HALO SPIN PARAMETER IN COSMOLOGICAL SIMULATIONS

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ABSTRACT

Using a cosmological Λ CDM simulation, we analyze the differences between the widely-used spin parameters suggested by Peebles and Bullock. The dimensionless spin parameter λ proposed by Peebles is theoretically well-justified but includes an annoying term, the potential energy, which cannot be directly obtained from observations and is computationally expensive to calculate in numerical simulations. The Bullock's spin parameter λ' avoids this problem assuming the isothermal density profile of a virialized halo in the Newtonian potential model. However, we find that there exists a substantial discrepancy between λ and λ' depending on the adopted potential model (Newtonian or Plummer) to calculate the halo total energy and that their redshift evolutions differ to each other significantly. Therefore, we introduce a new spin parameter, λ'' , which is simply designed to roughly recover the value of λ but to use the same halo quantities as used in λ' . If the Plummer potential is adopted, the λ'' is related to the Bullock's definition as $\lambda'' = 0.80 \times (1+z)^{-1/12} \lambda'$. Hence, the new spin parameter λ'' distribution becomes consistent with a log-normal distribution frequently seen for the λ' while its mean value is much closer to that of λ . On the other hand, in case of the Newtonian potential model, we obtain the relation of $\lambda'' = (1+z)^{-1/8} \lambda'$; there is no significant difference at $z = 0$ as found by others but λ' becomes more overestimated than λ or λ'' at higher redshifts. We also investigate the dependence of halo spin parameters on halo mass and redshift. We clearly show that although the λ' for small-mass halos with $M_h < 2 \times 10^{12} M_\odot$ seems redshift independent after $z = 1$, all the spin parameters explored, on the whole, show a stronger correlation with the increasing halo mass at higher redshifts.

Key words : cosmology: theory — halos — large-scale structure of universe — methods: numerical — methods: statistical — excursion set model

1. INTRODUCTION

The original spin parameter was proposed by Peebles (1969) who quantified the rotation of an object with a dimensionless parameter (λ) that requires information on the total energy, halo mass, and angular momentum. Although this spin parameter has widely been used in many literatures (Antonuccio-Delogu et al. 2010; Bett et al. 2007; D'Onghia & Navarro 2007; Gardner 2001), the measurement of potential energy is practically difficult and somehow inaccurate in both simulations and observations. On the other hand, an alternative spin parameter (λ') was proposed by Bullock et al. (2001), who used only two halo quantities, the total mass and angular momentum. This simplified version makes it much easier to measure the spin of simulated halos (Avila-Reese et al. 2005; Knebe & Power 2008; Stewart et al. 2013; Trowland et al. 2013) and observed galaxies (Burkert & D'Onghia 2004; Tonini et al. 2006).

These two spin parameters should be equivalent if a

halo fully virializes and its density profile follows the isothermal halo model (Mo et al. 1998; Maccio et al. 2007). However, simulated halos do not fully virialize and show a wide range of deviations in the radial density profile. Consequently, the basic assumption in the Bullock's definition should lead to some degree of scatters and a possible divergence in the $\lambda - \lambda'$ relation would be expected if halos are unvirialized.

Simple quantitative comparisons between Peebles' and Bullock's spin parameters have been made by Tonini et al. (2006) and Hetzner & Burkert (2006). Tonini et al. (2006) showed in their own galaxy model that the spin distribution of observed spirals peaks around $\lambda \simeq 0.03$ and $\lambda' \simeq 0.025$. Burkert & D'Onghia (2004) showed the median value of simulated λ is around 0.042 while Bullock et al. (2001) reported that the smaller median value is preferred for λ' ($\simeq 0.035$). Also the relation ($\lambda_{\text{med}} > \lambda'_{\text{med}}$) could also be confirmed in other papers. In the Millennium simulation, Bett et al. (2007) found that $\lambda_{\text{med}} \simeq 0.045$ for halos of $M \simeq 2 \times 10^{12} h^{-1} M_\odot$, and Knebe & Power (2008) found a lower value of $\lambda'_{\text{med}} = 0.035$ for the halo sample of $M_{\text{med}} = 5.25 \times 10^{12} h^{-1} M_\odot$.

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Moreover, Hetznecker & Burkert (2006) showed that the distribution of λ' seems to be redshift invariant while that of λ substantially shifts to a larger value as the evolution goes on. They argued that the different behavior may come from the difference in the distribution of spin transfer rate (the ratio of spin parameters after and prior to the accretion), implying that accretion may play a significant role in evolution of λ . And Munoz-Cuartas et al. (2011) found that there seems to be no obvious evolution of λ' distribution in the redshift range of $0 \leq z \leq 2$.

In spite of much effort, there are still large ambiguities in the halo potential. Although the definitions of halo spin are based on the Newtonian potential model, cosmological N -body simulations adopt the Plummer model to smooth out the divergence of small-scale Newtonian gravitational potential preventing violent scatterings between collisionless dark matter particles. Also the dynamical status of simulated halos may depend on the smoothing length, ϵ , in the Plummer model and the potential measured is significantly over- or underestimated if one adopts different value of ϵ in the spin measurement. Therefore, in this paper we study various systematic effects giving rise to the discrepancy between λ and λ' in detail.

This paper is organized as follows: In Section 2 and 3, we briefly describe the simulation specifics and show how to measure the physical quantities of simulated halos. Section 4 is devoted to show where the discrepancy between λ and λ' of the simulated halo comes from. Section 5 provides an empirical scaling factor to eliminate all the systematics. In Section 6 we show the time evolution of spin distributions for each spin parameter. We summarize and discuss the main results of this work in Section 5.

2. THE SIMULATION

We performed a WMAP 5-year cosmological simulation using 2048^3 particles in a cubic box of $L_{\text{box}} = 737.28 h^{-1} \text{Mpc}$ with the GOTPM code (Dubinski et al. 2004). The mean particle separation is $\bar{d} = 0.36 h^{-1} \text{Mpc}$ and the particle mass is $M_p = 3.37 \times 10^9 h^{-1} M_\odot$. The lowest halo mass (the collective mass of 30 particles) is $M_c \simeq 10^{11} h^{-1} M_\odot$. In this cosmology, the present density ratios of the matter and dark energy to the critical density are $\Omega_{m0} = 0.26$ and $\Omega_{\Lambda 0} = 0.74$, respectively. Then, a flat spatial curvature is ensured, or $\Omega_k = 0$. The Hubble constant is $H_0 = 100h \text{ km/second/Mpc}$, where we set $h = 0.72$. The starting redshift is $z_i = 120$ and the number of time steps is 3000 with an equal spacing in the Hubble expansion factor (a) all the way down to $z = 0$. The simulation force resolution, ϵ is fixed to $\epsilon = \bar{d}/10 = 0.036 h^{-1} \text{Mpc}$ in the comoving scale. Using the simulation output, we applied the parallel FoF (Friend-of-Friend) to identify cosmic virialized structures (or halos) with the standard linking length of 0.2 times the mean particle sep-

aration.

The angular momentum of a halo is given by

$$\mathbf{J} = \sum_i m_i \mathbf{r}_i \times \mathbf{v}_i, \quad (1)$$

where the subscript i is the member index, m_i is the mass of the particle, $\mathbf{r}_i$ is its relative position vector to the center of mass, and $\mathbf{v}_i$ is its relative velocity. The potential energy of a halo is half the sum of individual potentials of particles,

$$P = -\frac{G}{2} \sum_{ij} \frac{m_i m_j}{\sqrt{r_{ij}^2 + \epsilon^2}} = -G \sum_{i < j} \frac{m_i m_j}{\sqrt{r_{ij}^2 + \epsilon^2}}, \quad (2)$$

where r_{ij} is the mutual distance between particles of index i and j , and ϵ is the gravitational smoothing scale imposed to the simulated gravity. For setting $\epsilon = 0$, Eq. 2 reduces to the standard Newtonian potential as

$$P = -G \sum_{i < j} \frac{m_i m_j}{r_{ij}}. \quad (3)$$

The halo kinetic energy is simply measured as

$$K = \frac{1}{2} \sum_i m_i v_i^2, \quad (4)$$

where the velocity v_i incorporate the peculiar velocity and Hubble flow.

3. THE SPIN PARAMETERS

Peebles (1969) first proposed a dimensionless spin parameter of the form,

$$\lambda \equiv \frac{\sqrt{E} |\mathbf{J}|}{GM^{5/2}}, \quad (5)$$

where E is the total energy, M is the virial mass, and $\mathbf{J}$ is the total angular momentum of a halo. Among them, the total energy is hard to measure from both the observation and simulation because the potential energy requires the pre-knowledge of mass distribution far outside of the halo/galaxy boundaries (i.e., density profile). Therefore, a direct comparison of spin values between simulated halos and observed galaxies would be difficult in this definition.

An alternative spin parameter for dark matter halos was proposed by Bullock et al. (2001) as

$$\lambda' \equiv \frac{|\mathbf{J}|}{\sqrt{2} M R V}, \quad (6)$$

where R is the virial radius and V is the virial circular velocity defined as $V = \sqrt{GM/R}$. One should note that the Newtonian potential model is intrinsically employed in deriving λ' from λ and that only when a

system is virialized and its density profile follows the spherical isothermal (SI) model, the λ' is equivalent to the original spin parameter, λ . The virial radius can be determined using

$$M = \frac{4\pi}{3} R^3 \rho_c(z) \Delta_c(z), \quad (7)$$

where $\Delta_c \simeq (18\pi^2 + 82x - 39x^2)$ for a flat universe (Bryan & Normal 1998) and $x \equiv \Omega(z) - 1$. The matter content at redshift z is calculated as

$$\Omega(z) = \frac{\Omega_{m0}(1+z)^3}{\Omega_{m0}(1+z)^3 + 1 - \Omega_{m0}}. \quad (8)$$

And the critical overdensity is measured from the Hubble expansion as

$$\rho_c(z) \equiv \left(\frac{3}{8\pi G} \right) H^2(z), \quad (9)$$

where $H^2(z) = H_0^2 [\Omega_{m0}(1+z)^3 + 1 - \Omega_{m0}]$. In the denominator of Eq. 6, both the halo radius and circular velocity can easily be derived from the halo mass. Then, one needs to know the angular momentum and mass of a halo to calculate the spin parameter and this explains why Bullock's definition has been widely adopted in many literatures. Finally, the Bullock's spin parameter can be reduced to the form of

$$\lambda' = \left(\frac{1}{2G} \right)^{2/3} \frac{|\mathbf{J}|}{M^{5/3}} [H_0^2(1+z)^3 \Omega_{m0} \Delta_{vir}(z)]^{1/6}, \quad (10)$$

where $\Delta_{vir} \equiv \Delta_c/(1+x)$. In this equation, it is interesting to see that the Bullock's definition seems to have an apparent redshift dependence. However, this is just due to the virial overdensity which is defined over the critical density that changes with redshift (see Eq. 9).

4. DISCREPANCY IN SPIN VALUE BETWEEN THE DIFFERENT SPIN PARAMETERS

In Fig. 1, we show the difference between λ and λ' when the same Plummer potential model is adopted as used in the simulation. One can clearly see the significant offset of λ' above the diagonal line ($\lambda = \lambda'$) and this baseline offset becomes larger at higher redshift. Also significant scatters are apparent spreading over to higher value of λ' . In this section we investigate what effects drive these discrepancies.

4.1 Potential Model Effect

In N -body cosmological simulations of collisionless systems, zero-distance singularity in the Newtonian potential law of gravity ($\Phi \propto 1/r$) is present. To suppress the divergence of the Newtonian force at short distances, simulations have typically adopted Plummer

Fig. 1.— Scatter plots of λ'_P with respect to λ_P at $z = 0, 1, 2$, and 3 (from top left panel to bottom right panel). Halo potentials are measured with the same smoothing length (ϵ) as used in the simulation run.

potential model as a softened (or smoothed) Newtonian gravity as shown in Eq. 2. This implies that the gravitational potential is systematically biased in simulation if a non-zero softening length (or force resolution) is adopted. Note that in our simulation, we adopted the Plummer potential model and the force resolution is fixed by the mean particle separation $\bar{d}$ as $\epsilon = \bar{d}/10 = 0.036h^{-1}\text{Mpc}$.

In what follows, we describe how the potential model affects the measurement of original spin parameter (λ) of simulated halos. Fig. 2 shows the spin parameters for $\epsilon \neq 0$ (Plummer potential model; λ_P) and $\epsilon = 0$ (Newtonian potential model; λ_N), which shows the original spin parameter is not seriously affected by the force resolution, ϵ (or potential model). But there seems to be a slight redshift evolution of the spin relation between the potential models: at higher redshift, the λ of the Newtonian model tends to be larger than the Plummer model.

In Fig. 3 we show the relation between the original and Bullock's spin parameters in the Newtonian potential model at several redshifts ($z = 0, 1, 2$, and 3). In this figure, λ_N and λ'_N seem to be comparable to each other at $z = 0$, which is consistent with the findings in the previous studies. However, we found a slight redshift evolution of λ' becoming larger than λ . As a result, we conclude that the significant offset of λ'_P from λ_P mostly comes from the adopted potential model. At higher redshifts, a larger offset between λ_P and λ'_P is found, while λ'_N shows less redshift evolution. In Section 5, we will discuss in detail.

Now, we address how the Plummer potential model

Fig. 2.— Relations between λ_N and λ_P , which are measured with the Plummer (x -axis) and Newtonian (y -axis) potentials.

affects especially on the λ' in detail. First, we want to quantify how much the halo circular velocity is affected by the potential softening in the simulation. The analytic functional form for the circular velocity in the Plummer model can be found in the appendix of Gerner (1996) in the virialized isothermal case. By fitting the numerical estimations, we obtained the following fitting function,

$$V_P \simeq V_N \left[-0.39 \left(\frac{\epsilon}{R} \right) + 1 \right], \quad (11)$$

where V_P and V_N are the circular velocities in the Plummer and Newtonian models, respectively. This fitting relation holds for $\epsilon/R < 0.6$. Due to the finite force resolution of simulation, Bullock's spin parameter λ'_P is intrinsically overestimated with respect to the λ'_N (see Eq. 6) as

$$\begin{aligned} \lambda'_P &= \left(\frac{V_N}{V_P} \right) \lambda'_N \\ &\simeq \left[-0.39 \left(\frac{\epsilon}{R} \right) + 1 \right]^{-1} \lambda'_N. \end{aligned} \quad (12)$$

For a small halo of 30 particles, λ'_P would be higher than λ'_N by 10%. If $\epsilon = 0$ (Newtonian potential) or in the case of a very massive halo ($R \gg \epsilon$), λ'_P equals to λ'_N . For less massive halos, the offset could be substantially larger than λ . Otherwise specified, hereafter, we use λ and λ' as the ones measured with the Plummer potential model.

4.2 Virialization Effect

In this section, we explain the origin of large scatters above the λ - λ' line as shown in Fig. 1. If a system

Fig. 3.— Relation between λ_N and λ'_N in the Newtonian potential model at $z = 0, 1, 2$, and 3 .

virializes and its density profile follows the SI model, the Bullock's spin should be equivalent to the Peeble's spin in the Newtonian potential model. However, most of simulated halos are not fully virialized. Let us introduce a virialization parameter (C_v) of a halo as

$$C_v \equiv 2 \left(\frac{K}{P} \right) + 1, \quad (13)$$

where K and P are the kinetic and potential energies of the halo, respectively. Fig. 4 shows the spin difference ($\lambda'/\lambda - 1$) as a function of C_v for various redshifts. As a halo becomes virialized, the spin difference is reduced but a substantial offset is still seen at $z = 0$. For more unvirialized halos, the spin difference is getting greater. The scatter at higher values of λ shown in Fig. 1 corresponds to the diverging tail ($C_v < -0.4$) in this figure. Also Fig. 4 shows that most of massive halos at $z = 3$ have negative C_v , indicating that these halos still have higher kinetic energies and hat more time is needed for halos to reach the virialization. On the other hand, a large fraction of nearly virialized halos at $z = 0$ have positive C_v 's (or smaller kinetic energy compared to the potential energy). This redshift evolution of C_v may give us a hint for the evolution of internal dynamics of halos.

In Fig. 5 we show the Newtonian version of spin difference $(\lambda'_N - \lambda_N)/\lambda_N$. In this figure, we see that the average halo virialization parameter is growing with time and in the current epoch most halos have positive virialization parameter ($C_v > 0$). It is reasonable to assume that halos may oscillate around $C_v = 0$ with time. The overshoot of halo virialization parameter (C_v) at $z = 0$ is because the Newtonian halo potential, P_N , is overestimated compared with P_P leading to larger

Fig. 4.— Scatter plots of the relative spin difference as a function of $C_v \equiv 2K/P + 1$ for $z = 0, 1, 2$, and 3 . In this plot, we adopted the Plummer model to measure the halo potential.

values of C_v than the equilibrium state ($C_v = 0$).

Comparing Fig. 4 and 5, it would be important to note that which one is more proper in describing the dynamical status of halos. If same potential model is applied as used in the simulation, most of halos are assumed to be in the virialized region while on the other hand the Newtonian model predicts that halos overshoot the virialization condition due to larger value of potentials and that they are still unvirialized.

4.3 Density Profile Effect

Most of simulated halos are not only unrelaxed but also has a density profile substantially deviating from that of the SI model. Mo et al. (1998) and Maccio et al. (2007) pointed out that for a pure NFW-type halo (Navarro et al. 1997) the two spin parameters have a scaling relation as,

$$\lambda' = \lambda / \sqrt{g(c)}, \quad (14)$$

where c is the concentration index of the NFW profile and

$$g(c) = \frac{c [(1+c)^2 - 1 - 2(1+c) \ln(1+c)]}{2 [c - (1+c) \ln(1+c)]^2}. \quad (15)$$

For a cluster halo of $c \simeq 5$, $\lambda \simeq \lambda'$. But for $c \simeq 15$ (a galaxy scale), λ' can be underestimated with respect to λ by 20% even for a virialized system. The halo concentration index has been known to monotonically decrease with increasing redshift (Duffy et al. 2008; Bullock et al. 2001), which implies that the average spin

Fig. 5.— Same as Fig. 4 but for the Newtonian model case.

difference ($\langle \lambda' - \lambda \rangle$) increases with redshift. For example, for a typical halo of mass $M = 10^{12} h^{-1} M_\odot$, the average spin difference would be 35% between $z = 0$ and 3 , which is mainly due to the difference in the concentration index of the same mass halos. One should note that this relation is based on the Newtonian potential model.

4.4 Mass Resolution Effect of Simulation

Now, a question can be raised about how the measurements of virialization parameter and corresponding spin values are affected by the mass resolution of simulation, especially on low halo mass scales. Due to the under-sampling of member particles, smaller halos tend to have higher Poisson noise in the internal properties. Bett et al. (2007) investigated the Poisson effect using subhalos identified in two equivalent simulations of different mass resolutions (the mass ratio of particles is 64) and found that the higher resolution produces a few % lower λ than found in the lower-resolution simulation.

To address this issue, especially whether low-mass halo samples (around $M_c = 10^{11} h^{-1} M_\odot$ corresponding to the total mass of 30 particles) suffer from the low simulation resolution and show any obvious deviation from the relation projected from more massive halo samples, we investigate the distribution of virialization parameter as a function of halo mass. Fig. 6 shows the scatter plot of C_v as a function of halo mass at various redshifts. The median and quartiles of the sample distributions are shown as filled circles and error bars, respectively. The slope of median virialization parameter seems to be nearly flat at high redshift ($z = 3$) but show a negative slope with the increas-

Fig. 6.— Scatter plot of C_v versus the halo mass. The median and quartiles of the distribution are shown by filled circles and error bars in each mass bin, respectively.

ing halo mass can be observed at lower redshifts. A slope of C_v for more massive halo samples becomes more substantial at $z \leq 2$, which is partially because massive halos become to have lower C_v 's as the redshift decreases. Although the $1-\sigma$ distribution of C_v is increasing for smaller-mass samples, the median of C_v shows no significant change with increasing halo mass.

To investigate whether largely unvirialized halos suffer from the Poisson noise, we apply following criteria to divide the whole sample into the virialized and unvirialized halo samples by the critical value $C_v^c = -0.2$ at $z = 0$ or $C_v^c = -0.4$ at higher redshifts. In Figure 7, we show the distribution of spin difference for virialized halos ($C_v \geq C_v^c$), where less massive halos tend to have larger difference than more massive ones. The scatter seems to be wider for less massive halos but it is attributed to the sample-size effect because the $1-\sigma$ distribution does not change substantially. It is interesting to note that massive halos ($M \geq 10^{13} h^{-1} M_\odot$) tends to have higher difference at higher redshift and this is mainly due to the change of density profile with time.

Fig. 8 is for the unvirialized halo samples. The median and quartiles of distribution are shown as filled circles and error bars, respectively. The spin difference and its scatter gradually grow as the halo mass decreases and the difference becomes larger at lower redshifts. Although larger scatter in lower-mass samples seems to be due to the higher Poisson noise, the average trends of virialization parameter and the spin difference (except high-mass end at $z = 3$) show a monotonic dependence on the halo mass.

Fig. 7.— Distribution of spin difference for the virialized sample. The mean and $1-\sigma$ distribution are shown in the filled circles and error bars.

5. EMPIRICAL CORRECTION TO λ'

In Fig. 7, we see that even for virialized halos ($C_v \geq C_v^c$) there exist significant difference between λ and λ' and the amount of the difference changes with redshift. From Section 4, we know that most of the baseline offset in $\lambda' - \lambda$ distribution may come from the effects of finite force resolution, different density profile, and Poisson noise, becoming worse for higher redshift. To correct this systematic offset, we empirically fit the offset as a function of redshift and obtain a correction factor

$$f_P(z) = \left(\frac{0.26}{\sqrt{1+z}} \right)^{1/6} = 0.80 \left(\frac{1}{1+z} \right)^{1/12}. \quad (16)$$

In this fitting function, one should note that the constant factor (0.80) mainly reflects the effect of the smoothed gravitational force. Thus, by introducing a “modified spin parameter” as

$$\lambda'' \equiv f(z)\lambda', \quad (17)$$

we can get spin values comparable to the original spin parameter using with two halo quantities, halo mass and angular momentum. In case of the Newtonian potential, we get

$$f_N(z) = \left(\frac{1}{1+z} \right)^{1/8}, \quad (18)$$

from which the Newtonian model predicts better scaling relations. However, the Newtonian model has the problem in the virialization condition overestimating

Fig. 8.— Scatter plot of spin differences of unrelaxed halos. In this plot, we only select halos with $C_v < -0.2$ at $z = 0$ and $C_v < -0.4$ at other redshifts. The median and quartiles of the distribution are marked by the filled circles and error bars, respectively.

the halo potential making most of massive halos have substantially larger value of $C_v = 0$ at $z = 0$. From now on, we denote λ for the spin value measured in the Plummer potential model.

Fig. 9 shows the one-point distributions of the Peebles' λ (short-dashed), Bullock's λ' (long-dashed), and modified (solid) spin parameter (λ'') at $z = 0$. As many researchers have found, Bullock's definition shows a clear lognormal distribution (blue solid line), while Peebles' spin distribution has a shallower tail at large- λ , which is also confirmed by Bett et al. (2007). One should note that λ'' has the same shape of distribution as of λ' but the peak position is reduced towards a smaller value by about 20% that may be estimated using Eq. 11.

6. REDSHIFT & MASS DEPENDENCES OF THE SPIN PARAMETER

Now we study the evolution of spin distribution for different halo masses by dividing the whole halo sample into five subsamples according to the following mass criteria; $1 \leq M/M_{12} < 2$, $2 \leq M/M_{12} < 4$, $4 \leq M/M_{12} < 6$, $6 \leq M/M_{12} < 10$, and $10 \leq M/M_{12} < 50$ (where $M_{12} \equiv 10^{12} h^{-1} M_\odot$) at four redshift epochs ($z = 0, 1, 2$, and 3). In Fig. 10, we show the redshift evolution of spin distribution for the halo sample of $1 \leq M/M_{12} < 2$ (or particle members between 300 and 600). We see the overall redshift evolution with the peak moving to larger value until $z = 1$. After $z = 1$ the left slope of spin distribution seems to be

Fig. 9.— Distributions of λ (short-dashed), λ' (long-dashed), and λ'' (black solid) for the halo samples of mass ranges $2 \times 10^{12} h^{-1} M_\odot \leq M < 4 \times 10^{12} h^{-1} M_\odot$ at $z = 0$. The blue line is the lognormal fit to the Bullock's spin distribution.

Fig. 10.— Redshift evolution of λ distribution of the subsample of $1 \leq M/M_{12} < 2$.

almost fixed and, however, the other slope (including the high-end tail) of distribution shows a slight development with the slope less steeper and the tail more extended.

Fig. 11 shows the change of λ -, λ' -, and λ'' -distributions of halo subsamples except the subsample shown in Fig. 10. The peak position of spin distribution is increasing with redshift for all subsamples and more massive halos tend to have the lower value of spin

Fig. 11.— Redshift evolutions of λ distribution for mass samples of $2 \leq M/M_{12} < 4$, $4 \leq M/M_{12} < 6$, $6 \leq M/M_{12} < 10$, and $10 \leq M/M_{12} < 50$ (where $M_{12} \equiv 10^{12} h^{-1} M_{\odot}$) at $z = 0$ (solid), 1 (dotted), 2 (short-dashed), and 3 (long-dashed).

peak. This can further be confirmed in Fig. 12, where we plot the mean spin values, $\mu(\lambda)$, $\mu(\lambda')$, and $\mu(\lambda'')$ as a function of redshift for each subsample. The error bars are measured with the bootstrapping by dividing each halo sample into 30 equal-size subsamples. The resulting error bars are too small to be seen in the plot. Also it should be noted that the lognormal peak position is slightly larger than the mean spin value due to the significant high- λ tail of the distribution.

From this figure, several characteristics in the redshift evolution of spin parameters can be identified. First, one may see clear redshift evolution of spin distribution in all subsamples. Bullock’s λ' has higher mean spin values than λ or λ'' because the Plummer potential law is applied to measuring the spin value. In all cases, the mean values increase significantly with decreasing redshift until $z = 1$. Note that except the most massive sample ($10 \leq M/M_{12} < 50$; solid lines in Fig. 12), one can clearly see that the evolution slows down after $z = 1$. At lower redshifts than $z = 1$, $\mu(\lambda')$ of samples with $M \leq 4 \times 10^{12} h^{-1} M_{\odot}$ is almost fixed, which means that the redshift evolution seems to come to an end after $z = 1$ on this mass scale.

Fig. 12 shows the dependence of spin distribution on the halo mass. Less massive halos tend to have higher spin values but evolve less strongly with time than more massive ones. The mass dependence of the Peebles’ spin parameter λ shows somewhat different behavior from the others (λ' & λ''). The mean value of

Fig. 12.— Evolution of the average spin value of the λ (left panel), λ' (middle panel), and λ'' (right panel) distributions.

Fig. 13.— Evolution of the 1- σ width of halo spin distribution in case of λ (left panel), λ' (middle panel), and λ'' (right panel).

λ distribution seems to be less dependent on the halo mass compared to those of λ' and λ'' . After $z = 1$, the change of average spin value along with the halo mass seems to be reversed (see the left panel in Fig. 12); a higher mass sample tends to have a higher spin value and shows more significant evolution with time. The halo samples with $M/M_{12} < 4$ show less-stronger time evolution, which is consistent with previous findings (Hetznecker & Burkert 2006; Munoz-Cuartas et al. 2011).

Fig. 13 shows the evolution of 1- σ width (W_{λ} ; $16\% \leq \lambda \leq 84\%$) of spin distribution for the halo subsamples. More massive samples show shallower distributions and the distributional width gradually grows with time. Comparing with Fig. 12, we found that there is a positive correlation between $\mu(\lambda)$ and W_{λ} ; a distribution with a higher mean value tends to have a wider width.

7. SUMMARY & DISCUSSIONS

In this study, using a cosmological Λ CDM simulation we have shown that there is a substantial difference between λ and λ' and the amount of the difference varies with redshift. Less virialized halos tend to have bigger differences and a substantial difference persists even for the virialized halos.

We investigated all the possible systematics which affect the measurements, such as Poisson noise, halo virialization, and different force law applied to the calculation of spin parameters. In order to correct all those systematics we introduced a new spin parameter λ'' , which is related to λ' with the multiplication factor $f_P(z) = 0.8 \times (1+z)^{-1/12}$. In this relation the constant offset 0.8 reflects the effect of finite smoothing scale ($\epsilon = \bar{d}/10$) in the Plummer potential. For setting $\epsilon = 0$ (i.e., Newtonian model of gravity) to measure halo potential (in Eq. 2), a different correcting factor is obtained as $f_N(z) = (1+z)^{-1/8}$.

We argue here that it is important to apply the same smoothing scale (ϵ) to the potential model for λ' measure as used in the simulation. Otherwise, the halo potential may be different from the simulated value so that one may be mistaken about the dynamical status of halos.

We also found a different redshift evolution of λ from that of λ' . Our findings are similar to Hetzner & Burkert (2006). The unbounding process (Onions et al. 2013) does not significantly change the value of λ' of virialized halos while for the unrelaxed halos the difference between λ and λ' becomes significant as shown by Hetzner & Burkert (2006) and Maccio et al. (2007).

We found a clear evidence of the mass and redshift dependence of spin parameters. The average value of λ'' distribution, $\mu(\lambda'')$, changes over redshift and depends on the halo mass. We confirm that the λ'' distribution is much closer to the original λ distribution compared to the λ' . It shows significant redshift dependence when $z < 1$, especially for halos of the mass $M/M_{12} < 4$ as seen in the λ distribution and its halo mass dependence becomes weak compared to that of λ' .

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