Control of Phase and Phase Slope with Reinvestigation of 1D CRLH Transmission Lines

Dongho Jeon · Bomson Lee

Abstract

The conventionally classified waves are reclassified in this paper, including the waves associated with metamaterial (MTM) transmission lines. Various types of guided waves in periodic structures are analyzed for 1D Composite Right/Left Handed (CRLH) transmission lines (TL). In addition, dispersion diagrams for 1D CRLH, RH, and LH TLs are presented in terms of radiation conditions. We have refined the design equations of the conventional 1D CRLH TL and proposed a condition to realize the desired phase shift and its slope per unit cell. This condition has been applied to the conventional Wilkinson power divider and realized into MTM added 90° hybrid coupler. Based on the 10 dB return loss and the 90±10° phase shift, the bandwidths of the MTM added 90° hybrid coupler are 82.25 % and 103.5 % against the conventional 23 % and 52.5 %, respectively.

Key words: Metamaterial, CRLH, Phase Shift, Phase Slope.

I . Introduction

There has been intense research conducted on metamaterial based transmission lines. A tremendous amount of structures and theories of 1D CRLH have been studied and analyzed [1]~[6]. The conventional transmission lines, which support TEM(Transverse Electromagnetic) waves and follow the right hand rule, have been characterized by the distributed series inductance L (given in unit of H/m) and shunt capacitance C (given in unit of F/m). By adding a lumped series capacitance C_0 and shunt inductance L_0 periodically with a unit cell size of d (much smaller than one wavelength), the CRLH is constructed and a number of applications have then been conducted [7]~[12]. Usually, the lumped series capacitance C_0 is realized by transverse, inter digital cut or chip capacitor on the signal line, while the lumped shunt inductance is achieved by either shunt stub [13] line or chip inductor. In either instance of realizing lumped elements, some power will leak out of both the series and shunt cuts and the stub lines. These radiation losses are inevitable. Most applications, such as the phase shifter, power combiner, and power divider should carry the power to the load while minimizing the radiation losses. On the other hand, with some antenna applications such

as leaky wave antenna, this radiation effect needs to be increased and even controlled if possible [14].

In this paper, the radiation condition and types of guided waves are provided with EM and circuit simulations. We have presented dispersion diagrams for 1D CRLH, RH, and LH transmission lines considering back-fire, broadside, and endfire radiations. Furthermore, we have derived and provided the design equations for the 1D CRLH TL which has a desired phase shift and its slope per unit cell. The proposed methods are useful to widen the bandwidth of the phase shift between output ports. The phase shift of the conventional coupler is maintained only at the design frequency. However, the proposed method exhibits a significantly larger phase slope bandwidth. The effects of the closed-form equations are illustrated with a design example.

II . Design Equations for Lossless 1D CRLH Line

Fig. 1 shows the equivalent circuit of the 1D CRLH transmission line. For a lossless 1D CRLH TL(Fig. 1. $R_0=G_0=0$), the RH TL consists of distributed series capacitance C (F/m) and distributed shunt inductance L (H/m) and their product is given by

$$LC = \mu\epsilon_0\epsilon_e \quad (1)$$

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Fig. 1. Equivalent circuit of 1D CRLH transmission line.

where ϵ_0 and μ_0 are the permittivity and permeability of the free space, respectively, and ϵ_0 is the relative effective dielectric constant. Most applications using a lossless CRLH TL are commonly based on the matched condition.

$$Z_c = \sqrt{\frac{L}{C}} = \sqrt{\frac{L_0}{C_0}} \quad (2)$$

where Z_c is the characteristic impedance of the RH TL, C_0 is the series lumped capacitance and L_0 is the shunt lumped inductance. When the unit cell size, d , of the transmission line is much smaller than one wavelength, the phase shift per unit cell, $\phi(\omega)$, is given by

$$\phi(\omega_0) = -\beta d \approx -\left(\omega_0 \sqrt{LC}d - \frac{1}{\omega_0 \sqrt{L_0 C_0}} \right) = -kd + \frac{1}{\omega_0 \sqrt{L_0 C_0}} \quad (3)$$

where β is the propagation constant of the CRLH TL and k is the propagation constant of the RH TL. When a specific phase shift $\phi(\omega_0)$ per unit cell is required at the center frequency of ω_0 , the design formulas for lumped elements, C_0 and L_0 can be expressed as

$$C_0 = \frac{1}{\omega_0 Z_c [kd + \phi(\omega_0)]} \quad (4)$$

and

$$L_0 = Z_c^2 C_0. \quad (5)$$

The lumped capacitance C_0 can be realized by either a transverse cut or an inter-digital cut, and L_0 is commonly realized by a shunt shorted stub line or chip inductor. In the process of realizing lumped elements L_0 and C_0 , some power is leaked out and the radiation rate, η per unit cell due to a gap in the series and shunt line is the sum of each [3] and can be expressed as

$$\eta = \eta_{series} + \eta_{shunt} = \frac{R_0}{Z_c} + \frac{G_0}{Y_c} \quad (6)$$

Fig. 2 shows three particular radiation types as a function of frequency where d is the size of a unit cell and k_0 is the wave number in free space. If the phase shift,

 Fig. 2. Radiation types in accordance with a specific phase shift $\phi(\omega_0)$.

$\phi(\omega_0)$ are $k_0 d$ and $-k_0 d$ at a specific frequency, radiation takes place toward the backfire and endfire direction, respectively, as seen in Fig. 2(a) and (c). When $\phi(\omega_0)$ is equal to zero ($\beta=0$), the phase velocity along the line becomes infinite and the waves leaked out of the line due to the aforementioned discontinuities are directed toward the broadside as seen in Fig. 2(b). However, in either case of radiation the energy always flows to the right hand side. In other words, the group velocity is always positive. Since we have classified the radiated waves in Fig. 2 in the range of βd from $-k_0 d$ to $k_0 d$, we now describe the wave behaviors when $\beta d > k_0 d$. Fig. 3(a) shows the conceptual diagram for the transition between the endfire radiation and the RH TL. When the phase shift per unit cell, βd is between $k_0 d$ and kd as seen in Fig. 3(a) the wave travels through the air near the TL, and this structure is constructed when a LH(Left handedness) characteristic is added on the RH TL. However, when the phase shift per unit cell, βd is equal to the electrical length of RH TL, kd as seen in Fig. 3 (b) the wave tends to travel through the TL. Fig. 3(c) shows the conceptual diagram of a slow wave structure. Since the wave number in free space, k_0 needs to satisfy the equation.

$$k_0^2 = k_z^2 + k_x^2 \quad (7)$$

Fig. 3. The type of guided waves in array structures.

and $\beta d(k_0 d)$ is greater than kd for the slow wave structure, the normal component of the wave number k_x must be $-ja$ (attenuating). The slow wave structure is usually constructed by periodically adding series inductance and shunt capacitance [5]. Fig. 4 shows the dispersion diagrams for the CRLH, RH, and LH TL. Fig. 4(a) shows the dispersion diagram for a pure RH and LH TL. The RH TL has a linear phase characteristic and is shown only in the positive region (the solid line on the right side of the diagram). The LH TL, on the other hand, has an exponentially increasing phase characteristic and is shown only in the negative region (the symbol line on the left side of the diagram). A typical dispersion diagram for a zero phase shift ($\beta d=0$) 1D CRLH TL shown in Fig. 1 is depicted in Fig. 4(b), where the kd is 30° at 2 GHz. If the TL is designed to operate anywhere within the two light lines (dashed lines), radiation can occur. However, if the TL is designed to operate in the transition region between the light line and the RH TL, the radiation doesn't take place in the far field region and only the evanescent wave occurs in the near region while it travels along the TL. Fig. 4(c) presents the case of a design in which radiation does not occur, where kd is 30° and $\phi(\omega_0)$ is 49° at 2 GHz. If βd is equal to $-\pi$, the incident wave will be totally reflected and this is called a Bragg's total reflection. This is because the reflected waves are all constructively added in phase.

III. Analysis of the Condition for Realizing Desired Phase Shift and Phase Slope

(a) Dispersion diagram for pure RH and LH TL

(b) Dispersion diagram for radiating 1D CRLH ($\phi(\omega_0) = 0$ at 2 GHz) (c) Dispersion diagram for non-radiating 1D CRLH ($\phi(\omega_0) = 49^\circ$ at 2 GHz)

Fig. 4. Dispersion diagrams of 1D CRLH, LH, and RH.

The conventional design equation for 1D CRLH TL is reinvestigated in this chapter. For a given RH TL

characterized by Z_C and kd , a specifically required phase shift $\phi(\omega_0)$ per unit cell is realized by loading C_0 and L_0 as given by (4) and (5). When a specific phase slope at ω_0 is additionally required, we can still solve the problem by relaxing the condition for Z_C and kd . The first step may be to take the derivative of Eq. (3) in Chapter II.

$$\frac{d\phi}{d\omega_0} = \phi'(\omega_0) \approx -(\sqrt{LC}d + \frac{1}{\omega_0^2 \sqrt{L_0 C_0}}). \quad (8)$$

By simultaneously solving the equations (3) and (8), the electrical length kd of RH line becomes

$$kd = -\frac{1}{2} [\phi(\omega_0) - \omega_0 \phi'(\omega_0)]. \quad (9)$$

Then, the lumped capacitance C_0 and L_0 can be expressed as

$$C_0 = \frac{2}{Z_C} \cdot \frac{1}{\omega [\phi(\omega_0) - \omega \cdot \phi'(\omega_0)]} \quad (10)$$

and

$$L_0 = \frac{2 \cdot Z_C}{\omega_0 [\phi(\omega_0) - \omega_0 \cdot \phi'(\omega_0)]}. \quad (11)$$

Since the capacitance C_0 , inductance L_0 , and the electrical length kd must have positive values, the following condition should be satisfied in the design process.

$$|\phi(\omega_0)| < \omega_0 \cdot \phi'(\omega_0). \quad (12)$$

Fig. 5 shows the phase of S_{21} for the five cases that have different phase slopes $\omega \cdot \phi'(\omega)$ while the phase shift per unit cell $\phi(\omega)$ is fixed at zero-degree, as a function of frequency. When the kd is 45° ($C_0=1.76$ pF, $L_0=4.41$ nH), the slope of the S_{21} is shown to be steeper. However, it shows the most gradual slope of S_{21} when the kd is 1° ($C_0=79.3$ pF, $L_0=198.2$ nH) Fig. 6 shows the phase of S_{21} when the phase slope $\omega_0 \cdot \phi'(\omega_0)$ is fixed with $-\pi/2$ while the phase shift per unit cell varies from -60° ($C_0=5.29$ pF, $L_0=13.22$ nH, $kd=75^\circ$) to 60° ($C_0=1.06$ pF, $L_0=2.64$ nH, $kd=15^\circ$). It is verified in Fig. 6 that the slope of S_{21} for each unit cell can be maintained equally while the target phase shift is changing. The above results proved that any required phase shift and its slope can be achieved.

IV. Design of MTM added 90° Hybrid Coupler Based on Conventional Wilkinson Power Divider

The geometry of the MTM added 90° hybrid coupler

Fig. 5. Phase of S_{21} for different kd 's ($\phi(\omega) = 0^\circ$ fixed).

Fig. 6. Phase of S_{21} with different phase shift at ω_0 ($\omega_0 \cdot \phi'(\omega_0) = -\pi/2$ fixed).

provided as an example is shown in Fig. 7. Each branch line consists of two MTM TLs with series chip capacitors and shunt inductors. This proposed structure can maintain good characteristics of the conventional Wilkinson power divider (Conv. W. power divider), such as equal power splitting between two output ports and achieving high isolations of the output ports as well as excellent return loss at all ports. While those advantages are maintained, $+45^\circ$ and -45° MTM phase shifting lines are inserted along the upper and lower branches, respectively. The design of the two MTM TLs is based on the theories in [4], [5]. The proposed structure is constructed on Teflon substrate with a height of 1.6 mm and relative permittivity of 2.17. The width of the 50Ω RH microstrip TL is 5.3 mm and two $\lambda/4$ branches are designed with a characteristic impedance of $\sqrt{2} Z_0$ (70.7 Ω). The length d of MTM TL for the upper branch is 12 mm and the lower branch is 25.2 mm. Fig. 8 (a) shows

Fig. 7. Geometry of the MTM added 90° hybrid coupler.

the circuit/EM simulated magnitude of S_{11} , S_{21} , and S_{31} of a conventional 90° hybrid coupler (Conv. 90° hybrid) and the proposed MTM added 90° hybrid coupler. Based on the 10 dB return loss, the bandwidth of the conventional 90° hybrid coupler is shown to be 0.46 GHz (1.89~2.35 GHz) (23 %), whereas that of the proposed MTM added coupler based on circuit/ EM simulations is (2.54 GHz) (85.25 %), respectively. Also, the return loss of the port 2 and port 3 are shown to be below the 10 dB over very large bandwidth. In Fig. 8(b), excellent isolation is shown between port 2 and 3. Based on the 10 dB return loss bandwidth, the bandwidth of $|S_{32}|$ for the conventional 90° hybrid coupler is 0.8 GHz (0.163~2.43 GHz) (40 %) and that of the MTM added coupler is 1.9 GHz (0.87~2.77 GHz) (95 %) which is almost same as that of the conventional Wilkinson power divider.

In Fig. 8(c), we show the phase shift of S_{31} and S_{21} for the conventional 90° hybrid coupler and the proposed MTM added coupler (EM) as a function of frequency. Based on the $90 \pm 10^\circ$ phase shift, the bandwidth of the conventional 90° hybrid coupler is 1.04 GHz (1.71 ~ 2.75 GHz) (52.5 %) while the proposed MTM added design is 2.07 GHz (1.25 ~ 3.32 GHz) (103.5 %).

V. Conclusion

Various types of radiated and guided waves have been examined founded on the conventional MTM TL theory. In addition, some useful closed-form solutions have been derived by solving equations of phase shift, phase slope, and matching condition. Based on the solutions, a novel MTM added 90° hybrid coupler has been designed by placing $+45^\circ$ and -45° MTM TL sections at the ends of a conventional Wilkinson power divider. The performance of the proposed coupler has been greatly enhanced. Based on a 10 dB return loss, the bandwidth of the proposed coupler is 85.25 % against the conventional 23 %. The bandwidth of the proposed cou-

(a) Magnitude of S_{11} , S_{21} , and S_{31} of conventional and proposed hybrid coupler

(b) EM-simulated magnitude of S_{32}

(c) EM-simulated phase shift between port 2 and 3

Fig. 8. EM/circuit simulation results.

pler based on a $\pm 10^\circ$ phase shift between two output ports has been shown to be 103.5 % against the conventional 52.5 %.

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