parameters – galaxies: kinematics and dynamics – galaxies: spiral – distance scale

I. INTRODUCTION

The Tully-Fisher relation (Tully & Fisher 1977; hereafter the TF relation), i.e. the relation between luminosity and rotation velocity, is probably the most successful, and widely applied, distance estimator for spiral galaxies to date. Furthermore, the TF relation is a key relationship for studies of evolutionary processes. Its physical basis however is not well understood.

It is commonly assumed that mass plays an intermediary role between luminosity and rotation velocity. In this context, the TF relation is a combination of at least two independent relations: (1) a relation between the amount of luminous matter M_{lum} and the circular velocity V_{cir} , which is the outcome of the process of galaxy formation, and (2) a relation between the luminosity and luminous mass, which is the result of the history of star formation. In addition to the above-mentioned relationships, there must, of course, also be a connection between the distribution of luminous matter and that of all matter, that is, luminous plus dark. The slope of the $M_{lum} - V_{cir}$ relation might contain clues to the galaxy formation process. The mean trend of the $L - M_{lum}$ and $M_{lum} - V_{cir}$ relations determine the overall shape (slope) of the observed TF relation, while the scatter in each of the two relations contributes to the dispersion in the luminosity at a fixed maximum rotation velocity (see Rhee 1996; see also Djorgovski, de Carvalho & Han 1988).

There have been many attempts to explain the observed slope of the TF relation, the value 4 in particular

(see Sanders 1990 for a review of this subject). The observed TF relation slope of around 4 in the near infrared (e.g. Aaronson et al. 1979; Peletier & Willner 1993) has been particularly favoured. With the assumptions of constant M_{lum}/L and M_{dark}/M_{lum} , this slope of 4 can be naturally explained with an initial hierarchy of density perturbations that has a power spectrum slope ($n \simeq -2$) close to that of Cold Dark Matter (hereafter CDM) dominated cosmology (e.g. Faber 1982). However, the reality of the observed TF relation slope of 4 in the near infra-red waveband is questionable, mainly because of the complex nature of the hybrid definition of the H luminosity (H light inside one third of the *blue* diameter) introduced by Aaronson et al. (1979). Furthermore, Bernstein et al. (1994) have reported that the slope of the H band TF relation is found to be as small as 2.4, rather than the preferred value of 4, when they used *total* H luminosity. (But see Rhee (1996) for the total K-band TF relation with the slope 4.) Recent studies have suggested slopes of the I band TF relation of 3 to 3.2 (e.g. Pierce & Tully 1992; Mathewson et al. 1992). Note further that the wavelength dependence of the TF relation (Pierce & Tully 1992) and the existence of colour–magnitude relations (e.g. de Jong 1995) already indicate that the $L - V$ relation is not a *fundamental* relation. In fact, what theories predict is a $M - V$ relation with a preferred slope of 4, not a $L - V$ relation. Therefore, the key question is what is the slope of the more fundamental $M_{lum} - V_{cir}$ relation?

Another fundamental issue is the scatter in the TF

Table 1. GENERAL PROPERTIES OF THE SAMPLE GALAXIES.

Galaxy	T	D	h	R_{25}	R_l	M_{gas}	$L_B^{b,1}$	L_H	V_p	V_m	V_f	Ref.
(1)	(2)	Mpc (3)	kpc (4)	kpc (5)	kpc (6)	$10^{10} M_\odot$ (7)	$10^{10} L_{B\odot}$ (8)	$10^{10} L_{H\odot}$ (9)	km/s (10)	km/s (11)	km/s (12)	(13)
DDO 154	Irr	4.00	0.50	1.58	7.56	0.035	0.013		48	48	47	1,2
DDO 168	Irr	3.64	0.91	1.62	3.36	0.027	0.032		55	55	54	3
DDO 170	Sd	12.00	1.30	1.71	9.61	0.086	0.030		66	66	63	4
IC 2574	SBm	3.63	2.16	6.11	9.77	0.119	0.254	0.032	68	68	66	5
M 33	Sc	0.84	2.79	8.06	8.34	0.136	0.742	0.429	108	108	107	6,7
NGC 55	SBm	1.33	1.34	4.32	8.51	0.085	0.332		87	87	86	8
NGC 247	SBc	2.79	2.85	6.79	10.96	0.129	0.413	0.224	108	108	106	9,10
NGC 300	Sc	2.15	2.33	6.83	12.66	0.133	0.414		97	97	90	10,11
NGC 801	Sc	79.03	12.00	28.33	58.62	2.968	7.635		238	222	218	3,12
NGC 1003	Sc	11.80	1.90	8.81	33.04	0.841	1.490	0.449	110	110	108	13,14
NGC 1560	Sc	3.00	1.30	3.33	8.29	0.110	0.103	0.067	79	79	77	15
NGC 2403	SBc	3.52	2.22	11.15	21.09	0.500	1.277	0.984	136	136	134	16,17
NGC 2841	Sb	12.00	3.02	12.07	54.07	0.980	6.283	6.956	326	326	294	16,17
NGC 2903	SBbc	6.40	2.02	9.91	24.18	0.340	2.173	3.136	216	203	180	16,17
NGC 2998	SBc	67.16	5.40	25.66	46.56	3.370	7.605		214	214	209	3,12
NGC 3109	Irr	1.26	1.14	2.70	6.11	0.033	0.048		67	67	66	18
NGC 3198	SBc	9.36	2.63	9.46	29.92	0.690	1.727	0.821	157	157	149	16,17,19
NGC 5033	Sc	11.90	5.80	14.58	35.36	0.910	2.558	3.973	225	219	195	16,17
NGC 5533	Sb	55.72	11.60	23.85	74.29	3.273	6.923		302	273	239	3,14,20
NGC 5585	SBcd	7.55	1.54	6.00	11.71	0.288	0.539	0.142	92	92	91	21
NGC 6503	Sc	4.43	1.29	3.95	16.57	0.120	0.444	0.406	121	121	115	16,22
NGC 6674	SBb	49.34	8.37	29.81	69.37	3.984	7.112		291	266	241	3,14
NGC 7331	Sbc	14.90	4.48	20.15	36.72	1.580	9.646	19.878	257	242	238	16,17
NGC 7793	Scd	3.09	1.01	4.28	6.74	0.076	0.343	0.172	117	117	105	10,23
UGC 2259	Sc	9.80	1.33	3.76	7.60	0.031	0.071		90	90	90	17,24
UGC 2885	Sbc	79.09	13.00	51.08	72.86	4.960	7.339		310	298	298	12,25

Notes to Table 1: Sources of the references (column 13)

1 Carignan & Freeman (1985)	14 Broeils & Knapen (1991)
2 Carignan & Beaulieu (1989)	15 Broeils (1992a)
3 Broeils (1992b)	16 Begeman (1987)
4 Lake, Schommer & van Gorkom (1990)	17 Kent (1987a)
5 Martimbeau & Carignan (1994)	18 Jobin & Carignan (1990)
6 Kolkman (1995, unpublished)	19 Begeman (1989)
7 Kent (1987b)	20 Kent (1984)
8 Puche, Carignan & Wainscoat (1991)	21 Côté, Carignan & Sancisi (1991)
9 Carignan & Puche (1990b)	22 Wevers (1984)
10 Carignan (1985)	23 Carignan & Puche (1990a)
11 Puche, Carignan & Bosma (1990)	24 Carignan, Sancisi & van Albada (1988)
12 Kent (1986)	25 Roelfsema & Allen (1985)
13 Broeils (1995, unpublished)	

relation. While the slope of the TF relation has been predicted fairly well with the standard CDM model (and others), the predicted scatter is always rather larger (e.g. ~ 1 mag: Cole & Kaiser 1989; but see Cole et al. 1994 for a smaller value) than typical values observed (e.g. 0.35-0.4 mag in the I band: Strauss & Willick 1995; but see Federspiel et al. 1994 for a somewhat larger value, 0.6-0.7 mag in the I band). The low scatter observed (order of 0.3 mag) in the TF relation seems to lead to a fundamental question (e.g. Eisenstein & Loeb 1996): is the small scatter observed in the TF relation perhaps the direct result of cosmological initial conditions or the result of a feedback process that regulates gas dynamics and star formation in galaxies?

A radically different interpretation of the TF relation is given by the MOND (Modified Newtonian Dynamics; Milgrom 1983) theory. Milgrom (1983) predicts the exact mass-velocity relation as follow: $V_\infty^4 = a_0 GM$, where a_0 is a universal constant, with the dimension of acceleration, and M is the total luminous mass. In this context, the TF relation is no more than the $M_{lum} - L$ relation without any intermediary steps, and the scatter in the TF relation is directly related to the variation in M_{lum}/L among galaxies. However, a discussion of the issue of alternative gravity theories falls outside the scope of this paper (see Sanders (1990, 1997) for a discussion of this subject).

In this paper, we will focus on the slope of the $M_{lum} - V_{cir}$ relation, which can be determined from mass modelling of the observed HI rotation curves. Our main interest here is whether the slope of the $M_{lum} - V_{cir}$ relation is close to the canonical value of 4. Apart from this, we will also concentrate on the connection between dark and luminous matter; often the ratio M_{dark}/M_{lum} is assumed to be a constant. We will also briefly discuss the relation between the local rotation velocity and integrated luminosity inside the corresponding radius (i.e. local TF relation) and some more circumstantial evidence that might explain the small scatter observed in the TF relation. Finally we will try to obtain a better understanding of the physical basis of the TF relation based on the results of this paper and others.

II. SAMPLE

Our sample consists of 26 spiral and irregular galaxies which have both HI rotation curves, measured with either the WSRT or VLA, and surface photometry. Most of them are in common with Broeils (1992b). General properties for these galaxies are listed in Table I. Information in Table I requiring explanation is given below.

(1) Distance (column 3): We have used Cepheid-based distances when available: 0.84 Mpc for M 33 (Madore & Freedman 1991), 2.15 Mpc for NGC 300 (Freedman et al. 1992), 3.52 Mpc for NGC 2403 (Freedman, Lee & Madore 1996), and 1.26 Mpc for

NGC 3109 (Capaccioli, Piotto, & Bresolin 1992). We have adopted the M 101 group distance of 7.55 Mpc for NGC 5585 (Kelson et al. 1994), the M 81 group distance of 3.63 Mpc for IC 2574 (Freedman et al. 1994), and the Canes Venaticorum I (CVn I) group distance of 4 Mpc for DDO 154 (Carignan & Beaulieu 1989). The distances to the other galaxies have been calculated from the systemic velocity, corrected for the Virgo-centric flow following Kraan-Korteweg (1986), and a Hubble constant of $75 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

(2) Radii: h is the disc scale length (column 4) taken from the original papers (see column 13 and notes to Table I). R_{25} (column 5) is the isophotal radius at 25 mag arcsec $^{-2}$ in the B band taken from LEDA (Lyon-Meudon Extragalactic Database) and corrected for Galactic and internal extinction effects following Tully & Fouqué (1985). R_l (column 6) is the radius at the last measured point of the HI rotation curve.

(3) Mass of gas (column 7): M_{gas} is the mass of the gaseous (HI+HeI) component calculated from $M_{gas} = 4/3 \times M_{HI}$.

(4) Luminosities: $L_B^{b,i}$ (column 8) is the total blue luminosity from LEDA in unit of $L_{B\odot}$ using $M_{B\odot} = 5.48$ (Allen 1976). $L_B^{b,i}$ is corrected for Galactic and internal extinction effects following Tully & Fouqué (1985). L_H (column 9) is the near-infrared H luminosity (inside one third of the *blue* diameter) given by Tormen & Burstein (1995), expressed in units of $L_{H\odot}$ using $M_{H\odot} = 3.46$ (from Allen 1976 and Frogel et al. 1978).

(5) Rotation velocities: V_p (column 10) is the real peak velocity (often due to the bulge) of the HI rotation curve. $\dot{V}_m$ (column 11) is the maximum rotation velocity measured beyond two-thirds of the optical radius (R_{25}), to avoid the central peak (which might be due mainly to the bulge) in the rotation curve. V_f (column 12) is the rotation velocity of the HI rotation curve determined from the flat, outer part of the rotation curve. In case the rotation curve continues to be rising, V_f is an average of the outermost reliable points.

III. MASS MODELS WITH DARK HALOES

The amount of dark matter in spiral galaxies can be derived by comparing the predicted rotation curves, calculated from the observed density distributions of luminous matter (i.e. disc, bulge and gas) and assumed values of M/L , with the observed rotation curves; see Begeman (1987) and Broeils (1992b) for details of the modelling procedure. Unfortunately, different combinations of mass models for the luminous and dark matter produce equally good fits to a given observed rotation curve (see van Albada et al. 1985). In fact, we do not know the shape of dark haloes, e.g. density distribution and its extent, at all. We also do not know M/L a priori. How then can one determine properties of the dark and luminous matter?

The maximum-disc hypothesis (van Albada & Sancisi 1986), which chooses the maximum possible M_{disc}/L

Table 2. RESULTS OF THE MASS MODELS WITH ISOTHERMAL HALOES.

Galaxy	V_∞ km/s	r_c kpc	M_{halo} $10^{10} M_\odot$	r_{vir} kpc	M_{vir} $10^{10} M_\odot$	M_{disc} $10^{10} M_\odot$	M_{bul} $10^{10} M_\odot$	M_{lum} $10^{10} M_\odot$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Results for the maximum-disc								
DDO 154	58.7	2.04	0.11	45.13	1.20	0.006		0.041
DDO 168	97.0	2.70	0.70	74.65	5.44	0.021		0.048
DDO 170	74.5	2.19	0.80	57.34	2.47	0.022		0.108
IC 2574	114.0	9.50	1.67	87.27	8.70	0.070		0.189
M 33	160.0	11.50	4.52	122.67	24.16	0.900		1.036
NGC 55	146.0	6.53	1.53	112.24	18.51	0.241		0.326
NGC 247	179.2	11.97	5.69	137.46	33.99	1.070		1.199
NGC 300	153.4	8.98	3.68	117.80	21.40	0.502		0.635
NGC 801	302.0	72.99	45.75	220.80	140.88	24.954	4.15	32.072
NGC 1003	110.6	6.73	1.67	84.88	8.00	0.820		1.661
NGC 1560	184.5	10.68	1.24	141.67	37.21	0.140		0.250
NGC 2403	152.4	6.77	4.15	117.12	21.03	1.481		1.981
NGC 2841	313.2	14.97	15.64	240.72	182.56	15.980	5.27	22.226
NGC 2903	196.2	7.81	5.21	150.90	44.97	5.407		5.747
NGC 2998	242.7	24.20	18.59	185.30	83.26	13.741	0.52	17.629
NGC 3109	116.2	4.35	1.04	89.34	9.33	0.030		0.063
NGC 3198	156.1	7.59	5.36	119.94	22.58	3.110		3.800
NGC 5033	174.2	7.37	21.43	133.94	31.44	10.317		11.227
NGC 5533	255.0	46.53	48.90	190.77	90.87	30.050	5.80	39.123
NGC 5585	95.1	1.97	1.69	73.24	5.14	0.073		0.361
NGC 6503	115.5	1.91	2.00	88.94	9.21	0.623		0.743
NGC 6674	502.7	90.21	38.97	376.46	698.25	20.920	11.69	36.594
NGC 7331	982.0	103.00	20.61	749.16	5502.67	12.950	3.51	18.040
NGC 7793	87.0	2.57	0.69	66.94	3.93	0.610		0.686
UGC 2259	685.1	40.16	1.76	526.03	1904.92	0.399		0.430
UGC 2885	542.5	79.24	159.58	410.20	903.33	37.370	9.00	51.330
Results for the χ^2 -disc								
DDO 168	85.5	1.79	0.69	65.83	3.73	0.002		0.029
IC 2574	115.0	9.50	1.70	88.05	8.93	0.069		0.188
M 33	80.8	0.98	2.74	62.20	3.15	0.483		0.619
NGC 55	103.4	2.06	1.64	79.62	6.61	0.037		0.122
NGC 247	178.4	11.91	5.67	136.87	33.56	1.070		1.199
NGC 1003	107.6	5.61	1.81	82.69	7.40	0.724		1.565
NGC 1560	184.5	10.68	1.24	141.67	37.21	0.140		0.250
NGC 2403	142.8	5.07	4.35	109.86	17.35	1.306		1.806
NGC 2841	291.2	10.47	18.76	223.99	147.08	14.670	5.12	20.771
NGC 2903	196.2	7.81	5.21	150.91	44.97	5.407		5.747
NGC 2998	227.7	20.03	19.42	174.23	69.21	13.044	0.51	16.922
NGC 3109	96.4	2.90	0.96	74.17	5.34	0.010		0.043
NGC 3198	147.6	5.38	5.82	113.51	19.14	2.769		3.459
NGC 5033	174.2	7.37	21.43	133.93	31.44	10.317		11.227
NGC 5533	203.3	40.84	34.69	151.13	45.18	37.228	2.11	42.616
NGC 6503	113.5	1.49	2.08	87.40	8.74	0.552		0.672
NGC 6674	4704.7	964.26	37.05	3492.28	557420.06	20.920	13.46	38.367
NGC 7331	981.6	102.99	20.59	748.85	5495.91	12.957	3.50	18.039
NGC 7793	83.6	1.66	0.79	64.32	3.48	0.533		0.609
UGC 2259	685.1	40.16	1.76	526.03	1904.93	0.399		0.430

Table 2. Continued.

Galaxy	V_∞ km/s	r_c kpc	M_{halo} $10^{10} M_\odot$	r_{vir} kpc	M_{vir} $10^{10} M_\odot$	M_{disc} $10^{10} M_\odot$	M_{bul} $10^{10} M_\odot$	M_{lum} $10^{10} M_\odot$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Results for the Bottema-disc								
M 33	82.2	0.98	2.84	63.32	3.32	0.465		0.601
NGC 55	140.4	6.36	1.46	107.94	16.46	0.256		0.341
NGC 247	93.9	3.67	3.05	72.25	4.93	0.902		1.031
NGC 300	134.2	6.86	3.46	103.11	14.35	0.463		0.596
NGC 801	178.2	9.31	52.02	136.91	33.59	13.622	5.40	21.990
NGC 1003	104.2	4.28	2.00	80.16	6.74	0.593		1.434
NGC 2403	131.6	2.69	4.75	101.34	13.62	0.920		1.420
NGC 2841	266.3	4.04	25.67	205.05	112.83	9.267	5.61	15.860
NGC 2903	175.2	0.38	9.68	134.93	32.15	1.973		2.313
NGC 2998	177.6	1.49	26.05	136.73	33.45	6.582	0.36	10.315
NGC 3198	138.7	1.37	7.32	106.82	15.95	1.480		2.170
NGC 5033	181.0	0.04	30.91	139.41	35.46	3.788		4.698
NGC 5533	190.9	0.22	68.57	147.01	41.58	13.756	4.93	21.961
NGC 6503	110.8	0.61	2.32	85.29	8.12	0.335		0.455
NGC 6674	247.1	23.55	43.46	188.79	88.07	14.341	13.00	31.325
NGC 7331	233.4	9.26	24.68	179.49	75.68	6.819	5.09	13.492
NGC 7793	89.4	1.04	1.04	68.86	4.27	0.387		0.463
UGC 2259	81.2	1.17	1.17	62.51	3.20	0.221		0.252
UGC 2885	281.2	14.88	128.85	216.04	131.96	23.912	9.00	37.872

Notes:

1. DDO 154, DDO 170, NGC 300 & NGC 5585:

Negative values of M_{disc} have been found for the χ^2 -disc model

2. NGC 801 & UGC 2885: χ^2 algorithm failed to converge for the χ^2 -disc model

3. DDO 154, DDO 168, DDO 170, IC 2574, NGC 1560, NGC 3109 & NGC 5585:

Disqualified for the Bottema-disc model (see text)

column 2: Asymptotic velocity of the halo (see text)

column 3: Core radius of the halo (see text)

column 4: Mass of the halo measured inside seven disc scale lengths

column 5: Virial radius of the halo (see text)

column 6: Virial mass of the halo (see text)

column 7: Mass of the stellar disc

column 8: Mass of the bulge

column 9: Total luminous mass, $M_{lum} = M_{disc} + M_{bul} + M_{gas}$

Table 3. RESULTS OF THE MASS MODELS WITH HERNQUIST HALOES.

Galaxy	R_0 kpc	M_0 $10^{10} M_\odot$	M_{halo} $10^{10} M_\odot$	M_{disc} $10^{10} M_\odot$	M_{bul} $10^{10} M_\odot$	M_{lum} $10^{10} M_\odot$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Results for the maximum-disc						
DDO 154	38.48	15.62	0.11	0.006		0.041
DDO 168	382.08	2045.19	0.55	0.021		0.048
DDO 170	26.78	12.54	0.81	0.022		0.108
IC 2574	1167.65	7973.24	1.30	0.030		0.149
M 33	8.46	0.61	0.30	0.900		1.036
NGC 55	486.42	3688.37	1.32	0.200		0.285
NGC 247	365.92	1834.86	4.91	0.800		0.929
NGC 300	5896.72	438713.31	3.34	0.502		0.635
NGC 801	4441.15	107514.43	37.05	24.954	4.15	32.072
NGC 1003	91.91	100.61	1.61	0.820		1.661
NGC 1560	2542.96	85361.09	1.09	0.100		0.210
NGC 2403	332.04	2029.22	4.06	1.481		1.981
NGC 2841	98.39	637.89	19.95	13.820	4.66	19.456
NGC 2903	218.23	1339.98	4.96	5.407		5.747
NGC 2998	624.21	5367.22	17.50	13.741	0.52	17.629
NGC 3109	1504.25	26798.98	0.75	0.030		0.063
NGC 3198	130.33	336.93	5.16	3.110		3.800
NGC 5033	92.38	240.19	22.39	10.317		11.227
NGC 5533	341.71	1138.74	41.98	30.050	5.80	39.123
NGC 5585	26.39	20.30	1.71	0.073		0.361
NGC 6503	19.79	20.35	2.00	0.623		0.743
NGC 6674	25554.40	6867938.00	35.94	20.920	11.00	35.904
NGC 7331	2780.02	134147.97	16.70	12.957	3.50	18.039
NGC 7793	831.82	8138.55	0.58	0.610		0.686
UGC 2259	637.60	4829.01	1.00	0.399		0.430
UGC 2885	209.64	1171.73	107.36	37.370	7.50	49.830
Results for the χ^2 -disc						
M 33	10.77	3.73	1.55	0.649		0.785
NGC 55	877.42	15026.00	1.68	0.134		0.219
NGC 247	170.52	561.74	6.16	0.618		0.747
NGC 300	24.16	21.83	3.55	0.079		0.212
NGC 1003	56.95	51.74	1.86	0.649		1.490
NGC 1560	297.27	1379.82	1.22	0.049		0.159
NGC 2403	28.56	39.31	4.88	0.853		1.353
NGC 2841	98.38	637.86	19.95	13.820	4.66	19.456
NGC 2903	50.64	125.33	5.97	4.766		5.107
NGC 3198	35.82	55.86	6.44	2.288		2.978
NGC 5033	56.22	124.77	21.94	9.599		10.509
NGC 5533	442.89	1611.53	38.69	34.306	2.72	40.295
NGC 6503	15.60	15.73	2.12	0.534		0.654
NGC 6674	6030.48	465079.00	43.06	13.359	15.19	32.532
NGC 7331	4681.23	524667.00	23.23	9.479	4.10	15.158
NGC 7793	56.34	62.82	0.78	0.527		0.603
UGC 2259	6.50	4.10	1.42	0.026		0.057

Table 3. Continued.

Galaxy	R_0	M_0	M_{halo}	M_{disc}	M_{bul}	M_{lum}
	kpc	$10^{10} M_\odot$	$10^{10} M_\odot$	$10^{10} M_\odot$	$10^{10} M_\odot$	$10^{10} M_\odot$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Results for the Bottema-disc						
M 33	11.06	6.07	2.47	0.465		0.601
NGC 55	1031.97	13216.80	1.07	0.256		0.341
NGC 247	748.30	6127.66	4.13	0.902		1.031
NGC 300	5398.41	386629.00	3.51	0.463		0.596
NGC 801	119.42	321.59	54.84	13.622	5.40	21.990
NGC 1003	50.10	44.03	1.94	0.593		1.434
NGC 2403	32.08	45.12	4.80	0.920		1.420
NGC 2841	50.07	287.46	25.34	9.267	5.61	15.857
NGC 2903	8.57	23.97	9.29	1.973		2.313
NGC 2998	35.80	99.63	26.28	6.582	0.36	10.312
NGC 3198	19.47	31.85	7.53	1.480		2.170
NGC 5033	13.10	42.68	24.40	3.788		4.698
NGC 5533	34.38	121.99	60.21	13.756	4.93	21.959
NGC 6503	9.82	10.29	2.36	0.335		0.455
NGC 6674	1052.88	15949.70	44.32	14.341	13.00	31.325
NGC 7331	210.43	1527.43	25.69	6.819	5.09	13.489
NGC 7793	14.26	9.32	1.02	0.387		0.463
UGC 2259	18.90	12.11	1.32	0.221		0.252
UGC 2885	157.40	977.39	131.17	23.912	9.00	37.872

Notes:

1. DDO 154, DDO 168, DDO 170, IC 2574, NGC 801, NGC 3109, NGC 5585 & UGC 2885: Negative values of M_{disc} have been found for the χ^2 -disc model
2. NGC 2998: Negative values of M_{disc} & M_{bul} have been found for the χ^2 -disc model
3. DDO 154, DDO 168, DDO 170, IC 2574, NGC 1560, NGC 3109 & NGC 5585: Disqualified for the Bottema-disc model

column 2: Characteristic scale of the halo (see text)

column 3: Total mass of the halo (see text)

column 4: Mass of the halo measured inside seven disc scale length

column 5: Mass of the stellar disc

column 6: Mass of the bulge

column 7: Total luminous mass, $M_{lum} = M_{disc} + M_{bul} + M_{gas}$

(and M_{bulge}/L) ratio to match the predicted rotation curve to the observed one, is often adopted to estimate the amount of dark matter conservatively. Studies of rotation curves derived from optical spectra generally support the maximum disc method (see Freeman 1993). This should be considered as an indication of the importance of luminous matter inside the optical radius rather than as evidence against the existence of dark matter inside R_{opt} . Athanassoula et al. (1987) have applied spiral instability criteria to constrain the M_{lum}/L ratio and concluded that their maximum solutions are not much different from maximum disc solutions. The lowest possible value that has been found is about 50 % of the maximum disc value.

Bottema (1995) has studied stellar velocity dispersions of spiral galaxies and found a correlation between the disc velocity dispersion and maximum rotation velocity: the maximum circular velocity supplied by a disc is on average 63 % of the observed maximum rotation, depending on the assumed values of the intrinsic thickness of spiral disc, roughly independent of the size of the galaxy. For some of our models, we have scaled the maximum velocity of the disc component to be equal to 63 % of the observed maximum rotation velocity (avoiding the bulge contribution; see column 11 of Table I).

In addition to the maximum disc model and Bottema's result, we will use the χ^2 minimization solution (completely unconstrained fit of all free parameters (see below); hereafter the χ^2 -disc) to scale luminous matter.

Traditionally, a (modified) isothermal profile, implied by flat rotation curves, is used for the density profile of the dark halo:

$$\rho(r) = \rho_0 \left[1 + \left(\frac{r}{r_c} \right)^2 \right]^{-1}, \quad (1)$$

where ρ_0 is the central volume density of the halo and r_c is the core radius of the halo. The corresponding circular velocity is:

$$V_{halo}^2(r) = 4\pi G \rho_0 r_c^2 \left[1 - \frac{r_c}{r} \arctan \left(\frac{r}{r_c} \right) \right], \quad (2)$$

with the asymptotic circular velocity of $V_\infty = (4\pi G \rho_0 r_c^2)^{1/2}$.

For the isothermal halo model the mass does not converge. Therefore, integrated halo masses inside a specific radius, i.e. seven times the disc scalelength in our case, have been derived. Another selection of the halo mass, the so-called virial mass, has been derived from the following (Navarro et al. 1996):

$$M_{vir} = 200 \rho_{crit} (4\pi/3) r_{vir}^3, \quad (3)$$

where ρ_{crit} is the critical density ($= 3H_0^2/8\pi G$) of the Universe and the virial radius r_{vir} is the radius of the sphere (halo) encompassing a mean overdensity of 200. This definition has often been used to represent the

total halo mass in computer simulations (e.g. Navarro et al. 1996; Gelb & Bertschinger 1994).

On the other hand, recent high resolution computer simulations (e.g. Lemson 1995; Dubinski & Carlberg 1991) have shown that the density profiles of the simulated dark haloes can be fitted well by the Hernquist profile (Hernquist 1990) defined by:

$$\rho(r) = \frac{M_0 R_0}{2\pi r (r + R_0)^3}, \quad (4)$$

where M_0 is the total mass and R_0 is the characteristic scale of the halo. Furthermore, Lemson (1995) has pointed out that the basic model parameters, i.e. mass and characteristic scale, are strongly correlated ($M_0 \propto R_0^2$), which consequently implies that rotation curves must have a universal shape (see also Warren et al. 1992; Navarro et al. 1996).

Below we will use both forms of dark halo density profiles.

IV. RESULTS

We have performed six different methods to derive the parameters of the dark and luminous matter: the maximum disc, the χ^2 -disc, and the lowered maximum-disc model following Bottema (hereafter Bottema-disc), each with the isothermal and Hernquist halo. A non-linear least-squares fitting program has been used for the mass modelling.

We first consider the mass models with isothermal haloes. In general, all three disc models could reproduce the observed rotation curves reasonably well except for some cases (see below). The results of the maximum-disc, χ^2 -disc and the Bottema-disc fits with the isothermal halo are summarized in Table I.

For the maximum-disc fits, four galaxies (NGC 6674, NGC 7331, UGC 2259, and UGC 2885) show unrealistically large values (500 ~ 1000 km/s) of halo asymptotic velocities (and core radii to some degree; see Table I) and in turn these galaxies are often located at the most deviating positions when the halo parameters are plotted with other properties. The results of the maximum-disc fits are presented in Figure 13.

In case of the χ^2 -disc model, the χ^2 minimization process often failed to present a solution because of, from the numerical point of view, the somewhat complicated nature of the three or four dimensional χ^2 parameter space, especially when the bulge component is entered in the fitting procedure. The χ^2 minimization algorithm has failed to converge for NGC 801 and UGC 2885. For DDO 154, DDO 170, NGC 300, and NGC 5585, negative values for the disc mass were found. These galaxies have been excluded from further analysis. Note that in many cases the results of the χ^2 -disc model are close to those of the maximum-disc (see Figure 1). The results of the χ^2 -disc fits are presented in Figure 14.

Fig. 1.— Histograms of the ratio of luminous mass in the χ^2 -disc and the maximum-disc model. The histograms with the solid and dotted lines represent the results with isothermal and Hernquist halo, respectively.

The mass modelling using the Bottema-disc has failed to produce meaningful results for seven less massive galaxies (see Table I), with maximum rotation velocities smaller than say 100 km/s, as for these systems 63 % of the maximum rotation velocity already exceeds the maximum amplitude allowed by the fit. The minimum rotation velocity yielding an acceptable fit is of order of 120 km/s. The results of the Bottema-disc fits are presented in Figure 15. Note that recently Bottema (2002) generalized his relation, i.e. the disc maximum rotation velocity is now a function of surface brightness and colour. This new relation can be applied to smaller systems, too.

The mass models with the Hernquist halo also could reproduce the observed rotation curves reasonably well except for in some cases (see below). The results of the maximum-disc, χ^2 -disc and the Bottema-disc fits with the Hernquist halo are summarized in Table I.

For the maximum-disc fits using Hernquist haloes, we have used essentially the same values of M_{disc} and M_{bulge} as used for the isothermal halo model fits. For some cases like NGC 6674, NGC 7331 and UGC 2885, M_{disc} and M_{bulge} have been slightly changed to improve the fit. Note that for some less massive systems such as NGC 3109 and DDO 168, the resulting fits are not so successful. The results of the maximum-disc fits are presented in Figure 16.

For the χ^2 -disc model, negative values of disc masses have been found for eight galaxies: DDO 154, DDO 168, DDO 170, IC 2574, NGC 801, NGC 3109, NGC 5585 and UGC 2885. For NGC 2998, both disc and bulge masses were negative. These galaxies will not be considered in the further analysis of the χ^2 -disc fits. The results of the χ^2 -disc fits are presented in Figure 17.

The results of the Bottema-disc fits for the sample galaxies, except the seven disqualified ones (see above), are presented in Figure 15.

V. DISCUSSION OF THE RESULTS AND THE LUMINOUS MASS-VELOCITY RELATION

(a) Isothermal Haloes

One of the strong points in preference for an isothermal halo profile is that it leads to an asymptotically flat rotation curve. This in turn implies fine tuning of dark and luminous matter (and near equality of the observed asymptotic rotation velocity of a galaxy and of its surrounding halo).

Fig. 2.— (a) Comparison between the asymptotic halo velocities and the observed rotation velocities of the flat parts for the maximum-disc model with an isothermal halo. (b) Same as (a), but for the χ^2 -disc model. NGC 6674 ($V_{halo} = 4705$ km/s) is not shown. (c) Same as (a), but for the Bottema-disc model. The solid lines indicate equal results.

In Figure 2, the asymptotic halo velocities are compared to the observed rotation velocities of the flat parts. The asymptotic halo velocities can be found in Table I and the rotation velocities of the flat parts in Table I. The solid lines indicate equal results. It is clearly seen that, in the maximum-disc and χ^2 -disc model fits (Figure 2a and 2b), the rotation velocities of the dark matter haloes have not yet reached their

Fig. 3.— (a) Comparison between the halo core radii and the disc scale lengths for the maximum-disc model with an isothermal halo. (b) Same as (a) but for the χ^2 -disc model. (c) Same as (a) but for the Bottema-disc model.

asymptotic values in many cases and are still rising at the last measured points of the observed rotation curves. As a matter of fact, most of the less massive galaxies do not show any sign of a turning over of the halo rotation curve at all, not even at the last measured points. Even for galaxies with flat or falling rotation curves, the rotation curves become flat not by the asymptotically flat dark halo rotation velocities but by the inter-coupling between rising dark halo and falling luminous matter rotation velocities.

On the other hand, the resulting halo velocities of the Bottema-disc model show a strong one-to-one correspondence to the observed rotation velocities of the flat parts, as seen in Figure 2c. In this model the contribution of the luminous matter to the rotation curve is reduced by force and hence it is not surprising that the contribution of dark matter is important, even in the inner part.

In Figure 3, the halo core radii are plotted against the disc scale lengths. It is interesting to note that the relation between the halo core radius and disc scale length, another coupling indicator, shows the strongest

Fig. 4.— The Tully-Fisher relations (a) in the B band and, (b) in the H band. The rotation velocities of the flat parts are used. The solid lines represent the results of the least-squares fits.

trend for the maximum-disc and the weakest for the Bottema-disc model, while the opposite is true for the relations between rotation velocity parameters shown in Figure 2. Note also that the χ^2 -disc model lies in-between in both cases.

Let us now consider the TF relation. In Figure 4, B and H band luminosities are plotted against the observed rotation velocities of the flat parts. The solid lines represent the results of the least-squares fits (bisector of the two ordinary least-squares fits, Y on X and X on Y). The B band TF relation is found to have a slope of $3.63 (\pm 0.21)$ with a 1σ scatter of 0.27 dex, or 0.67 mag. Use of the maximum rotation velocities instead of the rotation velocities of the flat parts gives similar results (the slope of the TF relation becomes slightly shallower from 3.63 to 3.52). When only the galaxies with maximum velocities greater than 100 km/s are taken into account for the calculation, the slope of the TF relation is further reduced to 2.97, indicating non-linearity in the TF relation. The slope found here is somewhat larger than the conventional one (about 2-2.4) in the B band TF relation. However, Rubin et al. (1985) have also found the larger slope around 4 in the blue band when they used maximum rotation velocities from the optical rotation curves instead of HI linewidths.

For the H band TF relation, as seen in Figure 4b, we find a slope of $4.28 (\pm 0.42)$ and a 1σ scatter of 0.20 dex, or 0.50 mag. The scatter is further reduced to 0.12 dex, or 0.30 mag, when the two most deviating (and also the most luminous) points are not included in

Fig. 5.— (a) The luminous mass-rotation velocity relation for the maximum-disc model with an isothermal halo. (b) Same as (a), but for the χ^2 -disc model. (c) Same as (a), but for the Bottema-disc model. The solid lines represent the results of the least-squares fits. The rotation velocities of the flat parts are used.

the calculation. Use of the maximum velocities instead of the velocities of the flat parts resulted in a smaller TF relation slope of 4.06. These slopes of 4.28 and 4.06 are close to the previously determined values (e.g. Peletier & Willner 1993; Aaronson et al. 1979). As mentioned earlier, the reality of this slope has been questioned because of the hybrid definition of the H band luminosities. The newly determined TF relation slope of 2.40 from the *total* H band luminosities and more accurate HI linewidths by Bernstein et al. (1994) makes the situation more complicated. (But see Rhee (1996) for the total K-band TF relation with the slope 4.) Note that the H band luminosities used in this paper (taken from Tormen & Burstein 1995) are based on the hybrid definition (H light inside one third of the blue diameter).

To some extent, the slope of the TF relation depends on technical matters: related to the definition of rotation velocity and luminosity. The hybrid definition of the H band luminosity is a good example. Traditionally,

HI linewidths are used for the TF relation. However, different authors use different definitions and correction methods for the linewidths (see e.g. Bottinelli et al. 1990; Huchtmeier & Richter 1989). In recent years, rotation velocities determined from the optical rotation curves are often used for the TF relation (Mathewson et al. 1992; Schommer et al. 1993). Also CO linewidths are now being used as a substitute for the HI linewidth for galaxies in clusters (see Sofue 1994 for a general discussion). Rotation velocities from one-dimensional curves are used as well (Rhee 1996). In this paper, we use the rotation velocities derived from HI rotation curves from full HI velocity field. Use of different regression methods could result in somewhat different values for the slope of the TF relation. Furthermore, the selection of sample galaxies will affect the results as well. Therefore, these factors should be taken into account when comparing TF relation slopes from different studies.

Above all, true physical variations can affect the TF relation slope. Note that the slope of the TF relation is known to depend on wavelength (e.g. Pierce & Tully 1992). Therefore, there are many reasons to investigate the TF relation slope in a more fundamental way.

We now move to the main issue of this paper. In Figure 5, the total luminous mass (disc, bulge and gas components) is plotted against the velocity of the flat part of the rotation curve, separately for the three disc models. The solid lines represent the results of the least-squares fits. It is evident that there are well defined relations for all three models. For the maximum-disc model, we find a slope of $4.12 (\pm 0.16)$ and a 1σ scatter of 0.17 dex, for the χ^2 -disc model, we find a slope of $4.43 (\pm 0.26)$, with a scatter of 0.22 dex, and for the Bottema-disc model, a slope of $3.97 (\pm 0.25)$, with a scatter of 0.21 dex. Use of the maximum velocities instead of the velocities of the flat parts results in the slightly smaller slopes of 3.99, 4.12 and 3.83 for the maximum-disc, χ^2 -disc and Bottema-disc, respectively. It is remarkable that all three disc models produce a very similar slope, close to 4.

In our previous study (Rhee 1996), we have found a slope of around 3.7 for the $M_{lum} - V_{cir}$ relation, based on an analysis of optical rotation curves and I band photometry with one-component (only disc) maximum-disc models, compared to 4.0 found here for the maximum-disc model. The slope found previously could have been underestimated because the contribution from the bulge has not been explicitly taken into account, and because the maximum rotation velocities for the less massive galaxies may have been systematically underestimated due to the combination of the limited range of the distribution of HII regions (in general the optical rotation curve stops well inside the optical radius) and the extended and rising nature of the (HI) rotation curve of those galaxies. Therefore, the difference between our present result and the previous one is in the sense expected.

As stated before (see Introduction), the TF relation is a combination of two independent relations, i.e. a $M_{lum} - V_{cir}$ relation and a $L - M_{lum}$ relation. This can be expressed in a quantitative way. From the equation of centrifugal equilibrium, one obtains:

$$V_0^2 = \gamma \frac{GM}{R_c}, \quad (5)$$

where V_0 is the representative velocity (e.g. maximum rotation velocity), M is the total mass of a galaxy, R_c is the characteristic radius, G is the gravitational constant, and γ is a structural parameter depending on the shape of the mass distribution. The total mass M can be expressed as $M = M_{dark} + M_{lum}$. With introduction of the dark matter fraction parameter $\alpha (= M_{dark}/M_{lum})$ and the surface density parameter of the luminous matter $\mu_0 (= M_{lum}/R_c^2)$, equation 5 can be written in the form:

$$M_{lum} = V_0^4 [\gamma^2 (1 + \alpha)^2 G^2 \mu_0]^{-1}. \quad (6)$$

with M_{lum}/L (the second relation), equation 6 can be rewritten in the form of the TF relation as follows:

$$L = V_0^4 \left[\gamma^2 (1 + \alpha)^2 G^2 \mu_0 \left(\frac{M_{lum}}{L} \right) \right]^{-1}, \quad (7)$$

where L is the total luminosity in some wave band (see e.g. Djorgovski et al. 1988).

As is clearly expressed by equation 6, both the dark matter fraction α and the surface density parameter of luminous matter, μ_0 , must be constants or their products should be very close to a constant, in order to keep the slope of the $M_{lum} - V_f$ relation close to 4, with reasonably small scatter.

To investigate this, the mass ratios of the dark and luminous components are plotted against luminous mass in Figure 6. The filled circles represent the virial masses of the dark haloes (equation 3) and the open circles the masses inside seven disc scale lengths. Except for a few outliers, there is a clear general trend that less massive (in terms of the luminous mass) galaxies are characterised by larger mass ratios of the dark and luminous components. Similar results have already been found by various authors (see e.g. Persic & Salucci 1990; Broeils 1992b). For the Bottema-disc model, this effect is less dramatic but note that in that case the seven least massive galaxies have been dismissed *a priori*.

These observational results and the $M_{lum} - V_{cir}$ relation with a slope of about 4 naturally raise the question whether there exists an opposite trend between the mass surface density of the luminous matter and $\alpha (= M_{dark}/M_{lum})$ when plotted against luminous mass. Equation 6 suggests that this might help to reduce the scatter in the TF relation. One practical difficulty in connecting the observations to equation 6 is how to derive the characteristic radius R_c needed

Fig. 6.— (a) Comparison between the mass ratios of the dark and luminous components and luminous mass for the maximum-disc model with an isothermal halo. (b) Same as (a), but for the χ^2 -disc model. (c) Same as (a), but for the Bottema-disc model. The filled circles represent the virial masses of the dark haloes and the open circles the masses inside seven disc scale lengths.

in the definition of the surface density of the luminous matter. Ideally some sort of a mass radius is preferred for R_c . Since such an observational parameter is difficult to define, we have used the disc scale length h to define the mass surface density parameter of the luminous matter, assuming that h is a fair representative of R_c . In Figure 7, the mass surface density parameter of the luminous matter ($= M_{lum}/h^2$) is plotted against luminous mass. A trend opposite to that in Figure 6 is clearly present: the luminous mass surface densities of the less massive galaxies are systematically smaller than those of the more massive galaxies. The Bottema-disc model again shows a very weak trend, if any.

These results show that, apparently, the $M_{lum} - V_{cir}$ relation with a slope of about 4 is maintained by an anti-correlation between the mass surface density of luminous matter and the mass ratio of the dark and luminous components.

It is unfortunate that mass modelling has often failed

Fig. 7.— (a) Comparison between the luminous mass density parameters in units of $M_\odot/\text{pc}^2$ and luminous mass for the maximum-disc model with an isothermal halo. (b) Same as (a), but for the χ^2 -disc model. (c) Same as (a), but for the Bottema-disc model.

to produce solutions for many less massive galaxies, which would otherwise have been added to our analysis. These galaxies might shed light on the linearity of the $\log M_{lum} - \log V_{cir}$ relations. Note that the effective one-parameter model fits (usually M_{lum}/L), scaled often by the mean relations of the more massive galaxies, have not been particularly successful (Begeman, Broeils & Sanders 1991), suggesting somewhat different galaxy properties for those galaxies.

Before moving to the Hernquist halo models, it is worth commenting on the possibility of coupling effects among model parameters due to the mass modelling procedures. Clearly, some model parameters are not independently determined; for instance, there are built-in inter-relationships among halo parameters: the asymptotic velocity, central volume density and core radius of the halo are inter-connected through the definition of the halo density profile (see equations 1–3). Further, the disc (bulge) mass is scaled with the second power of the disc (bulge) rotation velocity ($\Delta M_{lum} \sim \Delta V_{lum}^2$). Thus, in the $M_{lum} - V_{lum}$ relation, the errors due to

Fig. 8.— The luminous mass–rotation velocity relation for the χ^2 -disc model with a Hernquist halo. The solid line represents the result of the least-squares fit. The rotation velocities of the flat parts are used.

incorrect scaling of the luminous matter do not entirely propagate to the Y-axis (M_{lum} axis) but depend on the difference between the slope of the true relation and the scaling slope, which is 2 in this case. These coupling effects among the model parameters generally lead to a small apparent scatter observed in a relation, while the true scatter in that relation might be somewhat larger. These effects could also affect the slope to some degree, depending on the size of the errors.

(b) Hernquist Haloes

Let us now move to the mass models with a Hernquist halo. Since the $M_{lum} - V_{cir}$ relations for the maximum-disc and Bottema-disc models with a Hernquist halo are basically the same as those with an isothermal halo, we only consider the $M_{lum} - V_{cir}$ relation for the χ^2 -disc model. In Figure 8, the total luminous mass is plotted against the rotation velocity of the flat part. The solid line represents the result of the least-squares fit. The $M_{lum} - V_f$ relation is found to have a slope of $4.32 (\pm 0.30)$ and a scatter about the relation is 0.25 dex. Use of the maximum rotation velocities instead of the rotation velocities of the flat parts gives a somewhat smaller slope: $4.07 (\pm 0.27)$. These values are very close to the values derived from the mass models with an isothermal halo.

As a next step, the surface density of luminous matter and the ratio of dark and luminous mass are plotted against luminous mass in Figure 9. The filled circles represent the total mass of the Hernquist halo and the open circles the integrated halo mass inside seven disc scale lengths. The results are similar to those for the isothermal halo fits.

A more careful look suggests that the ratio of dark and luminous mass beyond $10^{10} M_\odot$ is fairly constant around 27.4 for the χ^2 -disc model, except for two extreme outliers, i.e. NGC 6674 and NGC 7331. Note that NGC 6674 and NGC 7331 also happen to be the most extreme outliers in other relations involving parameters of the dark halo (see Figure 2 and also Tables I

Fig. 9.— (a) Comparison between the luminous mass density parameters in units of $M_{\odot}/\text{pc}^2$ and luminous mass for the χ^2 -disc model with a Hernquist halo. (b) Comparison between the mass ratios of the dark and luminous components and luminous mass for the χ^2 -disc model with a Hernquist halo.

and I).

The fact that the ratio of dark and luminous matter is fairly constant has already been noted by Sanders & Begeman (1994). They successfully fitted HI rotation curves with one parameter Hernquist models, borrowing some of the fundamental aspects of the MOND. Sanders & Begeman (1994) have also reproduced the TF relation in the form $V^4 \propto M_{\text{lum}}$ by showing the constancy of $M_{\text{dark}}/M_{\text{lum}}$ and the halo mass surface density parameter, $M_0/(2\pi R_0^2)$. Our results in this paper support their conclusions but only for the massive galaxies. (They did note however that one parameter Hernquist fits failed for the dwarf galaxies.) The χ^2 -disc model fits with a Hernquist halo in this paper also failed to produce results for most of the less massive galaxies. It is interesting to note that Moore (1994) argues that the standard CDM haloes, scaled with the results from the recent simulations, are too concentrated to be consistent with the observed halo parameters for dwarf galaxies.

The Hernquist halo can potentially provide an important constraint for mass models. Recent high resolution numerical simulations of cosmological models dominated by collisionless dark matter (e.g. Lemson 1995; Dubinski & Carlberg 1991) have shown that the density profiles of the simulated dark haloes can be fitted well with a Hernquist profile. Lemson (1995) has further pointed out that the basic model parameters, i.e. mass and characteristic scale of the halo, are

Fig. 10.— (a) Comparison between total halo mass and halo characteristic radius for the maximum-disc model with a Hernquist halo. (b) Same as (a), but for the χ^2 -disc model. (c) Same as (a), but for the Botttema-disc model. The filled circles represent the galaxies with luminous mass greater than $10^{10} M_{\odot}$ and the open circles the galaxies with luminous mass smaller than this limit. The solid lines represent the results of the least-squares fits and the dashed lines indicate the locus of constant halo mass surface density ($M_0 \propto R_0^2$).

strongly correlated ($M_0 \propto R_0^2$), as previously suggested by Sanders & Begeman (1994).

In order to investigate the constancy of M_0/R_0^2 , total halo masses M_0 are plotted against the characteristic radii of the haloes R_0 in Figure 10. It is evident that there are well defined relations for all cases. The filled circles represent the galaxies with luminous mass greater than $10^{10} M_{\odot}$ and the open circles the galaxies with luminous mass smaller than this limit. The solid lines represent least-squares fits. For the maximum-disc model, we find a slope of $1.89 (\pm 0.06)$ and a 1σ scatter of 0.23 dex. The $M_0 - R_0$ relations are found to have a slope of $1.79 (\pm 0.06)$ with a scatter of 0.22 dex for the χ^2 -disc model, and a slope of $1.62 (\pm 0.07)$ with a scatter of 0.27 dex for the Botttema-disc. The dashed lines indicate the locus of constant halo mass

Fig. 11.— The mass differences between the mass models vs the differences in radius. The open circles represent the differences in M_0 and R_0 between the maximum-disc and the χ^2 -disc model. The crosses represent the differences between the χ^2 -disc and the Bottema-disc model. The open triangles represent the differences between the Bottema-disc and the maximum-disc model. The solid line represents least-squares fit and the dashed line indicates the locus of constant halo mass surface density ($M_0 \propto R_0^2$).

surface density ($M_0 \propto R_0^2$). Both the maximum-disc and the χ^2 -disc models produce well defined $M_0 - R_0$ relations with a slope of almost 2. The Bottema-disc model, however, appears to be inconsistent with a slope of 2.

These results argue whether M_0 and R_0 can both be determined from mass model fits. Clearly there are coupling effects due to the mass modelling procedures in these diagrams. First of all, there is no apparent mass segregation among galaxies in Figure 10: dwarf and giant galaxies are well mixed. Secondly, some galaxies with unrealistically large values of M_0 and R_0 show no sign of a deviation from the mean relations. In Figure 11, mass differences between the mass models are plotted against the differences in radius. In this way, one can get some idea of the coupling effects between M_0 and R_0 due to the modelling procedures. The open circles represent the difference in dark halo mass between the maximum-disc and χ^2 -disc models. The crosses and triangles represent the difference in dark halo mass between the χ^2 -disc and the Bottema-disc models, and between the Bottema-disc and the maximum-disc models, respectively. The dashed line indicates the locus of constant halo mass surface density ($M_0 \propto R_0^2$). The solid line represents the results of the least-squares fit. The slope of the relation is found to be $1.62 (\pm 0.05)$. It is, therefore, evident that the relations seen in Figure 10 are to a large extent due to coupling effects. To make this point clear we re-write

Fig. 12.— (a) Comparison between the maximum halo velocities and the observed rotation velocities of the flat parts for the maximum-disc model with a Hernquist halo. (b) Same as (a) but for the χ^2 -disc model. (c) Same as (a) but for the Bottema-disc model. The solid lines indicate equal results.

equation 4 as follows:

$$V_{cir}^2 = \frac{GM(r)}{r} = \frac{GM_0 r}{(r + R_0)^2} \quad (8)$$

The maximum halo velocity $V_{halo} = (GM_0/(4R_0))^{1/2}$ is reached at $r = R_0$. It is clear from equation 8 that for a small radius ($r \ll R_0$), where the actual fitting is performed, V_{cir}^2 is determined by the ratio M_0/R_0^2 . Therefore, it might be difficult and less meaningful to measure M_0 and R_0 separately. On the other hand, M_0/R_0^2 itself can be measured quite accurately, as is demonstrated empirically in Figure 11.

Another interesting aspect of Hernquist models worth investigating is whether the predicted rotation curve of the Hernquist halo is more or less flat outside the last measured point. In Figure 12, maximum halo velocities, calculated from $V_{halo} = (GM_0/(4R_0))^{1/2}$, are plotted against the observed rotation velocities of the flat parts. The solid lines indicate equal results. Similar results as for the isothermal halo model are found.

For the maximum-disc, as seen in Figure 12a, the rotation velocities of the dark matter haloes are still rising at the last measured points of the observed rotation curves and have not yet reached their maximum values by far in many cases. Note also that R_0 , where the maximum value of the halo rotation velocity is reached, is about an order larger (after excluding extremes) than the radii of the last measured points of the HI rotation curves. As for the isothermal halo case, the observed rotation curves for the Hernquist halo models become flat not by the flat dark halo rotation velocities but by the inter-coupling between rising dark halo and falling luminous matter circular velocities. Therefore, a conspiracy between dark and luminous matter to produce a flat rotation curve (e.g. van Albada & Sancisi 1986) also occurs for the mass models with a Hernquist halo. As expected, the halo velocities of the Bottema-disc model show a strong one-to-one correspondence to the observed rotation velocities of the flat parts, as seen in Figure 12c, except for three extreme outliers. The χ^2 -disc model lies in-between these two.

If dark haloes have indeed density distributions close to a Hernquist profile, with a tight $M_0 - R_0$ relation with a slope of 2, as suggested by Sanders & Begeman (1994) and Lemson (1995), the Bottema-disc model with a slope of the $M_0 - R_0$ relation of about 1.62 (for massive galaxies) is not consistent with their results. On the other hand, the maximum-disc model with a slope of 1.89 and the χ^2 -disc model with a slope of 1.79 for the $M_0 - R_0$ relation are consistent with their results.

VI. THE TULLY-FISHER RELATION

On the basis of the results presented in the previous sections, we conclude that the physical basis of the Tully-Fisher relation lies in the $M_{lum} - V_{cir}$ relation with a slope of around 4. (This result does not depend on the assumed distribution of dark matter.) We have previously reached the same conclusion from an analysis of optical rotation curves with the maximum-disc model (Rhee 1996).

The $M_{lum} - V_{cir}$ relation possibly reflects the initial conditions of the galaxy formation epoch. Variations in the collapse factor would result in variations in mass ratio of the dark and luminous components and in turn lead to an increase of the scatter in the $M_{lum} - V_{cir}$ relation. A spread in the potential depths of galaxy haloes due to differences in their formation time would increase the scatter in the $M_{halo} - V_{halo}$ relation (Eisenstein & Loeb 1996) and consequently the scatter in the $M_{lum} - V_{cir}$ relation. Therefore, the intrinsic scatter in the $M_{lum} - V_{cir}$ relation, if properly measured, will tell us about the initial conditions of galaxy formation. However, estimating the intrinsic scatter in the $M_{lum} - V_{cir}$ relation is extremely difficult, as it requires an accurate determination of the scatter due to observational errors. The scatter observed in the $M_{lum} - V_{cir}$ relation is of the same order as that

in the H band TF relation (see previous section). We have previously found a factor of 2 smaller scatter in the $M_{lum} - V_{cir}$ relation than in the I band TF relation for 775 spiral galaxies (Rhee 1996). It seems that the larger (relative) scatter in the $M_{lum} - V_{cir}$ relations found in this paper is related to the larger random errors in the adopted distances for the sample galaxies in this paper. Note that most of the sample galaxies used in this paper are much nearer than those in Rhee (1996).

Even though there have been no other systematic studies of the $M_{lum} - V_{cir}$ relation than our own work (Rhee 1996), some related studies can be found in the literature. Salucci, Frenk & Persic (1993) have found that the contribution to the circular velocity from the luminous disc alone correlates better with luminosity than does the total velocity, which contains an additional contribution from the dark halo. Chiba & Yoshii (1995) have reported that the disc scale length is tightly correlated with a combination of the disc central surface brightness and the rotation velocity at a characteristic radius of 2.15 scale lengths from the center. In both studies new distance indicators have been proposed based on these results. Basically, both studies proposed the use of the disc alone rotation velocity (the rotation velocity at 2.15 scale lengths, roughly corresponds to the disc-alone rotation velocity for the (maximum) Freeman exponential disc).

While the slope of the $M_{lum} - V_{cir}$ relation is relatively well defined as shown in this paper, the determination of the slope of the TF relation needs one more step. The mean trend of the $L - M_{lum}$ relation determines the overall shape (slope) of the TF relation (see equations 6 and 7). A recent study of the optical and near-infrared colours of face-on spiral galaxies by de Jong (1995) indicates substantial M_{lum}/L differences both within galaxies and among galaxies. De Jong (1995) has also pointed out that late-type galaxies ($T > 6$) have lower metallicities and are younger on average than earlier types and have therefore an entirely different M_{lum}/L in most passbands. Therefore, the key problem is how to measure the $L - M_{lum}$ relation.

Finally, we discuss the scatter in the TF relation. As discussed in section 1, the slope of the TF relation has been predicted fairly well with many leading theories like the standard CDM model, but the predicted scatter is rather larger than typical values observed. How small is it? Strauss & Willick (1995) have estimated 0.35-0.4 mag in the CCD R and I band TF relation's from an analysis of over 3000 galaxies collected from various sources. However, Federspiel et al. (1994) have found 0.6-0.7 mag in the I band TF relation after careful consideration of the sample selection effects. On the other hand, a scatter of about 0.1 mag is reported (Bernstein et al. 1994) for the galaxies in the Coma cluster. Therefore, the debate on the scatter in TF relation is continuing. It should also be noted that estimating the *intrinsic scatter* is not always straightforward, because determinations of the scatter due to the purely observa-

Fig. 13.— Maximum-disc models with isothermal haloes. The dotted and dashed lines represent the rotation curves of the gas and stellar components. The dot-dashed lines show the rotation curves of the dark matter halo. The solid lines indicate the contribution of all the components.

Fig. 14.— χ^2 -disc models with isothermal haloes. The dotted and dashed lines represent the rotation curves of the gas and stellar components. The dot-dashed lines show the rotation curves of the dark matter halo. The solid lines indicate the contribution of all the components.

Fig. 15.— Bottenga-disc models with isothermal haloes. The dotted and dashed lines represent the rotation curves of the gas and stellar components. The dot-dashed lines show the rotation curves of the dark matter halo. The solid lines indicate the contribution of all the components.

Fig. 16.— Maximum-disc models with Hernquist haloes. The dotted and dashed lines represent the rotation curves of the gas and stellar components. The dot-dashed lines show the rotation curves of the dark matter halo. The solid lines indicate the contribution of all the components.

Fig. 17.— χ^2 -disc models with Hernquist haloes. The dotted and dashed lines represent the rotation curves of the gas and stellar components. The dot-dashed lines show the rotation curves of the dark matter halo. The solid lines indicate the contribution of all the components.

Fig. 18.— Bottema-disc models with Hernquist haloes. The dotted and dashed lines represent the rotation curves of the gas and stellar components. The dot-dashed lines show the rotation curves of the dark matter halo. The solid lines indicate the contribution of all the components.

tional (measurements and correction) errors are often subjective. However, note that the scatter in the TF relation may not be as large as one magnitude, at least for normal galaxies in *clean* cluster environments (i.e. no significant substructures).

It should also be kept in mind that, to some extent, the scatter in the TF relation is affected by correlation of errors. For instance, the errors in the circular velocity and luminosity are not independent: they are correlated through the inclination correction for the circular velocity and the extinction correction (again via inclination) for the luminosity. Ignoring this effect would result in an underestimation of the intrinsic scatter.

There may also be a physical basis for a small scatter in the TF relation (see also the discussion by Djorgovski 1992). The existence of a local Tully-Fisher relation, i.e. a relation between the local rotation velocity and the integrated luminosity inside the corresponding radius (Madore & Woods 1987), supports the idea of halo-regulated star formation suggested by Wyse & Silk (1989). In their model, Wyse & Silk (1989) introduce the local angular frequency ($\Omega = V(r)/r$) as an additional parameter to the gas density to describe the star formation rate. In this way, the local star formation rate is related to the potential of the surrounding halo via the rotation curve and this may lead to a relation between the local circular velocity and local luminosity. This is exactly what Madore & Woods (1987) have found. Petrou (1981) already found a similar relation, but of a different form: the local mass-to-light ratio in disc galaxies is proportional to the local surface brightness. The relation between the shape of the rotation curve and the slope of the metallicity gradient found by Zaritsky (1992) in some spiral galaxies, also supports Wyse & Silk's (1989) arguments. Kennicutt (1989) also found a dependence of the star formation threshold on the local angular frequency.

We argue that these physical relationships may offer an explanation for the small scatter observed in the TF relation in the sense that there may exist a physical connection between local and global properties. As a result, one may expect a well defined $M_{lum} - L$ relation, which we believe is one of the key relations determining the size of the scatter in the TF relation. Furthermore, they may have a bearing on the apparent connection between young and old stellar populations in spiral galaxies. In this way, a TF relation for a wide range of the wavelengths, as observed, may result. Therefore, it could well be possible that the small scatter observed in the TF relation is the results of a feedback process that regulates gas dynamics and star formation in galaxies, rather than the direct result of the cosmological initial conditions. Recent observational results (Forbes et al. 1996) from the HST and Keck telescope indicate that the TF relation for moderate redshift galaxies show similar scaling relations as for local galaxies, with a modest brightening of ~ 1 magnitude, giving more weight to our explanation of a feedback-based small scatter in the TF relation.

VII. CONCLUDING REMARKS

It is interesting to note that the mass models for both the maximum-disc and the Bottema-disc (and also the χ^2 -disc model), with both isothermal and Hernquist halo, produce rather similar relations (of course, with different zero points), especially for the $M_{lum} - V_{cir}$ relation. The Hernquist halo is attractive because of its mathematical simplicity (e.g. finite mass) and because it finds a physical basis in cosmological simulations. However, as a drawback, especially for the maximum-disc model, in some cases the predicted rotation curves rise outside the last measured points with physically implausible values of maximum halo velocities. The same is true for the isothermal halo, which also shows many very high values of the asymptotic halo velocities.

We conclude that the *conspiracy* between luminous and dark matter exists in a *local* sense (producing a flat or smooth rotation curve) and in a *global* sense (affecting M_{dark}/M_{lum}). Apparently, these conspiracies maintain a $M_{lum} - V_{cir}$ relation with slope 4. We, therefore, conclude that the physical basis of the TF relation lies in the $M_{lum} - V_{cir}$ relation. While the slope of the $M_{lum} - V_{cir}$ relation is fairly well defined, no matter what the mass of the dark matter halo is or how the dark matter is distributed, the zero-point of the relation is still to be determined. The key parameter needed to determine the zero point of the $M_{lum} - V_{cir}$ relation obviously is M_{lum}/L .

We propose a multi-wavelength study of a number of galaxies which happen to be target galaxies for the HST key project (Kennicutt, Freedman & Mould 1995) to solve unanswered questions in this paper, i.e. the zero-point of the $M_{lum} - V_{cir}$ relation, and of the TF relation studies in general. With about 30 nearby galaxies with independently determined Cepheid-based distances, good quality HI rotation curves, multi-colour photometry (UV to FIR), and good modelings, one can most likely constrain M_{lum}/L of these galaxies and, in turn, the zero-point of the $M_{lum} - V_{cir}$ relation in an *absolute* sense, independent of the Hubble constant.

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