Revision on Material Strength of Steel Fiber-Reinforced Concrete

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Abstract: Many studies have been performed on steel fiber-reinforced normal/high-strength concrete (SFRC, SFRHC) for years, which is to improve some of the weak material properties of concrete. Most of equations for material strengths of SFRHC, however, were proposed based on relatively limited test results. In this research, therefore, the material test results of SFR(H)C were extensively collected from literature, and material tests have conducted on SFR(H)C; compressive strength tests, splitting tensile tests, and modulus of rupture tests. Based on the extensive test data obtained from previous studies and this research, a database of SFR(H)C material strengths has been established, and improved equations for material strengths of SFR(H)C were also proposed. Test results showed that both the splitting tensile strength and the modulus of rupture of SFR(H)C increased as the volume fraction of steel fiber increased, while the effect of the steel fiber volume fraction on the compressive strength of SFR(H)C were not clearly observed. The proposed equations for the splitting tensile strength and the modulus of rupture of SFR(H)C showed better results than the previous equations examined in this study in terms of not only accuracy but also safety/reliability.

Keywords: fiber, SFRC, compressive strength, splitting tensile strength, modulus of rupture.

1. Introduction

Due to the continuous increase in demand for high-strength and high-performance construction materials, many ongoing researches are being conducted to improve the structural performance of concrete material that has been widely used in buildings, bridges, and other structures.¹⁻¹⁰ Many studies are especially focusing on steel fiber-reinforced concrete (SFRC), because it has excellent characteristics as a structural material such as higher tensile strength and shear strength than conventional concrete, and also excellent crack control performance. Its supply and application are comparatively easy as well.¹⁻¹⁰

In order to actually apply SFRC to the practical field, however, its material strength should be properly estimated with the adequate margin of safety and reliability. Many studies have been conducted to clarify the characteristics of SFRC material, but questions still remain on the reliability and safety of material strengths of SFRC.^{11,12} There is also a lack of experimental studies on high-strength concrete mixed with steel fibers, which has recently been used in the construction field. Therefore, this study provides experimental results on compressive strength, tensile strength, and modulus of rupture of SFRC mixtures containing steel fiber-reinforced high strength concrete (SFRHC). This study also proposes the evaluation equations for the material strengths of

SFRC and SFRHC that consider adequate margin of safety and reliability, which are evaluated by the experimental test results obtained from this study together with those from literatures.

2. Review of previous researches

Narayanan and Darwish¹³ conducted splitting tensile strength tests on SFRC with variables of concrete compressive strength (f_{ck}) and fiber shape factor (α_f). Based on their experimental results and those of existing studies, the splitting tensile strength (f_{spf}) of SFRC was proposed as:

$$f_{spf} = \frac{f_{cuf}}{20 - \sqrt{F_1}} + 0.7 + \sqrt{F_1} \quad (1)$$

where, f_{cuf} is the cubic compressive strength of the SFRC, F_1 is the fiber factor which is defined as $V_f L_f \alpha_f / D_f$. Here, V_f is the volume fraction of steel fibers, L_f is the length of the steel fiber, and D_f is the diameter of the steel fiber.

Wafa and Ashour¹⁴ conducted many material tests with volume fraction of steel fibers (V_f) as the key variable, and the compressive strength (f_{ckf}), splitting tensile strength (f_{spf}), and modulus of rupture (f_{rt}) of SFRC were proposed, respectively, as:

$$f_{ckf} = f_{ck} + 3.53 V_f \quad (2-1)$$

$$f_{spf} = 0.58 \sqrt{f_{ck}} + 3.02 V_f \quad (2-2)$$

$$f_{rt} = 0.99 \sqrt{f_{ck}} + 3.83 V_f \quad (2-3)$$

where, f_{ck} is the compressive strength of concrete.

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Eren and Celik¹⁵ made a total of thirty mixtures with various ranges of volume fraction of steel fibers (V_f), fiber factor (F_1) and silica fume addition. Based on test results, they proposed the splitting tensile strength (f_{spf}) of SFR(H)C as follows:

$$f_{spf} = A(V_f) + B \quad (3-1)$$

$$f_{spf} = C(\sqrt{f_{ck}}) + D \quad (3-2)$$

Equation (3-1) and equation (3-2) considers the effect of the volume fraction of steel fibers (V_f) and the compressive strength of the concrete (f_{ck}), respectively. A, B, C, and D are coefficients that were determined based on regression analysis, and were proposed by Eren and Celik with different values according to the characteristics of SFRC.¹⁵

Song and Hwang¹⁶ also reported test results on SFRHCs having various volume fraction of steel fibers (V_f), and proposed compressive strength (f_{ckf}), splitting tensile strength (f_{spf}) and flexural modulus of rupture (f_{rf}) as follows, respectively:

$$f_{ckf} = f_{ck} + 15.12 V_f + 4.71 V_f^2 \quad (4-1)$$

$$f_{spf} = 0.63 \sqrt{f_{ck}} + 3.01 V_f - 0.02 V_f^2 \quad (4-2)$$

$$f_{rf} = 0.69 \sqrt{f_{ck}} + 3.43 V_f + 0.32 V_f^2 \quad (4-3)$$

Thomas and Ramaswamy¹⁷ conducted a large number of material tests on SFR(H)Cs, and proposed compressive strength (f_{ckf}), splitting tensile strength (f_{spf}) and flexural modulus of rupture (f_{rf}) of SFR(H)C, respectively, as follows:

$$f_{ckf} = 0.83 f_{cu} + 0.046 f_{cu} F_2 + 1.02 F_2 \quad (5-1)$$

$$f_{spf} = 0.63 \sqrt{f_{cu}} + 0.288 \sqrt{f_{cu}} F_2 + 0.052 F_2 \quad (5-2)$$

$$f_{rf} = 0.97 \sqrt{f_{cu}} + 0.295 \sqrt{f_{cu}} F_2 + 1.117 F_2 \quad (5-3)$$

where, f_{cu} is the cubic compressive strength of conventional concrete, and F_2 is defined as, $F_2 = V_f L_f / D_f$.

Oh⁵ performed parametric studies on the modulus of rupture of SFRHC (f_{rf}), based on his test results, and proposed the modulus of rupture as follows:

$$f_{rf} = 0.63 \sqrt{f_{ck}} + 0.75 F_1 \sqrt{f_{ck}} \quad (6)$$

As described above, many experimental studies have been conducted on the material characteristics of SFRC and SFRHC,

and different equations for the material strengths were proposed. Most of the existing proposed equations for material strengths, however, are based on quite limited experimental results, and those need to be verified with comparisons of a larger number of material test results available by now. In addition, there has hardly been any consideration on the margin of safety of material strengths based on reliability of the characteristics of SFR(H)C. Therefore, additional material experiments on SFR(H)C were conducted in this study, and the experiment results from this study and those collected from literatures were used for a reliability analysis. Based on the analysis results and calibrations, material strength equations for SFR(H)C were proposed with a proper margin of safety.

3. Experimental programs

A total of sixteen SFR(H)Cs were mixed with concrete compressive strength (f_{ck}) and volume fraction of steel fibers (V_f) as primary variables to investigate the material strength of SFRC. For each mix, three $\phi 100 \text{ mm} \times 200 \text{ mm}$ cylinders for compressive strength test, three $\phi 150 \text{ mm} \times 300 \text{ mm}$ splitting tensile strength test specimens, and a $150 \text{ mm} \times 150 \text{ mm} \times 530 \text{ mm}$ modulus of rupture test prism were cast and tested.

Table 1 shows the concrete mixtures used in this study. Mixture 1 and Mixture 2 were designed to have target compressive strengths of 50 MPa and 70 MPa, respectively. Ordinary Portland cement, graded crushed gravel, and river sand were used in this mixture, where a poly-carbonic acid type of AE water reducing agent was added to enhance workability. Table 2 shows the detail properties of steel fiber used in this study. The steel fiber was a hooked type, 30 mm in length (L_f), 0.5 mm in diameter (D_f), and had a tensile strength of 1,200 MPa.

As shown in Table 3, the volume fraction of steel fibers of test specimens were 0.0%, 0.5%, 1.0%, 1.5% and 2.0% and concrete compressive strengths were 50 MPa and 70 MPa. Many of the specimens of groups A and B had the same variables, but they were divided into two groups because they were cast from different batches. A1 and A5, and B1 and B5 from the two groups were control specimens with 0% volume fraction of steel fibers (V_f), that is, normal concrete.

3.1 Test results of compressive strength

Experimental results of compressive strength of SFRC (f_{ckf}) conducted in this study are shown in Table 4 and Figure 1. The values of f_{ckf}/f_{ck} in Table 4 show the relative increase(or decrease) of test specimens compared to A1, A5, B1 and B5, which are control specimens with 0% volume fraction of steel fibers (V_f) for each of SFR(H)C series. It is also noted that the compressive strength of SFRC shown in Table 4 is the average

Table 1 Concrete mix proportion.

No.	W/C (%)	S/a (%)	Unit weight (kgf/m ³)					AE(%)
			W	C	S	G	Si	
1	33.2	40.6	194	584	635	945	-	-
2	24.0	40.2	145	600	673	1068	55	1.5

W/C water to cement ratio, S/a : fine aggregate ratio, W : water, C : cement, S : fine aggregate, G : coarse aggregate, Si : silica fume, AE : AE water reducing agent

Table 2 Properties of steel fibers.

Shape	Length (L_f , mm)	Diameter (D_f , mm)	L_f / D_f	Tensile strength (f_{sfur} , MPa)
Hooked	30	0.5	60	1,200

Table 3 Test variables of specimens.

Name	$f_{ck}(f_{ckf})$	V_f (%)	Name	$f_{ck}(f_{ckf})$	V_f
A1	50	0.0	B1	50	0.0
A2	50	0.5	B2	50	0.5
A3	50	1.0	B3	50	1.0
A4	50	1.5	B4	50	1.5
A5	70	0.0	B5	70	0.0
A6	70	0.5	B6	70	0.5
A7	70	1.0	B7	70	1.0
-	-	-	B8	70	1.5
-	-	-	B9	70	2.0

f_{ck} : Compressive strength of RC, f_{ckf} : Compressive strength of SFR(H)C,
 V_f : Volume fraction of fiber

Table 4 Test results of compressive strength.

Name	f_{ckf} (MPa)	f_{ckf}/f_{ck}	Name	f_{ckf} (MPa)	f_{ckf}/f_{ck}
A1	49.1	1.00	B1	62.7	1.00
A2	46.4	0.95	B2	57.3	0.91
A3	55.1	1.12	B3	66.3	1.06
A4	62.0	1.26	B4	56.3	0.90
A5	73.5	1.00	B5	62.2	1.00
A6	69.1	0.94	B6	69.9	1.12
A7	84.4	1.15	B7	79.0	1.27
-	-	-	B8	97.0	1.56
-	-	-	B9	105.8	1.70

value of the three compressive strength (f_{ckf}) of test cylinders.

As shown in Figure 1, there is no clear tendency of strength increase in the 50 MPa specimen group according to increase of volume fraction of steel fibers (V_f). This is the same result as that reported by Eren and Celik¹⁵. However, with the exception of the A6 specimen, the compressive strengths (f_{ckf}) of the 70 MPa group clearly increase as volume fraction of steel fibers (V_f) increase, and this is the same observation reported from Narayanan and Darwish.¹³ Thus, the compressive strength of SFRC showed different tendency between the groups as the volume fraction of steel fibers (V_f) changes. This result seems to imply that the compressive strength of SFRC are affected by the volume fraction of steel fibers (V_f) at high-strength concrete while its effect becomes little as concrete strength decreases. There are, however, many different observations on the effect of volume fraction of steel fibers (V_f) on the compressive strength of SFRC^{3,15,18}, and thus, the results of this study are not sufficient to reach general conclusion on the compressive strength of SFRC.

3.2 Test results of splitting tensile strength

Table 5 and Figure 2 show the average values of the three splitting tensile strengths cast and tested for each of the mixtures.

(a) Design compressive strength of 50MPa

(b) Design compressive strength of 70MPa

Fig. 1 Test results of compressive strength.**Table 5** Test results of splitting tensile strength.

Name	f_{spf} (MPa)	f_{spf}/f_{sp}	Name	f_{spf} (MPa)	f_{spf}/f_{sp}
A1	4.12	1.00	B1	3.77	1.00
A2	4.23	1.03	B2	4.24	1.12
A3	4.41	1.07	B3	4.53	1.20
A4	5.13	1.25	B4	6.04	1.60
A5	4.28	1.00	B5	3.81	1.00
A6	4.68	1.09	B6	4.58	1.20
A7	5.92	1.38	B7	4.89	1.28
-	-	-	B8	6.26	1.64
-	-	-	B9	7.93	2.08

The values of f_{spf}/f_{sp} in Table 5 are the splitting tensile strength of SFRC specimens (f_{spf}) divided by splitting tensile strength of a specimen without any steel fiber (f_{sp}) that are the control specimens. As shown in Table 5 and Figure 2, the splitting tensile strength of SFRC (f_{spf}) increased as the volume fraction of steel fibers (V_f) increased. For the group A with a compressive strength of 50 MPa, the splitting tensile strength of SFRC (f_{spf}) increased by 3%, 7%, and 25% each, compared to the control specimens, when the volume fraction of steel fibers (V_f) were 0.5%, 1.0%, and 1.5%, respectively. For the group A with a compressive strength of 70 MPa, the splitting tensile strength also significantly increased by 9% and 38% each, compared to the control specimens, when the volume fraction of steel fibers were 0.5% and 1.0%, respectively. For the group B with a compressive strength of 50 MPa, the

(a) Design compressive strength of 50MPa

(b) Design compressive strength of 70MPa

Fig. 2 Test results of splitting tensile strength.

splitting tensile strength (f_{sp}) increased by 12%, 20%, and 60 % each, when the fiber mixing rates (V_f) were 0.5%, 1.0%, and 1.5%, respectively. For the group B with a compressive strength of 70 MPa, the splitting tensile strength (f_{sp}) significantly increased by 20%, 28%, 64% and 104% each, when the volume fraction of steel fibers were 0.5%, 1.0%, 1.5% and 2.0%, respectively. Different from the compressive strength of SFRC described in chapter 3.1, there was a clear increase in splitting tensile strength of SFRC (f_{sp}) compared to normal concrete. This is because the fibers transfer the tensile stress between the cracks. The strengthening effects of steel fiber was found to be larger in high-strength concrete than normal strength concrete, which is considered to be due to the larger bonding stress of steel fibers in high-strength concrete. It should be noted that the fibers were pulled out at failure before reaching their tensile strength because of the very high tensile strength of fibers.

3.3 Test results of modulus of rupture

Table 6 and Figure 3 show the test results of the modulus of rupture. The values of f_c used in Table 6 are each test results of the control specimens A1, A5, B1 and B5, which have no steel fibers. With the exception of specimen A4 and group A specimens with a compressive strength of 70 MPa, the modulus of rupture of SFRC (f_{rf}) increased as the volume fraction of steel fibers (V_f) increased. In the group A with compressive strength of 50 MPa, the modulus of rupture of SFRC specimens with 0.5% and 1.0% volume fraction of steel fibers (V_f) increased by 15% and 45%, compared to the specimens without fiber, respectively. For group B with a compressive strength of 50 MPa, the modulus of rupture of SFRC

Table 6 Test results of modulus of rupture.

Name	f_{rf} (MPa)	f_{rf} / f_r	Name	f_{rf} (MPa)	f_{rf} / f_r
A1	4.51	1.00	B1	4.07	1.00
A2	5.17	1.15	B2	4.60	1.13
A3	6.53	1.45	B3	7.00	1.72
A4	4.10	0.91	B4	7.72	1.90
A5	6.10	1.00	B5	3.51	1.00
A6	5.17	0.85	B6	4.43	1.26
A7	5.70	0.93	B7	6.46	1.84
-	-	-	B8	13.35	3.80
-	-	-	B9	7.33	2.09

(a) Design compressive strength of 50MPa

(b) Design compressive strength of 70MPa

Fig. 3 Test results of modulus of rupture.

(f_{rf}) increased by 13%, 72% and 90% each, compared to the conventional concrete specimens, when the volume fractions of steel fibers (V_f) were 0.5%, 1.0% and 1.5%. For the group B specimens with a compressive strength of 70 MPa, the modulus of rupture was increased by 26%, 84%, 280% and 109%, when the volume fractions of steel fibers (V_f) were 0.5%, 1.0%, 1.5% and 2.0%, respectively.

4. Proposed expressions on material strength

As pointed out in the previous section, it was not clear whether the compressive strength of SFRC increases as the volume fraction of steel fibers (V_f) increases, on which many researchers show different opinions.^{12,14,17,19} Therefore, further studies are required to clarify the strengthening effects of steel fiber on the

compressive strength, and the scope of this study on compressive strength of SFRC is limited to presenting the experiment results. In this study, the strength equations on splitting tensile strength and modulus of rupture for SFRC were proposed, which were also compared to test results and calibrated to provide a proper margin of safety.

4.1 Expression on splitting tensile strength

The results of this study confirmed that the splitting tensile strengths (f_{spf}) of SFRC increase as the volume fractions of steel fibers (V_f) increase. However, since the number of specimens tested in this research was very limited, the experiment results were collected from previous studies,^{1,3,6,13-18,21,22} and a total of 134 test data was used for analysis. As shown in Table 7, the range of collected data included the volume fractions of steel fibers (V_f) from 0.25 to 3.00%, a fiber shape factor (aspect ratio, L_f/D_f) from 55 to 133, and fiber types such as the hooked, crimped, and indented type.

Figure 4 shows the results of parametric analysis using the collected data. As shown in Figure 4(a) and (b), the tensile strength of SFRC increased as the volume fraction of steel fibers (V_f) or fiber factor (F_1) increased. Especially, the increasing rate of the tensile strength of SFRC can be more clearly seen against the fiber factor (F_1), because it reflects both the volume fraction of steel fibers (V_f) and the fiber shape factor (α_f). The fiber shape factor (α_f) indirectly reflect the bond strength of different types of steel fibers. Thus, considering the fiber factor (F_1) as an important influential factor, the basic form of splitting tensile strength (f_{spf}) of SFRC was derived from a regression analysis as follows:

$$f_{spf} = (32.57F_1 + 0.6853)\sqrt{f_{ck}} \quad (7)$$

Figure 5 shows the ratio of test results to calculated splitting tensile strength (f_{spf}) using eq. (7) ($f_{spf(test)}/f_{spf(Eq. 7)}$) according to the fiber factor (F_1). The proposed splitting tensile strength of SFRC evaluated the test results accurately with a mean of 1.00, a standard deviation of 0.21 and a COV of 0.21. However, as shown

Fig. 4 Parametric study on splitting tensile strength.

in Figure 5, splitting tensile strengths of SFRC (f_{spf}) calculated by equation 7 provided the unconservative results on many specimens. Therefore, considering the characteristics of SFRC, such as balling of steel fibers, there is a need to consider the un-conservative results in order to secure not only accuracy but also sufficient margin of safety. Researchers can have different opinions on how much safety level to be secured, and various international codes actually adopt the different safety level. The fractile level (F) for

Table 7 Database of splitting tensile strength and modulus of rupture test of SFR(H)C.

Author	Number of specimen	V_f (%)	L_f/D_f	Shape [†]
Mansur et al. ²²⁾	4(4)*	0.50 ~ 0.80	60	H
Narayanan et al. ¹³⁾	19(0)*	0.30 ~ 3.00	100 ~ 133	C
Li et al. ¹⁸⁾	2(2)*	1.00	60 ~ 100	H
Wafa and Ashour ¹⁴⁾ Ashour et al. ²¹⁾	24(24)*	0.25 ~ 1.50	75	H
Eren and Celik ¹⁵⁾	27(0)*	0.50 ~ 2.00	60 ~ 83	H
Song et al. ¹⁶⁾	4(4)*	0.50 ~ 2.0	64	H
Yun and Park ⁶⁾	17(17)*	0.50 ~ 2.00	56-80	H,C,I
Thomas and Ramaswamy ¹⁷⁾	9(9)*	0.50 ~ 1.50	55	H
Kim et al. ³⁾	24(12)*	0.50 ~ 2.00	60	H
Oh ⁵⁾	0(12)*	1.00 ~ 3.00	62 ~ 73	H
Karl et al. ¹⁾	4(4)*	0.50 ~ 2.00	60	H
Total	134(88)*	0.25 ~ 3.00	55 ~ 133	H,C,I

*() : No. of modulus of rupture test specimens

[†]H: Hooked type, C: Crimped type, I: Indented type

Note : No. of specimen was counted excluding the conventional concrete specimens.

Fig. 5 Evaluation of Eq.7.

Fig. 6 Fractile level based on Eq. (7). (normal distribution of data assumed)

the compressive strength of conventional concrete, which is tested using cylinder specimens, specified in U.S. code²³ is 9%, and Kim et al.¹¹ evaluated the shear strength equations of reinforced concrete based on the fractile value of 10%. Thus, a similar fractile level (F) of 10% was considered for this study. Then, with an assumption that the ratios of the splitting tensile strength obtained from test results to the calculated values by equation 7 ($f_{spf}(test)/f_{spf}(Eq. 7)$) approaches a normal distribution, the value of the x axis where the fractile level is 0.1 (10%) is about 0.74. Thus, if the average value of ($f_{spf}(test)/f_{spf}(Eq. 7)$) is moved as shown in Figure 7 and 0.74 is multiplied to equation (7) so that the fractile level (F) is 1.00 when the x axis value is 1.00, the splitting tensile strength (f_{spf}) of SFRC is simply expressed as follows:

$$f_{spf} = (24F_1 + 0.5) \sqrt{f_{ck}} \quad (8)$$

In other words, there is less than 10% probability of the splitting tensile strength (f_{spf}) of SFRC being under the value calculated in equation (8). Also, when no fibers are included in mixture, the fiber factor (F_1) in equation (8) becomes 0, and it results in the splitting tensile strength of conventional concrete presented by the ACI code²³, that is $f_{sp} = 0.5 \sqrt{f_{ck}}$. Therefore, equation (8) not only reflects an appropriate safety margin based on reliability analysis, but can also be applied to the estimation of splitting tensile strength of both conventional concrete and steel fiber-reinforced concrete.

Fig. 7 Fractile level based on the calibrated mean value from Eq. (8). (normal distribution of data assumed)

Fig. 8 Evaluation of proposed splitting tensile strength equation (Eq. (8)) for SFR(H)C.

Figure 8 and Figure 9 show the evaluation results of the splitting tensile strength of SFRC by proposed equation (8) in this study and equations proposed by other researchers, respectively. The results of equation (8) showed a mean of 1.37, SD of 0.28, and COV of 0.21, which is the lowest COV value among the evaluation equations studied in this study, and thus it provided better safety margin and accuracy than the other equations. Equation (1) presented by Narayanan and Darwish¹³ was found to reflect the effects of the fiber factor (F_1) comparatively well, and generally showed safe results. However, the equation proposed in this study only has un-conservative results for under 10%, whereas the equation proposed by Narayanan and Darwish¹³ has the fractile level of 0.21 (21%) giving larger numbers of un-conservative estimations of the splitting tensile strength (f_{spf}). Equation (2-2) presented by Wafa and Ashour¹⁴ provided very conservative results, because the effect of fiber factor (F_1), which is an important influencing factor to splitting tensile strength (f_{spf}) of SFRC, was not included. Consequently, it tended to be more conservative as the fiber factor (F_1) got higher. The evaluation results of equation (4-2) proposed by Song and Hwang¹⁶ were also very similar to that proposed by Wafa and Ashour¹⁴. The equation (5-2) proposed by Thomas and Ramaswamy¹⁷ had the lowest value of standard deviation out of the many different equations, and it considered the effects of the fiber factor (F_1) reasonably well. On the other hand, however, it provided the largest number of unsafe data among all other equations considered

Fig. 9 Evaluation of splitting tensile strength equations for SFR(H)C by other researchers.

in this research, which implies that the consideration of safety appeared to be an issue.

4.2 Expression on modulus of rupture

Test results of eighty eight SFRC modulus of rupture were collected from previous studies as shown in Table 7.^{1,3,5,6,14,16-18,21,22} The range of collected data included the volume fraction of steel fibers (V_f) from 0.25% to 3.00%, a fiber aspect ratio (L_f/D_f) from 55 to 100, and different types of fiber, such as hooked, crimped, and indented types.

Figure 10 (a) and (b) show the influence of the volume fraction of steel fibers (V_f) and the fiber factor (F_1) on the modulus of rupture of SFRC (f_{rf}), using the collected data shown in Table 7. It shows that the modulus of rupture of SFRC (f_{rf}) increases as the volume fraction of steel fibers (V_f) or fiber factor (F_1) increase, and especially, the increase trend is more clearly seen for fiber factor (F_1). Thus, based on the observation, this study considered the fiber factor (F_1) as the primary influencing parameter, and modulus of rupture (f_{rf}) of SFRC was derived as follows:

$$f_{rf} = (57.03F_1 + 0.8434)\sqrt{f_{ck}} \quad (9)$$

Figure 11 shows the ratio of test results to the calculated modulus of rupture by equation 9 ($f_{rf}(test)/f_{rf}(Eq. 9)$) against the fiber factor (F_1). The values of $f_{rf}(test)/f_{rf}(Eq. 9)$ show a mean of 1.00, SD of 0.26, and COV of 0.27, which are considered to imply that equation (9) reasonably well estimates the modulus of rupture of SFRC (f_{rf}). However, just as the case with the splitting tensile

Fig. 10 Parametric study on modulus of rupture of SFRC.

Fig. 11 Evaluation of Eq. (9).

strength evaluation equation, there were many cases that the results of the modulus of rupture calculated from equation (9) were un-conservative compared to the actual experimental values. Therefore, assuming that ratio of $f_{rf}(test)/f_{rf}(Eq. 9)$ approaches a normal distribution, $f_{rf}(test)/f_{rf}(Eq. 9)$ is around 0.65 when the fractile level¹¹ is 0.1 as shown in Figure 12. Thus, as shown in Figure 13, equation (9) can be modified in simple form to provide the unsafe estimation less than 10%, that is:

$$f_{rf} = (37F_1 + 0.55) \sqrt{f_{ck}} \quad (10)$$

Fig. 12 Fractile level based on Eq. (9). (normal distribution of data assumed)

Fig. 13 Fractile level based on the calibrated mean value from Eq. (10). (normal distribution of data assumed)

Fig.14 Evaluation of proposed modulus of rupture equation (Eq. (10)) for SFR(H)C.

The equation (10) is considered to have sufficiently reflected the uncertainty of the material strength, and also be applied to estimate the modulus of rupture for both conventional concrete and steel fiber-reinforced concrete. For conventional concrete without steel fibers, equation (10) gives $f_r = 0.55 \sqrt{f_{ck}}$, which is slightly different from that of conventional concrete specified in the ACI code.²³

Figure 14 and Figure 15 present the evaluation results of equation 10 proposed in this study and the equations from other studies^{5,14,16,17} compared to the experiment results. The equation (10) provided the mean 1.54, the SD 0.42, and the COV 0.27, which is the most accurate estimation of the modulus of rupture of SFRC (f_{rf}) among the equations presented in this study. Also, as mentioned above, it should be noted that sufficient safety is secured based on the reliability analysis. Equation (2-3) by Wafa and Ashour¹⁴ and equation (4-3) by Song and Hwang¹⁶ did not consider the effects of the fiber factor (F_1) properly. As a result, the modulus of rupture of SFRC (f_{rf}) is largely underestimated as the fiber factor (F_1) becomes larger, and a large portion of estimated results by equation (2-3) are also un-conservative. Equation (5-3) by Thomas and Ramaswamy¹⁷ overestimated the modulus of rupture (f_{rf}) of SFRC specimens providing no margin of safety for a large number of specimens. The equation (6) proposed by Oh⁵ well considered the effects of the fiber factor (F_1) on the modulus of rupture of SFRC (f_{rf}), but there were many cases of un-conservative estimation. Therefore, it is shown that the modulus of rupture of SFRC proposed in this study is more proper than the existing equations in terms of safety as well as accuracy.

5. Conclusions

In this study, experimental tests have been conducted on the material characteristics of SFRC with the primary variables of concrete compressive strength (f_{ck}) and volume fraction of steel fibers (V_f). An extensive literature review has also been performed, from which a database of material test results on SFRC has been established. Based on parametric and reliability analysis, improved equations for the material strengths of SFRC have been proposed. The following conclusions were obtained from this study.

1) The splitting tensile strength of SFRC (f_{spf}) increased as the volume fraction of steel fibers (V_f) increased, whose increment was observed to be up to approximately twice as much as conventional concrete.

Fig. 15 Evaluation of modulus of rupture equations for SFR(H)C by other researchers.

2) The modulus of rupture of SFRC (f_{rf}) significantly increased as the volume fraction of steel fibers (V_f) increased, and it was significantly enhanced up to about four times compared to conventional concrete.

3) The equations for evaluation of splitting tensile strength and modulus of rupture of SFRC proposed in this study are not only accurate, but also appear to be rational in terms of reliability or safety.

4) Compared to conventional concrete, the compressive strength of SFRC increased at high-strength as the volume fraction of steel fibers (V_f) increased, whereas no increase of compressive strength was observed at normal strength in this study. However, as different researchers have different opinions about the effect of steel fibers on the compressive strength, further studies are required on this issue.

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