VS-FCM: Validity-guided Spatial Fuzzy c -Means Clustering for Image Segmentation

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Abstract

In this paper a new fuzzy clustering approach to the color clustering problem has been proposed. To deal with the limitations of the traditional FCM algorithm, we propose a spatial homogeneity-based FCM algorithm. Moreover, the cluster validity index is employed to automatically determine the number of clusters for a given image. We refer to this method as VS-FCM algorithm. The effectiveness of the proposed method is demonstrated through various clustering examples.

Key Words : Fuzzy clustering, Color clustering, Fuzzy c -means

1. Introduction

The objective of color clustering is to divide a color set into c homogeneous color clusters, which is used in color image segmentation and recognition. Color clustering is an inherently ambiguous task because color boundaries between objects are often blurred and distorted due to the image acquisition process [1]. Since Bezdek first reported his fuzzy set theoretic image segmentation algorithm for aerial images [1], numerous reports have appeared detailing the superior manner in which the fuzzy c -means (FCM) algorithm handles uncertainties [1, 2, 3, 4, 5, 6, 7]; FCM is less prone to falling into local optima than crisp clustering algorithms.

In spite of the advantages of the fuzzy clustering approach, the traditional FCM-type methods have a number of limitations:

Limit 1. The initialization step is important because different selections of the initial cluster centroids can potentially lead to different local optima or different partitions. However, there is no general agreement regarding which initialization scheme gives the best results [1, 3]. Most algorithms developed to date assign the random locations to the initial centroids because such a selection will definitely converge in the iterative process.

Limit 2. Traditional spatial FCM algorithms are limited in formulating the influence of the spatial neighborhood

of a data point on calculating its degree of membership to a given cluster. As Liew pointed out [6], in the original spatial FCM algorithm put forwarded by Tolias and Panas [5], the influence of neighboring color points on a central color point is binary in that it is implemented through the addition and subtraction of a fixed constant. The adaptive spatial FCM algorithm of Liew still does not provide an explicit mathematical formulation for handling homogeneous regions.

Limit 3. The FCM requires the user to pre-define the number of clusters; however, it is not always possible to know the number of clusters in advance.

In this study, we present a new fuzzy approach to the color clustering problem that effectively handles the aforementioned limitations.

2. The Proposed Method

2.1 Initial Centroids Selection

To solve the initialization problem, we exploit the work of Kim et al. [8] for selecting initial centroids of the FCM. We briefly describe the method as follows.

The key concept of the method is that the dominant and vivid data in a given data set are unlikely to belong to the

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same cluster; these distinguishable dominant data are then used to guess the initial cluster centroids. To obtain the dominant colors from a color set $x \in X$, the following two factors should be considered; (1) a set of reference data and (2) a measure of similarity between a data point and the reference color. Let us suppose that a set of reference data containing all the major features in a given data set can be compiled. For example, when applied to the image segmentation, the reference data might be composed of a set of distinguishable colors such as red, green, and blue with which to measure similarity between color data.

Secondly, the similarity between a color data and a reference color is calculated by the degree of membership of each color point to a reference color. Suppose a color point, denoted by $x = (x_L, x_a, x_b) \in X$, is a point in the CIELAB color space. Likewise, the i -th reference color, denoted by $r_i = (r_L^i, r_a^i, r_b^i) \in R$, is a point in the CIELAB space. The distance between x and r_i , denoted by $\delta(x, r_i)$, is calculated from the CIELAB color difference formula [9]:

$$\delta(x, r_i) = \sqrt{(x_L - r_L^i)^2 + (x_a - r_a^i)^2 + (x_b - r_b^i)^2} \quad (1)$$

Then, the degree of membership $\mu_{r_i}(x)$ of x to the reference color r_i is defined as:

$$\mu_{r_i}(x) = \left(\sum_{j=1}^k \frac{\delta(x, r_i)}{\delta(x, r_j)} \right)^{-1} \quad (2)$$

where $r_i \neq r_j, r_j \in R$. If x exactly coincides with r_i , it has $\mu_{r_i}(x) = 1.0$. Conversely, if x exactly coincides with another reference color r_j , then $\mu_{r_i}(x) = 0.0$. When x does not exactly coincide with any reference colors, the membership value is determined using Eq. 2.

To determine the dominant colors, we compute the membership degrees between all $x_j \in X$ and $r_i \in R$. Each r_i has two additional attributes, denoted μ_i and p_i . μ_i indicates the highest membership degree obtained by computing $\mu_{r_i}(x_j)$ for all $x_j \in X$, and p_i indicates the closest color point x_j to r_i . For $x_j \in X$, we compute the membership degree for each of the reference colors, and update μ_i and p_i . When the computation is completed, the reference colors are sorted by μ_i in decreasing order, which is represented by $R^s = (r_{s1}, r_{s2}, \dots, r_{sk})$ where r_{s1} has the highest value of μ_i and r_{sk} has the lowest one.

Given R , R^s , and the number of clusters c , the set of dominant colors is defined as $D = \{d_i \mid d_i = r_{si}, 1 \leq i \leq c\}$. Thus D consists of the first c reference colors in the sorted list. Having established the dominant colors, the initial cluster centroids ($V_0 = \{v_1, v_2, \dots, v_c\}$) are assigned to the color point $x_i (= p_i)$ that is closest to the dominant color d_i . Formally given input $X = \{x_1, \dots, x_n\}$, $R = \{r_1, \dots, r_k\}$, and c , the initialization step is described as follows:

S1: Compute the membership degree $\mu_{r_i}(x_j)$ for all $x_j \in X$

S2: Update μ_i and p_i for all $r_i \in R$

S3: Sort $r_i \in R$ in descending order of μ_i to obtain R^s

S4: Determine d_i from R^s and assign $p_i \in d_i$ to v_i

2.2 S-FCM: Spatial Homogeneity FCM

To exploit the spatial clustering, we employ the notion of homogeneity extracted from spatial domain. Homogeneity is a local information that reflects the uniformity of a region [10]; when a region is homogeneous, data in the homogeneous region are likely to have similar characteristics. For each color point, the homogeneity value for each color point is computed in terms of two components, variance and discontinuity in the CIELAB space. In this scheme, the higher the homogeneity value of color $x_j \in X$, the greater the likelihood that x_j and its spatial neighborhood belong to the same region.

The variance is a measure of the differences of color values in a predefined spatial neighborhood. Let x_j be a color point in the CIELAB space, and let S_j denote a spatial neighborhood in two-dimensional space, specified to be a $d \times d$ window size centered at x_j . Then, the variance $\sigma^2(x_j)$ of color x_j is defined as:

$$\sigma^2(x_j) = \frac{1}{d^2} \sum_{x_k \in S_j} (x_k - m_j)^2 \quad (3)$$

where

$$m_j = \frac{1}{d^2} \sum_{x_k \in S_j} x_k \quad (4)$$

m_j is the mean value of the color data in S_j .

The discontinuity $g(x_j)$ of color x_j is obtained through the first derivatives at spatial coordinate x_j , which is defined as the magnitude of the gradient between x_j and S_j [10]. When the data to be clustered are given in two dimensional space, the discontinuity means the first derivative gradients for the horizontal ($G_{x_j,h}^2$) and vertical ($G_{x_j,v}^2$) directions at x_j :

$$g(x_j) = [G_{x_j,h}^2 + G_{x_j,v}^2]^{1/2} \quad (5)$$

$$= \left[\left(\frac{\partial g}{\partial x_{j,h}} \right)^2 + \left(\frac{\partial g}{\partial x_{j,v}} \right)^2 \right]^{1/2} \quad (6)$$

The two measures, $\sigma^2(x_j)$ and $g(x_j)$, have different scales that needs to be reconciled through a normalization by their maximum values. These normalized values of the variance, $N(\sigma^2(x_j))$, and discontinuity, $N(g(x_j))$, are now used to define the homogeneity value $h(x_j)$ of data x_j :

$$h(x_j) = 1 - N(\sigma^2(x_j)) \times N(g(x_j)) \quad (7)$$

where $0 \leq h(x_j) \leq 1$.

The goal of the present study is to cluster a set of color data $X = \{x_1, x_2, \dots, x_n\}$ into c homogeneous groups. To achieve this, we propose a spatial homogeneity-based fuzzy c -means (S-FCM) algorithm. The objective of the S-FCM algorithm is to cluster the data X into c clusters ($F_1, F_2, \dots, F_c$) by minimizing J_m :

$$J_m(U, V : X) = \sum_{i=1}^c \sum_{j=1}^n (\mu_{F_i}(x_j))^m \|x_j - v_i\|^2 \quad (8)$$

where

$$\mu_{F_i}(x_j) = \frac{1}{n_j} \left(\mu_{ij} + \sum_{x_k \in S_j} \mu_{ik} h(x_k) \right) \quad (9)$$

and

$$\mu_{ij} = \left(\sum_{z=1}^c \left(\frac{\|x_j - v_i\|^2}{\|x_j - v_z\|^2} \right)^{\frac{1}{m-1}} \right)^{-1} \quad (10)$$

subject to $0 \leq \mu_{F_i}(x_j) \leq 1$, $\sum_{i=1}^c \mu_{F_i}(x_j) = 1$, and $0 < \sum_{j=1}^n \mu_{F_i}(x_j) < n$. In Eq. 9, μ_{ij} is obtained by computing relative distances between clusters in a color feature space. $x_k \in S_j$ is a spatial neighborhood of x_j , defined as a $d \times d$ window. $n_j (= d^2)$ is the number of x_j and its neighboring data, and $h(x_k)$ is the homogeneity value of color x_k .

We can see that $\mu_{F_i}(x_j)$ is influenced by the relations (μ_{ik}) between $x_k \in S_j$ and F_i and, furthermore, the extent of this influence is controlled by the degree of homogeneity $h(x_k)$. When x_j is located in a homogeneous region, it is useful to exploit the relations between the neighborhood and the centroid because data in a homogeneous region are likely to belong to the same cluster. In contrast, when $x_k \in S_j$ lies in a nonhomogeneous region such as an edge, the influence of x_k on x_j should be made as small as possible. Thus, the degree to which x_k influences x_j is determined through $h(x_k)$. Formally given input $X = \{x_1, \dots, x_n\}$, $R = \{r_1, \dots, r_k\}$, c , m , ϵ , and d , the S-FCM procedural steps are described as follows:

- S1: $J(0) \leftarrow \infty$, $t \leftarrow 1$, run the initialization procedure
- S2: Compute $h(x_j)$ and $\mu_{F_1}(x_j)(t), \dots, \mu_{F_c}(x_j)(t)$
- S3: Compute the objective function $J(t)$
- S4: while $|J(t) - J(t-1)| > \epsilon$ do (S5, S6, S7, S8)
- S5: $t \leftarrow t + 1$
- S6: $v_i(t) \leftarrow \sum_{j=1}^n (\mu_{F_i}(x_j)(t))^m x_j / \sum_{j=1}^n (\mu_{F_i}(x_j)(t))^m$
- S7: Update $\mu_{F_1}(x_j)(t), \dots, \mu_{F_c}(x_j)(t)$ for all $x_j \in X$
- S8: Compute $J(t)$ and go to S4

2.3 VS-FCM: Validity-Guided SFCM

Most of clustering methods including the S-FCM algorithm require the user to pre-define the number of clusters; however, it is not always possible to know how many color image segments are within a color image in advance. Therefore, to exploit the clustering task in a full automatic way, the clustering algorithm should contain a procedure to determine the optimal number of clusters from data itself without the user's specification.

To achieve this, the cluster validity index, v_{os} , proposed by Kim et al. [11] is applied to the clustering results obtained through the S-FCM algorithm to validate the quality of the partition. The v_{os} index searches for the optimal number of clusters using two properties: an overlap measure and a separation measure. A good cluster result is expected to have a low degree of overlap and a large separation distance. We refer to this procedure as a validity-guided S-FCM (VS-FCM) algorithm. This procedure is repeated with respect to the possible candidates for the number of clusters. Of the partitions considered, the partition with the minimum value of v_{os} is identified as the optimal clustering result. Formally given input $X = \{x_1, \dots, x_n\}$, $R = \{r_1, \dots, r_k\}$, m , ϵ , and d , the VS-FCM procedural steps are described as follows:

- S1: $c \leftarrow 2$, $c_{max} \leftarrow \sqrt{n}$, and $v_{os}^{min} \leftarrow 0$;
- S2: while $c \leq c_{max}$ do (S3, S4, S5)
- S3: $F^c \leftarrow$ the partition obtained by the S-FCM
- S4: compute $v_{os}(F^c) = \text{Overlap}(F^c) / \text{Sep}(F^c)$
- S5: $c \leftarrow c + 1$ and go to S2
- S6: Report F^c with $\min v_{os}$ as the optimal partition;

3. Experiments and Conclusion

To test the effectiveness of the proposed scheme, the FCM, S-FCM, and VS-FCM were applied to image segmentation problems; the weighing exponent was set to be $m = 2.0$ since it have showed the overwhelming favorite [1]; the termination criteria was set to be $\epsilon = 0.001$; in FCM, the initial centroids were selected by random; the color data $x_j \in X$ in an image were represented in the CIELAB color space due to its superior performance to other spaces [9]; 14 distinguishable colors were chosen from the the Munsell Color system to compose the reference data set [12].

The size of the spatial neighborhood S_j has influence on the value of the homogeneity. As Cheng et al. pointed out [10], a small value of S_j is limited to reflect local information in the calculation of the homogeneity. In contrary, a large value of S_j requires a high computation time. Thus,

in this study we chose a 3x3 window for S_j by weighing the trade-off.

Fig. 1 Comparison of the segmentation results using FCM and S-FCM: (a) original image “pants” (b) FCM (c) S-FCM

Fig. 2 Comparison of the segmentation results using FCM and S-FCM: (a) original image “clown” (b) FCM (c) S-FCM

Fig. 3 Comparison of the segmentation results using FCM and S-FCM: (a) original image “house” (b) FCM (c) S-FCM

Firstly, we compared the clustering performance of the S-FCM with the FCM method. Figure 1(a) shows an original image (“pants”) containing four colors ($c = 4$): a white for the background, a gray for the left-side pants, a red for the right-side pants, and black for the rectangular mark on the red pants. (Please note that this figure must be viewed in color to appreciate the color segmentation results.) An ideal clustering would result in four segmented objects. Many color points on the gray pants are close to those of the rectangular mark on the red pants in the color space. Thus, clustering based on the FCM algorithm [Figure 1(b)] led to an over-segmentation in the left-side (gray) pants due to the large number of creases in those pants, and the black mark is not clear. In contrast, those components were clearly segmented by the S-FCM algorithm [Figure 1(c)]; it prevented the FCM from falling into incorrect local optima.

Figure 2(a) shows an original image (“clown”) containing various colors. Each clustering algorithm was applied to segment the image into six colors ($c = 6$): white, black, red, brown, yellow, and green. The FCM algorithm failed to give correct clustering results [Figure 2(b)]; for example, the green hair was misclassified as yellow color in the final image. In contrast, the S-FCM algorithm [Figure 2(c)] successfully segmented the six colors where the hair was clearly classified since the green color was identified as one of the dominant colors from the image.

Figure 3(a) shows an original image (“house”) where each clustering algorithm was applied to segment the image into six clusters ($c = 6$): white, blue, red, black, gray, and green. The FCM algorithm failed to give a correct clustering result [Figure 3(b)]; the bottom (green) grass was over-segmented and the (gray) roof of the house was misclassified as white color. In contrast, the S-FCM algorithm provided a better segmentation result [Figure 3(c)]; the grass, the roof and the (black) shade were more clearly classified.

Table 1. Cluster validity values for the “clown” image for the VS-FCM with respect to $c = 2, \dots, c_{\max} = 7$. The bold face indicates the optimal result for the given validity index.

c	v_{PC}	v_{PE}	v_{XB}	v_K	v_{SC}	v_{OS}
2	0.57	0.23	0.29	4743.95	-0.78	1.12
3	0.54	0.29	0.12	1969.15	-1.40	0.58
4	0.49	0.35	0.23	3729.60	-2.10	0.49
5	0.50	0.37	0.18	3009.77	-2.63	0.39
6	0.49	0.40	0.16	2584.05	-3.10	0.37
7	0.44	0.44	0.40	6577.64	-4.28	0.41

Fig. 4 Comparison of the segmentation results using VS-FCM for different values of c : (a) VS-FCM with $c = 5$ (b) VS-FCM with $c = 6$ (c) VS-FCM with $c = 7$

Secondly, to show the potential of the VS-FCM, we tested several comparisons with other validity indexes for $c = 2, \dots, c_{\max}$. The detailed explanation of each validity index can be found in [11]. For the “clown” image [Figure 2(a)], we find that the VS-FCM using $c = 6$ generates the optimal clustering result [Table 1]; only v_{OS} correctly recognizes $c = 6$ whereas all other indexes point to $c = 2$ or $c = 3$ as the optimal partition. Figure 4 shows three of the six clustering results for the VS-FCM for different values of c ; From the figure, we find that VS-FCM with $c = 6$,

in figure 4(b), shows better result than the other two cases of VS-FCM with $c = 5$ [fig. 4(a)] and VS-FCM with $c = 7$ [fig. 4(c)].

From the experiments, we see that the results presented above highlight the potential of the proposed approach to image segmentation; the VS-FCM makes it possible for user not to explicitly specify the number of clusters a priori.

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