Hydrodynamic Responses of Spar Hull with Single and Double Heave Plates in Random Waves

S.Sudhakar¹ and S. Nallayarasu²

¹Research Scholar, Department of Ocean Engineering, IIT Madras, Chennai, Tamilnadu, 600036, India.

²Professor, Department of Ocean Engineering, IIT Madras, Chennai, Tamilnadu, 600036, India

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Abstract

Heave plates have been widely used to enhance viscous damping and thus reduces the heave response of Spar platforms. Single heave plate attached to the keel of the Spar has been reported in literature (Tao and Cai 2004). The effect of double heave plates on hydrodynamic response in random waves has been investigated in this study. The influence of relative spacing L_d/D_d (D_d -the diameter of the heave plate) on the hydrodynamic response in random waves has been simulated in wave basin experiments and numerical model. The experimental investigation has been carried out using 1:100 scale model of Spar with double heave plates in random waves for different relative spacing and varying wave period. The influence of relative spacing between the heave plates on the motion responses of Spar are evaluated and presented. Numerical investigation has been carried out to investigate effect of relative spacing on hydrodynamic characteristics such as heave added mass and hydrodynamic responses. The measured results were compared with those obtained from numerical simulation and found to be in good agreement. Experimental and numerical simulation shows that the damping coefficient and added mass does not increase for relative spacing of 0.4 and the effect greater than relative spacing on significant heave response is insignificant.

Keywords: Classic Spar, Double heave plate, Viscous damping, Added mass, Random waves, Significant motion responses

1. Introduction

Deep water offshore structures such as Spar platforms have been considered as the most reliable and cost effective system for offshore oil and gas exploration. The basic configuration usually referred to as the classic Spar comprises of a floating vertical cylinder of large draft. It consists of a top and bottom tank separated by a midsection. The top tank ("hard tank") provides the buoyancy. A square or rectangular center well run through its length and there is a flooded skirt ("mid section") hanging from the hard tank. The mid section is used for oil storage. The bottom tank ("soft tank") mainly con-

tains solid ballast in order to provide stability. The Spar hull generally has helical strakes to prevent VIV (Vortex-Induced Vibration) in current. It is normally moored using taut or catenary system of usually six to twenty lines anchored to the sea floor. The heave natural period of classic Spar platforms are relatively long, i.e. above 25 sec which is sufficiently outside the wave period range. Hence the amplitude of oscillations in the vertical plane is generally insignificant. This permits installation of dry trees and rigid risers. Therefore the classic Spar provides an attractive and reliable design solution for offshore oil and gas exploration in the regions where the most severe sea states exists and also in the regions where crude oil storage is required.

However, the classic Spar platforms are susceptible to resonant behaviour in sea states with long

^{*}Corresponding author. Tel.: +91-44-2257-4819, Fax+91 44 2257 4802,
E-mail address: nallay@iitm.ac.in
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swell condition having peak period lying in the range of 23 to 25 sec. They may undergo large heave motions, up to 8 to 10 times (Rho *et al.* (2002)) of incident wave amplitude at resonance. Such large heave motions can damage both risers and mooring systems. An efficient means to reduce the amplitude of responses is to shift the heave natural period of the proposed Spar outside the range of wave spectrum.

The heave natural period ($T_{N,3}$) of floating cylinder of uniform cross sectional area without mooring effects can be derived from dynamic analysis as

$$T_{N,3} = 2\pi \sqrt{\frac{h(1 + C_a)}{g}} \quad (1)$$

where h is the draft of the cylinder, C_a being the added mass coefficient and g is the acceleration due to gravity. The equation (1) indicates that the long heave natural period can be achieved by increasing the draft, the added mass of the system or decreasing the water plane area. However, increasing the draft and consequently increase in the system mass may lead to many other considerations in the design, thus it is not a viable economic solution for increasing the heave natural period further.

Another efficient means of reducing the heave response is to increase the heave damping of system. This can be achieved by introducing active damping systems in the form of disk (Fig. 1) at the keel of the Spar. The heave plate (disk) attached to the keel has the principal advantage that the viscous excitation from waves is limited due to exponential decay with depth. The usage of heave plates not only increases the heave damping but also provides supplementary added mass which results in longer heave natural period (Eqn (1).) as well as reduced motion. Hence the optimization of heave response is an important issue which needs to be addressed for efficient operation of Spar platforms.

2. Literature Review

The various methods to control the motion response of Spar have been investigated by several researchers. Thiagarajan *et al.* (1998) conducted model tests to examine the effect of adding an appendage in the form of disk to Tension Leg Platform columns and the influence of a small uniform

current. The results showed that the heave damping induced by an attached disk is linear with the amplitude of oscillation. Fischer *et al.* (1998) presented the importance of heave characteristics of Spar platforms that have been gleaned from wave basin model tests, numerical simulations and combination of the two. It was concluded that the optimum spacing between the heave plates is approximately one cylinder diameter which is based upon only from damping coefficient obtained from free decay tests. Tao *et al.* (2004) investigated the hydrodynamics of heaving vertical cylinder with a single disk attached at the keel. Numerical experiments showed that the disk extension should be at least four times the typical heave amplitude to achieve the optimum drag effect. Hong *et al.* (2005) carried out model tests on four types of Spar models in order to understand the influence of heave augmentation devices on response characteristics. The positive effects of strake and heave damping plates were confirmed. It was observed that unstable roll and pitch motions were occurred when the frequency of encountered wave is closer to the heave resonant frequency and half of the roll/pitch natural frequency. Tao *et al.* (2007) carried out numerical simulations using finite difference approach to investigate the effect of relative spacing (L_d/D_d) on added mass and damping coefficients in a heaving vertical cylinder attached with two circular disks. It was concluded that the disk should be placed in the L_d/D_d independent region in order to achieve the maximum benefit of motion suppression due to increased damping. Kirthi *et al.* (2007) carried out experimental investigation on motion response of Spar attached with single heave plate. The experimental results were compared with numerical model and the results obtained from WAMIT and ORCAFLEX. It was concluded that the heave plate attached to the keel of the Spar can reduce the heave response considerably to an extent as much as 50% for a heave plate diameter ratio of 1.5. Muthuchelvi *et al.* (2010) conducted experimental and numerical investigation on the use of non circular hull forms for the Spar platforms. It was concluded that the non circular hull forms are effective in reducing the pitch response but the heave response remains same as that of circular Spar. Sudhakar and Nallayarasu (2011) conducted experimental and numerical investigation on the heave response of classic Spar with circular heave plates of different diameter in regular waves. The

recommended diameter of the heave plate is 20% to 30% larger than the diameter of the Spar in order to achieve optimum heave, surge and pitch responses. Manusha *et al.* (2012) conducted experimental and numerical investigations on Spar models with different hull shapes for understanding the motion response characteristics and comparative analysis with classic Spar. It was concluded that the Spar with enlarged base was found to be more stable in pitch and less stable in heave than the Spar with circular disk at keel. Nimmy *et al.* (2012) carried out numerical simulation of the flow around circular disks of three different configurations, attached to a Spar buoy. In addition to the numerical simulation, flow visualisation and measurement using Particle Image Velocimetry (PIV) has also been carried out for comparison and evaluation. It was concluded that the added mass was found to be maximum for the Spar with double disk, thereby reducing the excitation force, making the system more stable in heave.

It can be observed that limited experimental studies have been reported in the literature on the hydrodynamic response of Spar attached with double heave plate. Hence, an attempt is made in this study to measure experimentally the damping characteristics and the motion responses of model Spar with different relative spacing of double heave plate in random waves. The numerical model is also validated with measured results for specific cases. The present study will try to explore optimum spacing between the heave plates for the considered heave plate geometry based upon experimental studies. This can be effectively used for the improved hy-

drodynamic design of floating platforms such as classic Spar.

3. Objective and Scope

The objective of the present study is to investigate the influence of spacing between the heave plates on the hydrodynamic response of Spar hull in random waves. The detailed scope of the present study is summarized below:

- Experimental studies to measure hydrodynamic responses of a Spar hull with double heave plate in random waves.
- Numerical simulation of hydrodynamic responses of Spar with double heave plates in random waves

4. Damping Elements Considered for the Present Study

The review of literature indicates that the appendages in the form of circular disks seemed to be widely used in the past which has effective means of limiting the heave motion. Hence the present study deals with addition of heave plates in the form of circular disk to the Spar hull in two different configurations like, Spar with single heave plate at keel ($D_d/D_s = 1.3$) and Spar with double heave plate at certain spacing. In case of Spar with double heave plate, there are two heave plates with fixed disk diameter ratio (D_d/D_s) of 1.3 (Sudhakar and Nallayarasu (2011)) and the relative spacing between heave plates is varied from 0.1 to 0.5. The details of Spar with double heave plates with different relative spacing are shown in Table 1. The Spar models with different heave plate configurations are shown in Fig. 2.

Fig. 1. Classic Spar with heave plate

Fig. 2. Spar models with single and double heave plate Configuration

Table 1. Geometric Parameters of Spar with double heave plates

Heave plate configuration	Geometric parameters	Relative Spacing between heave plates (L_d/D_s)				
		0.1	0.2	0.3	0.4	0.5
Spar with double heave plate	Heave plate diameter ratio (D_d/D_s)	1.30	1.30	1.30	1.30	1.30
	Heave plate diameter (cm)	32.5	32.5	32.5	32.5	32.5
	Heave plate thickness (cm)	0.5	0.5	0.5	0.5	0.5
	Relative Spacing (cm)	3.25	6.5	9.75	13.00	16.25

5. Experimental Investigation

The experimental studies were carried out to investigate the influence of relative spacing between the heave plates on hydrodynamic responses (heave, surge and pitch) of the Spar in random waves.

5.1 Model Details

The Prototype Spar was designed for a water depth of 300 m with a payload of 10,000 tonnes. The prototype was scaled down to 1:100 to obtain the models. Froude scaling was adopted to arrive the model dimensions. The scale model was fabricated using acrylic material. The principal parts of the classic Spar model are vertical hollow cylinder, a deck plate at the top and a detachable heave plate at the bottom. The concentric acrylic cylinder of outer

diameter 25 cm, an inner diameter 24 cm with closed bottom and top ends formed the main part. The bottom of the Spar is closed with a circular acrylic plate of 2 cm thick with threaded connection to facilitate the fixing of steel plate which is used as fixed ballast. The total length of the Spar is 125 cm of which the draft and free board were kept as 110 cm and 15 cm respectively. A steel deck plate of size 35 cm x 35 cm x 1 cm was used as the deck plate to attain the topside weight. The details of prototype and scale model of classic Spar including dimension, payload and its hydrostatic properties are summarized in Table 2. The fabricated model of Spar and Spar with double heave plate is shown in Fig. 3.

(a) Spar

(b) Spar with double heave plate

Fig. 3. Fabricated scale models

Fig. 4. Photographic view of the wave basin at DOE, IIT Madras

5.2 Instrumentation

The resistance type wave probe of 50 cm long comprises of two thin parallel stainless steel electrodes were employed to measure wave surface elevation. The wave probe is suspended at a distance of 1 m from the model in the upstream side in order to eliminate the disturbance caused due to the diffraction/ radiation from the model. When immersed in water, the change in conductivity of the instantaneous water column between the two electrodes is measured. This change is proportional to the variation of the water surface elevation between the electrodes. Inductive type single axis accelerometers were used to measure accelerations of the model in surge, heave and pitch modes.

5.3 Test Facility

The experimental investigations were carried out in a wave basin measuring 30 m x 30 m with a wa-

ter depth of 3 m, equipped with Multi Element Wave Maker (MEWM) on one side and wave absorbers on the opposite side. The MEWM consists of 52 paddles, each of 0.5 m width, with a hinge at the bottom. Each paddle pivots independently according to the servo actuator motion. The front of the 52 paddles is covered by an elastic membrane, which ensures that the rear of the wave maker is dry. The MEWM is capable of generating regular waves and random waves of pre-defined spectral characteristics. This wave maker is controlled by the Wave Synthesizer (WS4), an application software package for wave generation and data acquisition and processing. The photographic view of wave basin is shown in Fig. 4. The elevation and plan view of the wave basin showing the wave makers along with the location of the Spar model for the response measurement studies is presented in Fig. 5.

Fig. 5. Experimental setup of Spar with double heave plate in wave basin (N.T.S.)

5.4 Experimental Setup

The scale model of the Spar with single and double heave plates was placed at a distance of 15 m from MEWM (Fig. 5(b)) with a mooring system which comprises of four slack mooring lines of strand type twisted steel wire rope of 3mm diameter. One end of each mooring line was connected to the fair leader points (mid distance between center of gravity and center of buoyancy) on the Spar model and the other end to a rigid concrete block. The measured responses were recorded with a help of a

data acquisition system. The sampling rate of data acquisition was set to 25 Hz in random wave tests.

5.5 Heave Oscillation Tests

Heave oscillation tests were carried out to determine the natural period and damping ratio of three different configurations such as Spar, Spar with single heave plate at keel and Spar with double heave plate. The model was given an initial vertical displacement and the subsequent motions were recorded.

Table 2. Details of prototype Spar and scale model

Description	Prototype	Scale model (1:100)
Water depth	300 m	3 m
Material	Steel	Acrylic
Unit weight	78.5 kN/m ³	12 kN/m ³
Deck size	35x35x1 m	0.35x0.35x0.01 m
Topside weight	98100 kN	98.1N
Draft	110 m	1.1 m
Free board	15 m	0.15 m
Diameter	25 m	0.25 m
Self weight	71613 kN	71.62 N
Weight due to ballast	357055 kN	357.1 N
Buoyancy force (B)	542944 kN	543 N
Vertical center of gravity from keel (VCG)	60.01 m	0.60 m
Vertical center of buoyancy from keel (VCB)	55 m	0.55 m
Metacentric height (GM)	5.38 m	0.0535 m
Wall thickness	95 mm	5 mm
Heave natural period	22 sec	2.24 sec
Pitch natural period	42 sec	4.2 sec

Table 3. Measured natural period and damping ratio of Spar and Spar with single heave plate

Heave plate configuration	Parameters	Measured values
Spar	Heave natural period $T_{N,3}$ (sec)	2.24
	Heave damping ratio ξ_3 (%)	4.30
Spar with single heave plate ($D_d/D_s=1.3$)	Heave natural period $T_{N,3}$ (sec)	2.39
	Heave damping ratio ξ_3 (%)	7.00

Table 4. Measured natural period and damping ratio of Spar with double heave plates

Heave plate configuration	Parameters	Relative Spacing between heave plates (L_d/D_d)				
		0.1	0.2	0.3	0.4	0.5
Spar with double heave plate	Heave natural period $T_{N,3}$ (sec)	2.396	2.41	2.415	2.416	2.420
	Heave damping ratio ξ_3 (%)	7.96	8.47	9.36	10.13	10.47

Fig. 6. Measured time history of Spar with double heave plate ($L_d/D_d = 0.5$) in random waves

5.6 Estimation of Damping Ratio

Logarithmic decrement method is used to find the damping ratio of an under damped system in the time domain. The logarithmic decrement is the natural log of the amplitudes of any two peaks:

$$\delta = \frac{1}{n} \ln \left(\frac{x_0}{x_n} \right) \quad (2)$$

where x_0 is the greater of the two amplitudes and x_n is the amplitude of a peak n periods away. The damping ratio is then found from the logarithmic decrement:

$$\xi = \frac{1}{\sqrt{1 + \left(\frac{2\pi}{\delta} \right)^2}} \quad (3)$$

The damping ratio can then be used to find the undamped natural frequency ω_n of vibration of the system from the damped natural frequency ω_d :

$$\omega_d = \frac{2\pi}{T} \quad (4)$$

where T , the period of oscillation, is the time between two successive amplitude peaks. The natural frequency and natural time period can then be easily found:

$$\omega_n = \frac{\omega_d}{\sqrt{1 - \xi^2}} \quad T_n = \frac{2\pi}{\omega_n} \quad (5) \& (6)$$

The calculated natural period and damping ratio using the above procedure are summarized in Tables 3 and 4.

5.7. Random Wave Tests

Random wave tests were carried out with peak wave period (T_p) ranging from 1 sec to 2.75 sec at an interval of 0.25 sec along with significant wave height (H_s) equal to 5 cm. A fully developed sea state represented by Pierson-Moskowitz (P-M) spectrum model is chosen for the present study and WS4 software is used to input significant wave height and peak wave period. Each random wave test was run for a period of 90 sec with a sampling rate of 25 Hz. The typical time series for surge and heave acceleration of Spar with double heave plate ($L_d/D_d = 0.5$) for $H_s = 5$ cm and $T_p = 1.5$ sec obtained from random wave test is presented in Fig. 6. The acquired acceleration (surge, heave and pitch) time series from random wave tests is converted in

to spectral density functions using Fast Fourier Transform technique using MATLAB code. This acceleration spectrum is divided by the square of the corresponding individual frequency component (ω^2) to obtain response energy spectrum.

5.8 Random Wave Responses and Statistics

The random wave studies have been performed by choosing Pierson-Moskowitz (P-M) spectrum, which describes a fully developed sea state. The expression for P-M spectrum is written as

$$S_{\eta\eta}(\omega) = \frac{5}{6} H_s^2 2\pi \frac{\omega^2}{\omega_0^5} \exp \left[-\frac{5}{4} \left(\frac{\omega}{\omega_0} \right)^4 \right] \quad (7)$$

Where H_s is the significant wave height, ω_0 is the circular frequency at which spectral energy is maximum $= 2\pi f_0$, $f_0 = 1/T_P$ where T_P = Peak period and ω is the circular wave frequency. The response spectrum is defined as the response energy density of a floating system due to the incident wave energy density spectrum. The response spectrum of a floating body with six degrees of freedom can be expressed as

$$S_{rr,i}(\omega) = [RAO(\omega)]^2 S_{\eta\eta}(\omega) \quad (8)$$

Where $i = 1$ to 6 (six degrees of freedom), $S_{rr}(\omega)$ and $S_{\eta\eta}(\omega)$ are measured response and wave surface elevation spectral density respectively, $RAO(\omega)$ is the response amplitude operator and η is the wave surface elevation. The motion response of the system is also expressed in terms of statistical parameters. The area under the response spectrum is statistical variance (m_0), the zeroth moment and other statistical parameters are defined based on the statistical variance. The zeroth moment can be defined as:

$$m_0 = \int_0^{\infty} S_{rr}(\omega) d\omega \quad (9)$$

The statistical parameters considered in the analysis are average double amplitude of the response (S_{avg}), maximum double amplitude of the response

(S_{max}), root mean square double amplitude of the response (S_{rms}), significant double amplitude of the response (S_s) and average of the highest 10% of the response ($S_{1/10}$). The expressions for these statistical parameters are given below:

Average height of the response

$$S_{avg} = 2.5 \sqrt{m_0} \quad (10)$$

Maximum height of the response

$$S_{max} = 7.44 \sqrt{m_0} \quad (11)$$

Root mean square height of the response

$$S_{rms} = 2 \sqrt{2m_0} \quad (12)$$

Significant height of the response

$$S_s = 4 \sqrt{m_0} \quad (13)$$

Average height of the highest 10% of the response

$$S_{1/10} = 5.09 \sqrt{m_0} \quad (14)$$

The comparison of measured and target wave surface elevation spectra for Spar with different double heave plate configurations and different wave periods are shown in Figs. 7 and 8 respectively.

6. Numerical Investigation

The numerical simulation of the model Spar alone and Spar with single and double heave plates have been carried out using ANSYS AQWA. It is a diffraction radiation program based on linear potential theory. It works on the principle of panel methods. The incident wave acting on the body is assumed to be harmonic and of small amplitude in comparison to its length. The fluid is also assumed to be ideal, incompressible and irrotational, hence potential flow theory is used. The hydrostatic fluid forces combined with hydrodynamic forces and body mass characteristics were used to calculate the small amplitude rigid body response about an equilibrium mean position.

(a) $L_d/D_d = 0.2$ (a) $T_p = 1.5$ sec, $H_s = 5$ cm (b) $L_d/D_d = 0.3$ (b) $T_p = 2$ sec, $H_s = 5$ cm (c) $L_d/D_d = 0.5$ (c) $T_p = 2.5$ sec, $H_s = 5$ cm

Fig. 7. Comparison of measured and target wave elevation spectra in case of Spar with double heave plate for different relative spacing ($T_p = 1.25$ sec, $H_s = 5$ cm)

Fig. 8. Comparison of measured and target wave elevation spectra for different T_p in case of Spar with double heave plate ($L_d/D_d = 0.3$)

Table 5. Range of dimensionless parameters

Heave plate configuration	Parameters	Range
Spar and Spar with heave plate	Wave steepness parameter (H/L)	0.0036-0.032
	Diffraction parameter (D/L)	0.018-0.16
	Depth parameter (d/L)	0.214-1.92

Table 6. External input values for Spar with single heave plate ($D_d/D_s = 1.3$) in numerical simulation

Parameters	Numerical input values
Center of gravity	$C_{xx} = 0$
	$C_{yy} = 0$
	$C_{zz} = -61.22$ cm
Radius of gyration	$R_{xx} = 79.07$ cm
	$R_{yy} = 79.07$ cm
	$R_{zz} = 10.15$ cm

6.1 Wave Parameters

The numerical simulation was carried out with peak wave periods ranging from 1.0 sec to 3.0 sec along with significant wave height equal to 5cm which corresponds to a wave length of 1.56 m to 14.04 m in a water depth of 3 m. Based on parameters such as wave length (L), water depth (d), wave height (H) and the characteristics body dimension along the horizontal plane (Diameter, D), the non dimensional ratios can be formed such as steepness parameter (H/L), depth parameter (d/L) and scattering or diffraction parameter (D/L) can be formed and the details are summarized in Table 5.

6.2 Computational Methodology

The numerical simulation of model Spar and Spar with single and double heave plates in random waves has been carried out in time domain using in built P-M spectrum in AQWA NAUT module. The surface profiles of floating body are simulated through AQWA DESIGN MODELER which uses solid modeling technique. The simulated 3D surface is shown in Fig. 9. The input such as mass of the body, radius of gyration is provided to the program externally. The input used for numerical simulation of hydrodynamic response of the Classic Spar with single heave plate ($D_d/D_s = 1.3$) are presented in Table 6. The hydrodynamic analysis was performed with the chosen element size of 2 cm in the model configurations which is arrived based upon convergence studies. The software considers

only radiation damping and the effect of viscous damping are not automatically generated. Hence to include the effect of viscous damping, the external heave damping values obtained from heave oscillation tests are included in the simulation using the external damping input option available in the software. The typical external damping inputs used in Spar with double heave plates at heave resonant condition are presented in Table 7. The mooring line attached to the system is modeled as a linear elastic weightless spring, with constant line stiffness. The properties of the mooring lines are specified in the input file as their outstretched lengths, end nodes on respective bodies and their load/extension characteristics. The measured time history of surge, heave and pitch responses were converted to spectral density functions using FFT technique.

7. Results and Discussion

The experimental studies were carried out using scale model of Spar and Spar with single and double heave plates in random waves. The numerical simulations had been carried out for model Spar and other configurations in ANSYS AQWA. The measured and simulated responses were presented in the form of energy density spectrum for the scale model. The simulated significant motion responses were compared with those of measured significant motion responses and presented.

Table 7. External damping (%) for Spar with double heave plate at heave resonant condition

Heave plate configuration	Heave plate Diameter ratio (D_d/D_s)	Relative Spacing (L_d/D_d)	Total Damping Experiment (%)	Simulated radiation damping (%)	External Damping input to Numerical simulation (%)
Spar with double heave plate	1.3	0.1	7.96	0.124	7.846
	1.3	0.2	8.47	0.125	8.345
	1.3	0.3	9.36	0.125	9.235
	1.3	0.4	10.13	0.126	10.004
	1.3	0.5	10.47	0.126	10.344

7.1 Comparison of Measured and Simulated Results

The measured and simulated motion response statistics for surge, heave and pitch for peak wave period (T_p) of 1.5 sec and significant wave height (H_s) of 5 cm for the Spar with single and double heave plate in random waves are shown in Tables 10-12. It is observed that, the measured and simulated heave significant response values matches well with a maximum difference of 5%, 4% and 7% at relative spacing (L_d/D_d) equal to 0.1, 0.3 and 0.5 respectively. Similarly, the measured and simulated significant surge and pitch responses are found to be in close agreement with a maximum difference of 4% and 3% respectively for the Spar with double heave plate ($L_d/D_d = 0.3$). Similar trends are followed for peak wave period (T_p) of 2 sec.

7.2 Effect of Relative Spacing on Heave Added Mass

The variation of heave added mass with wave period obtained from the numerical simulation for the Spar alone, Spar with single and double heave plates is presented in Fig. 10(a). It is observed that, the variation of heave added mass with wave

period is constant. This may be attributed to the fact that the heave added mass effect is felt near the keel of the Spar, which is much far from the wave surface. Hence it is least affected by the wave periods. As the relative spacing (L_d/D_d) between the plates increases, the heave added mass also increases for all the wave periods. The simulated peak heave added mass expressed in terms of relative spacing is also shown in Fig. 10(b). The peak heave added mass of the Spar increases from 4.11 Kg to 7.69 Kg in case of Spar with single heave plate of heave plate diameter ratio 1.3. However, by employing additional heave plate, the peak heave added mass is further increased to 9.43 Kg and 9.60 Kg at relative spacing equal to 0.2 and 0.5 respectively. Hence the percentage increase is 22.6% and 24.8% respectively in comparison with Spar with single heave plate. It is also found that the heave added mass vs. relative spacing curve become flatter beyond relative spacing of 0.3. i.e., the percentage increase in heave added mass is marginal beyond this limit. Hence it is concluded that the heave added mass will become independent on spacing between the heave plates beyond relative spacing of 0.3.

Fig. 9. Numerical models (a) Spar with single heave plate (b& c) Spar with double heave plate

Table 8. Comparison of Heave added mass coefficient for Spar and Spar with single heave plate

Heave plate configuration	Heave plate Diameter ratio (D_d/D_s)	Theoretical		Numerical simulation	
		$A_{33}(\text{Kg})$	C_a	$A_{33}(\text{Kg})$	C_a
Spar	1.0	4.184	0.0703	4.111	0.0691
Spar with single heave plate (at keel)	1.3	7.394	0.1170	7.693	0.1211

Table 9. Comparison of Heave added mass coefficient for Spar with double heave plate

Heave plate configuration	Heave plate Diameter ratio (D_d/D_s)	Relative Spacing (L_d/D_d)	Theoretical		Numerical simulation	
			$A_{33}(\text{Kg})$	C_a	$A_{33}(\text{Kg})$	C_a
Spar with double heave plate	1.3	0.1	8.499	0.1310	9.090	0.1387
	1.3	0.2	9.469	0.1436	9.427	0.1431
	1.3	0.3	10.165	0.1525	9.518	0.1443
	1.3	0.5	10.194	0.1530	9.602	0.1454

In order to verify the added mass generated in the numerical simulation an attempt was made to verify the same using theoretical equation. The heave added mass for the cylinder with closed end can be taken as $A_{33} = 2.09\rho r^3$, ρ is the density of sea water and r is the radius of cylinder respectively. The heave added mass of cylinder + heave plate is computed theoretically (Tao and Molin (2007)) using the following relationship

$$m_a = A_{33} = \frac{1}{12} \rho \left[\frac{2D_d^3 + 3\pi D_d^2 z - \pi^3 z^3}{-3\pi D_s^2 z} \right] \quad (15)$$

Where, $z = \frac{1}{\pi} \sqrt{D_d^2 - D_s^2}$, D_s is the diameter of the Spar and D_d is the diameter of the heave plate respectively. The heave added mass coefficient of a cylinder + heave plate is then obtained as the ratio of the heave added mass (m_a) to the displaced mass ($m + m_a$). The heave added mass of cylinder + two heave plates is calculated (Tao & Molin (2007)) using an analytical expression as given below

$$m_{2a} = A_{233} = \frac{1}{48} \rho \left[\frac{8D_d^3 + 12\pi D_d^2 z + 12\pi D_d^2 L_d}{-\pi^3 L^3 - 4\pi^3 z^3 - 12\pi D_s^2 z} \right] \quad (16)$$

The heave added mass coefficient of a cylinder + two heave plates is calculated as the ratio of heave added mass (m_{2a}) to the displaced mass ($m_1 + m_{2a}$). The obtained heave added mass and heave added mass from numerical simulation and those of calculated values using the above theoretical equations are presented in Tables 8 and 9. It is observed that, the simulated heave added mass coefficient closely matches with the proposed method.

7.3 Effect of Relative Spacing on Viscous Damping

The variation of heave damping ratio for the Spar with different relative spacing of double heave plate is presented in Fig. 10(c). The heave damping ratio of Spar alone ($D_d/D_s = 1.0$), increases from 4.3% to 7% compared to that of the Spar with single heave plate of heave plate diameter ratio 1.3. In case of Spar with double heave plate, the heave damping ratio is further increased to 9.36% and 10.47% at the relative spacing equal to 0.3 and 0.5 respectively. Hence the percentage increase is 33.7% and 50% respectively in comparison with Spar with single heave plate. As the spacing between the heave plates increases, the interaction of the vortices produced by the two plates will be less. So the net vortex shedding process will be more and hence the significant increase in percentage of damping is observed. But the increase in viscous damping is marginal, beyond the relative spacing of 0.4. Hence it is concluded that the viscous damping becomes independent on relative spacing beyond this limit.

7.4 Effect of Geometry on Significant Motion Response in Random Waves

The motion response (Surge, heave and pitch) spectra for Spar with single and double heave plates for significant wave height (H_s) of 5 cm and peak wave period (T_p) of 2 sec are shown in Fig. 11. The comparison of measured motion response spectra for different peak wave periods ($T_p=1.5$ and 2 sec) and significant wave height ($H_s=5$ cm) is also presented in Fig. 12. It can be observed that the peak of spectral density shifts towards higher wave period. This could be attributed to the fact that the spectral energy is concentrated at their peak period in random waves. Hence peak of the spectrum occurs at the respective peak period. The observations on measured significant response characteristics in surge, heave and pitch mode which are calculated based on area under the above spectral density curves (Tables 10 to 12) is summarised below.

The significant surge response of the Spar with single heave plate reduces by 13% and 17% compared to the Spar with double heave plate of relative spacing equal to 0.3 and 0.5 respectively. The reduction in significant surge responses may be attributed to increase in surge added mass and surge damping.

The significant heave response of a Spar with single heave plate ($D_d/D_s=1.3$) attached with slack mooring is 0.857 m for $T_p=2$ sec. The significant heave response of the Spar with heave plate is reduced from 0.857 m to 0.771 m on the addition of another heave plate at $0.3D_d$ from the keel. Hence the reduction in heave response is about 17 %. However, when the additional heave plate is attached at $0.5D_d$ from keel, the significant heave response is further reduced to 0.666 m. i.e., the percentage of reduction is only about 22.3%. The reduction in significant responses in heave mode is mainly due to increase in viscous damping, but also to the fact that the increase in heave added mass leading to increase in heave natural period resulting in reduced heave motion. It is also observed from the Table 12, the significant pitch response of the Spar with single heave plate is reduced by 15 % and 17 % on the addition of another heave plate at a relative spacing of 0.3 and 0.5 respectively. This reduction in significant responses in pitch mode may be attributed to the increase in

pitch added mass as well as the increase in pitch damping.

7.5 Effect of Relative Spacing on Significant Motion Responses in Random Waves

The variation of significant heave, surge and pitch responses with the relative spacing in case of Spar with double heave plate for significant wave height (H_s) of 5 cm and peak wave period (T_p) of 2.0 sec is shown Fig. 14. As the relative spacing increases, the significant response characteristics in the heave, surge and pitch mode decreases. However this decrease in trend is steep up to relative spacing of 0.4 beyond which, the reduction in significant responses will be minimum. Hence the spacing between the heave plates not more than 40% diameter of the heave plate ($D_d/D_s=1.3$) is preferred as it will not enhance reduction of significant motion responses. Hence the relative spacing of 30% to 40% D_d is found to be very effective in the reduction of significant responses even in random waves.

8. Conclusions

The hydrodynamic response of a classic Spar, Spar with single and double heave plates was investigated in a laboratory wave basin and numerical simulation. Experimental studies have been conducted in a laboratory wave basin with a 1:100 scale model of Spar with single and double heave plates in random waves. Numerical simulations were carried out using the software based on panel method and compared with measured significant motion responses in random waves. The following conclusions were drawn from this study.

- The heave damping factor increases by 34% and 50% for increase of relative spacing 0.3 and 0.5 respectively.
- The peak heave added mass increases by 24.8% for the Spar with double heave plate ($L_d/D_d=0.5$) than the Spar with single heave plate. However, the increase in heave added mass is steep only up to relative spacing of 0.3. The heave added mass vs. relative spacing curve become flatter beyond this limit.

Table 10. Comparison of simulated and measured motion response statistics (Surge)

Heave plate configuration	Relative Spacing (L_d/D_d)	Parameters	$T_p = 1.5$ sec		$T_p = 2.0$ sec	
			Measured	Simulated	Measured	Simulated
Spar with single heave plate (at keel) ($D_d/D_s=1.3$)	-----	S_{avg}	0.248	0.236	0.275	0.261
		S_{max}	0.737	0.702	0.817	0.776
		S_{rms}	0.280	0.267	0.311	0.295
		S_s	0.397	0.377	0.439	0.417
		$S_{(1/10)}$	0.505	0.480	0.559	0.531
Spar with double heave plate	0.1	S_{avg}	0.220	0.216	0.254	0.232
		S_{max}	0.654	0.642	0.756	0.690
		S_{rms}	0.249	0.244	0.288	0.262
		S_s	0.352	0.345	0.407	0.371
		$S_{(1/10)}$	0.448	0.439	0.517	0.472
Spar with double heave plate	0.3	S_{avg}	0.208	0.202	0.240	0.219
		S_{max}	0.619	0.600	0.715	0.651
		S_{rms}	0.235	0.228	0.272	0.248
		S_s	0.333	0.322	0.384	0.350
		$S_{(1/10)}$	0.423	0.410	0.489	0.446
Spar with double heave plate	0.5	S_{avg}	0.195	0.193	0.227	0.208
		S_{max}	0.579	0.575	0.676	0.620
		S_{rms}	0.220	0.219	0.257	0.236
		S_s	0.312	0.309	0.363	0.334
		$S_{(1/10)}$	0.396	0.394	0.462	0.424

Table 11. Comparison of simulated and measured motion response statistics (Heave)

Heave plate configuration	Relative Spacing (L_d/D_d)	Parameters	$T_p = 1.5$ sec		$T_p = 2.0$ sec	
			Measured	Simulated	Measured	Simulated
Spar with single heave plate (at keel) ($D_d/D_s=1.3$)	-----	S_{avg}	0.482	0.476	0.535	0.520
		S_{max}	1.435	1.417	1.593	1.547
		S_{rms}	0.545	0.539	0.606	0.588
		S_s	0.771	0.762	0.857	0.832
		$S_{(1/10)}$	0.981	0.970	1.090	1.058
Spar with double heave plate	0.1	S_{avg}	0.431	0.408	0.489	0.474
		S_{max}	1.283	1.214	1.454	1.412
		S_{rms}	0.488	0.462	0.553	0.537

Spar with double heave plate	0.3	S_s	0.690	0.653	0.782	0.759
		$S_{(1/10)}$	0.878	0.831	0.995	0.966
		S_{avg}	0.375	0.358	0.444	0.433
		S_{max}	1.114	1.067	1.322	1.289
		S_{rms}	0.424	0.405	0.503	0.490
Spar with double heave plate	0.5	S_s	0.599	0.573	0.711	0.693
		$S_{(1/10)}$	0.762	0.730	0.905	0.882
		S_{avg}	0.355	0.330	0.417	0.408
		S_{max}	1.056	0.983	1.239	1.213
		S_{rms}	0.402	0.374	0.471	0.461
		S_s	0.568	0.528	0.666	0.652
		$S_{(1/10)}$	0.723	0.672	0.848	0.830

Table 12. Comparison of simulated and measured motion response statistics (Pitch)

Heave plate configuration	Relative Spacing (L_d/D_d)	Parameters	$T_p = 1.5$ sec		$T_p = 2.0$ sec	
			Measured	Simulated	Measured	Simulated
Spar with single heave plate (at keel) ($D_d/D_s=1.3$)	-----	S_{avg}	0.309	0.303	0.258	0.244
		S_{max}	0.918	0.900	0.768	0.725
		S_{rms}	0.349	0.342	0.292	0.276
		S_s	0.494	0.484	0.413	0.390
		$S_{(1/10)}$	0.628	0.616	0.525	0.496
Spar with double heave plate	0.1	S_{avg}	0.290	0.286	0.237	0.229
		S_{max}	0.864	0.851	0.705	0.682
		S_{rms}	0.328	0.324	0.268	0.259
		S_s	0.464	0.458	0.379	0.367
		$S_{(1/10)}$	0.591	0.582	0.483	0.466
Spar with double heave plate	0.3	S_{avg}	0.274	0.265	0.219	0.212
		S_{max}	0.815	0.789	0.653	0.630
		S_{rms}	0.310	0.300	0.248	0.239
		S_s	0.438	0.424	0.351	0.339
		$S_{(1/10)}$	0.558	0.540	0.446	0.431
Spar with double heave plate	0.5	S_{avg}	0.263	0.259	0.215	0.207
		S_{max}	0.782	0.772	0.640	0.615
		S_{rms}	0.297	0.293	0.243	0.234
		S_s	0.421	0.415	0.344	0.331
		$S_{(1/10)}$	0.535	0.528	0.438	0.421

(a) Heave added mass

(a) Surge

(b) Peak heave added mass

(b) Heave

(c) Heave damping ratio

(c) Pitch

Fig. 10. (a) Variation of heave added mass with wave period (b & c) peak heave added mass and heave damping ratio as a function of relative spacing

Fig. 11. Measured motion response spectra of Spar with single and double heave plate at ($T_p = 2.0$ sec & $H_s = 5$ cm)

(a) Surge

(b) Heave

(c) Pitch

Fig. 12. Comparison of measured response spectra for different peak wave period for the Spar with double heave plate ($L_d/D_d = 0.3$)

(a) Surge

(b) Heave

(c) Pitch

Fig. 13. Variation of Significant heave, surge and pitch responses with relative spacing for the Spar with double heave plate

- The significant heave response of the Spar with single heave plate is reduced by 17% and 22% on the addition of another heave plate at $0.3D_d$ and $0.5D_d$ from the keel in random waves. The reduction significant heave response is achieved mainly due to increase in viscous damping than the increase in heave added mass.
- For a disk diameter ratio of 1.3, the spacing between heave plates will enhance both heave added mass and vortex shedding process. As the spacing increases, the heave added mass and vortex shedding process increases and beyond relative spacing equal to 0.4, both the will become independent heave plates.
- A recommended relative spacing between the heave plates is 30% to 40% larger than the diameter of the Spar with heave plate diameter ratio of 1.3 in order to achieve optimum significant surge, heave and pitch responses in random waves.

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