Flows over Concave Surfaces: Development of Pre-set Wavelength Görtler Vortices

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Abstract

The development of pre-set wavelength Görtler vortices are studied in the boundary-layer flows on concave surfaces of 1.0 and 2.0 m radius of curvature. The wavelengths of the vortices were pre-set by thin wires of 0.2 mm diameter placed 10 mm upstream and perpendicular to the concave surface leading edge. Velocity contours were obtained from velocity measurements using a single hot-wire anemometer probe. The most amplified or dominant wavelength is found to be 15 mm for free-stream velocity of 2.1 m/s and 3.0 m/s on the concave surface of $R = 1$ m and 2 m, respectively. The velocity contours in the cross-sectional planes at several streamwise locations show the growth and breakdown of the vortices. Three different regions can be identified based on the growth rate of the vortices. The occurrence of a secondary instability mode is also shown in the form of mushroom-like structures as a consequence of the non-linear growth of the Görtler vortices. By pre-setting the vortex wavelength to be much larger and much smaller than the most amplified one, the splitting and merging of Görtler vortices can be respectively observed.

Keyword: Görtler vortices, pre-set wavelength, concave surface, boundary-layer flow

1. Introduction

Flows over concave surfaces will be subjected to centrifugal instability which may result in streamwise counter-rotating Görtler vortices, named after Görtler [1] who first theoretically predicted the existence of the vortices, as sketched in Fig. 1. Such vortices will occur if the Görtler number G_θ , as defined by (Smith, 1955):

$$G_\theta = (U_\infty \theta / \nu) \sqrt{\theta / R} \quad (1)$$

exceeds a critical value $G_{\theta cr}$, where ν is the fluid kinematic viscosity, θ the momentum thickness based on Blasius boundary-layer profile, U_∞ the free-stream velocity, and R the concave surface radius of curvature. The vortices will be amplified downstream resulting in a three-dimensional boundary-layer due to distribution of streamwise momentum which causes spanwise variation in the boundary-layer thickness, and consequently will form the “upwash” region, where low momentum fluid moves away from the wall, and the “downwash” region, where high speed outer fluid moves towards the wall (Fig.1). At the “upwash”, the boundary-layer is thicker and the shear stress is lower than those at the “downwash”.

Since concave surfaces can be found in many fluid engineering applications, such as turbine blades and aerofoils, the effects of Görtler vortices on boundary layer development, heat transfer and deposition (if applicable) can not be ignored. Recently, flows over dimpled surfaces, which are three-dimensional concave surfaces [2], have attracted much attention due to the potential applications in heat transfer enhancement [3] and drag reduction.

The wavelength selection of Görtler vortices is not clearly understood yet, but the onset and development of Görtler vortices are known to be influenced by the initial disturbances. Also, the leading edge receptivity influences the resulting vortex wavelengths that appear in the experiments. That is, Görtler instability amplifies the wavelength imposed by the rig facilities on the incoming flow [4], and a competition of perturbations with different amplification rates, as the only wavelength selection mechanism of Görtler vortices, resulting in non-uniform wavelengths in naturally developing Görtler vortices. Swearingen and Blackwelder [5] used spectral analysis to determine the wavelength of the disturbance that virtually gave the same estimation as the average wavelength as estimated from the smoke wire visualization of the naturally developed Görtler vortices.

Fig. 1 Sketch of Görtler vortices on a concave surface: (a) Flow configuration, (b) Spanwise distribution of mean streamwise velocity U showing definitions of vortex wavelength λ , downwash (peak), and upwash (valley) regions.

Assuming that only the most amplified vortices will appear in experiment, a method based on the Görtler vortex stability diagram (of Smith [6], for example) can be used to predict the experimental wavelength of Görtler vortices. In this method, the non-dimensional wavelength parameter Λ is defined as:

$$\Lambda = (U_\infty \lambda / \nu) \sqrt{\lambda / R} \quad (2)$$

where λ is the most amplified Görtler vortex wavelength, and Λ represents a family of straight lines which cross the Görtler vortex stability diagram of G_θ versus $\alpha\theta$. ($\alpha\theta$ is called dimensionless wave number and $\alpha = 2\pi/\lambda$). Luchini and Bottaro [7] found that the most amplified wavelengths are for $\Lambda = 220$ to 270 , which agrees well with Floryan [8], Smith [6], and Meksyn [9] who respectively proposed $\Lambda = 210$, 272 , and 227 .

Since Görtler [1] first predicted this vortex phenomenon, there have been numerous works in visualizing naturally developing Görtler vortices. Early visualizations of Görtler vortices were mainly carried out to prove the existence of these vortices, and can be classified into: surface flow visualizations, suspended particle flow visualizations, and electrolytic flow visualizations, which will be briefly reviewed below.

1.1 Surface Flow Visualization

Gregory and Walker [10] were the first to visualize the existence of Görtler vortices using china clay surface flow visualization on the concave (pressure) surface of a Griffith aerofoil. With this method, a solid layer of china clay was spread on the aerofoil surface to form a smooth white absorbent film, and a liquid of the same refractive was spread onto this layer which made it transparent. The white china clay would reappear after evaporation of the liquid had been completed. As turbulent and laminar regions in a boundary layer set a differential rate of evaporation (due to differential rate of heat transfer), the transition line would be indicated quite sharply due to colour contrast between turbulent and laminar regions. McCormack *et al.* [11] also visualized Görtler vortices using this method but used a solution of methyl-alcohol saturated with naphthalene. Since the sprayed solution was removed faster in the region of vortices, distinct longitudinal streaks appeared after some time on the concave surface (of $R = 5$ inches) of the test section after 2 minutes of running at $U_\infty = 22$ ft/s. The average spanwise wavelength was measured to be 0.2 inches which gives $\Lambda = 445$.

Highnett and Gibson [12] observed Görtler vortices as a large number of fine streaks on the pressure (concave) surface of a fan blade by means of dust deposit surface flow visualization. Kemp [13] also used china clay to observe Görtler vortices on the pressure surface of a turbine blade in cascade. The maximum vortex wavelength was about 2.5 mm and the minimum about 0.8 mm. The resulting surface flow pattern was then used to guide a Pitot probe to measure the streamwise velocity across the vortices in the spanwise direction. Kemp [13] also tried the particle deposit surface flow visualization using talcum powder and chalk particles and found that the talcum powder to be most satisfactory.

1.2 Suspended Particle Flow Visualization

Kahawita and Meroney [14] visualized Görtler vortices using smoke in a wind tunnel to study the effect of heating on concave surface boundary layer flow.

Smoke visualization of naturally developed Görtler vortices was also reported by Swearingen and Blackwelder [15], while more extensive visualization study had been conducted by Ito [16, 17] who injected smoke from the slits on a concave surface of $R = 1.0$ m, as well as used smoke wire method for $U_\infty = 2.5$ m/s. The development and breakdown of Görtler vortices are shown in the cross-sectional and top views.

Petitjeans *et al.* [18] visualized Görtler vortex structures in liquid flow by using laser induced fluorescence and found that the height of the mushroom-like structures observed can still increase as the amplitude of the instability decreases under the non-linear saturation.

Fig. 2 Sketch of the wind tunnel, curved test section and concave test surface (all dimensions are in mm).

1.3 Electrolytic Flow Visualization

Visualization by using electrolytic method: tellurium and hydrogen bubble techniques had been respectively tried by Wortmann [19] and Bippes and Görtler [20].

Wortmann [19] visualized Görtler vortices along a concave surface of 20 m radius of curvature in a curved rectangular water tunnel of 0.2 m x 0.8 m cross-section using tellurium flow visualization method, which is similar to the hydrogen bubble method. The working fluid is water with addition of a small quantity of potassium chloride (KCl) to increase its conductivity. Two electrodes are introduced with a thin wire of tellurium as the cathode placed in the flow region under investigation. By applying a voltage between the electrodes, tellurium ions (with a double negative charge) are separated from the surface of the cathode. As oxygen is present in the working fluid, the tellurium ions are brought into a state of colloidal suspension that appears in the form of a black dye in the fluid. However, the required voltage increases with decreasing conductivity of the working fluid.

Bippes and Görtler [20] used hydrogen bubble technique to visualize Görtler vortices in boundary layer on a concave surface of $R = 1.0$ m in a water tunnel. This work was the first attempt to measure the normal and spanwise velocity components of Görtler vortices by using photogrammetric evaluation of stereographs. Winoto and Crane [21] also visualized Görtler vortices using hydrogen bubbles in a small curved water channel to supplement their laser Doppler anemometer measurements.

Since the wavelengths of naturally developed Görtler vortices are not uniform, Peerhossaini and Bahri [22], Ajakh *et al.* [23], Toe *et al.* [24] and Mitsudharmadi *et al.* [25] used a series of thin wires placed upstream and perpendicular to concave surface leading edge to pre-set or “force” the wavelength of Görtler vortices. The resulting wavelengths of Görtler vortices are uniform and equal to the spanwise distance of the wires. Peerhossaini and Bahri [22] and Ajakh *et al.* [23] also visualized the pre-set wavelength Görtler vortices by using smoke injected from the concave surface and showed the formation of the mushroom-like structures, indicating the appearance of the secondary instability varicose mode.

In this work, the developments of pre-set most amplified wavelength Görtler vortices in concave surface boundary-layer flows were studied by means of hot-wire anemometer measurements. The vortex structures were visualized by plotting the contours of the streamwise velocity component data obtained from extensive hot-wire measurements.

2. Description of Experiment

The experiments were carried out in curved perspex 60° and 90° bend test sections (ducts), connected to a low speed, blow down type wind tunnel. A smooth concave test surface is mounted inside the curved test section at 0.05 m from its bottom surface (Fig. 2): the test surface with $R = 2$ m in the 60° bend and the test surface with $R = 1$ m in the 90° bend. Each test section is of rectangular (0.15 m x 0.60 m) cross-section, and 13 vertical wires of 0.2 mm diameter are placed at 10 mm prior and perpendicular to the concave surface leading edge to pre-set the Görtler vortex wavelength. The Reynolds number based on wire diameter Re_d is 28 for $U_\infty = 2.1$ m/s and 40 for $U_\infty = 3$ m/s. Hence, for both cases, the von Kármán vortex streets were not generated behind the wires.

Since the most amplified wavelength of the vortices can be predicted using the Görtler vortex stability diagram by setting Λ in the range of 200 – 270, the spanwise distance between the wires was first set at 15 mm for $U_\infty = 2.1$ m/s ($R = 1$ m) and $U_\infty = 3.0$ m/s ($R = 2$ m) so that $\Lambda = 249$ and 252 respectively. The free-stream turbulence level in the test section is about 0.35 % for U_∞ ranging from 1.0 to 6.0 m/s.

Fig. 3 Mean velocity (U/U_∞) contours in y - z plane showing the development of Görtler vortices in boundary layer flow on concave surface of $R = 2$ m for $U_\infty = 3$ m/s and wire spacing of 15.0 mm.

A single hot-wire probe was used to traverse along a spanwise distance of 60 mm to obtain the streamwise mean and fluctuating velocity component values. The probe was operated in a Constant Temperature Anemometer (CTA) mode, which was coupled to a signal conditioner. The signal was low passed filtered at 3000 Hz and sampled at 6000 Hz for the duration of 21 seconds. The data collected were digitized using an analog to digital (A/D) converter card DT3016 installed in a personal computer, and were further analyzed using the HPVEE software.

A calibrated pressure transducer (against a micro-manometer) was used together with a Pitot-static tube placed in the free-stream region for hot-wire calibration. During velocity measurements, the Pitot-static tube was positioned in free-stream region to monitor the local free-stream velocity. At the end of data acquisition, the probe was placed in the free-stream region and calibration was re-checked. A more than 2 % drift for the sensor was not acceptable and the data would be rejected, resulting in the calibration and data acquisition process to be repeated.

The hot-wire probe and Pitot-static tube were mounted on a traversing mechanism. Two stepper motors controlled the movement of the mechanism along the normal (y) and spanwise (z) directions with an accuracy of ± 0.01 mm. Measurements in

the y-z plane were done with the step size of 1.0 mm along the z-direction, while along the y-direction the step size was 0.5 – 1.0 mm depending on the boundary-layer thickness at the measurement station. Reference spanwise position $z = 0$ is in the middle of y-z plane in the measurement domain.

3. Results and Discussion

The development of Görtler vortices on concave surface of $R = 1$ m and 2 m are quite similar, that is, started with linear region, non-linear region, and then followed by the decay of mushroom structures prior to turbulence. The main differences are on the location of the onset of linear and non-linear regions which are related to the growth rate of the vortices as shown by the maximum value of streamwise disturbance amplitude κ_{umax} (see Eqn. (3)). Hence, discussion of the mean velocity contours and velocity profiles showing the development of Görtler vortices is focused on the case of $R = 2$ m only, and analysis on vortex growth rates will be further discussed for both $R = 1$ m and 2 m.

Figure 3 shows the U/U_∞ contours in cross-sectional (y-z) plane at some streamwise (x) locations. For reference, the corresponding Blasius boundary-layer thickness δ is indicated on the ordinate for each streamwise location. The contours were obtained from the data using a commercial “Tecplot” software. Figure 3(a) shows that at $x = 200$ mm ($G_0 = 2.393$), the contours are wavy in the spanwise direction, indicating the occurrence of Görtler vortices at this location. The waviness becomes more pronounced as the flow develops downstream indicating amplification of the vortices, for example, at $x = 500$ mm (Figs. 3(c)) and when transform into horseshoe vortices at $x = 600$ mm (Figs. 3(d)). The transformation of the horseshoe vortices, which propagate downstream, into mushroom-like structures before breaking down, is the consequence of the non-linear growth of Görtler vortices [26]. It shows the occurrence of the varicose mode instability as a secondary instability. As shown in the figures, the mushroom-like vortices (as the spanwise structures) are clearly shown at $x = 700$ mm (Fig. 3(e)) until $x = 852$ mm (Fig. 3(h)). The vortices start to decay at $x \approx 800$ mm, however at $x = 904$ mm (Fig. 3(i)) the mushroom shapes are still maintained. The breakdown of the spanwise structures could be attributed to the increase of mixing due to the onset of turbulence. The mushroom-like structures shown in Figs. 3(e)-(i) reveal the structures of finite amplitude Görtler vortices and the streamwise region in which the mushroom-like structures are coherent and dominating the flow. The development of the mushroom-like structures is caused by the strong non-linearity in the cross-sectional (y-z) plane [26]. Close to the “stem” of the mushroom, low momentum fluid is ejected from the wall and returned back to the wall in the region of maximum shear to form the shape of mushroom hat.

The U/U_∞ contours in the x-z plane in Fig. 4 show the formation of streaks of low velocity (upwash) regions in green colour. The high velocity (downwash) regions occur in the area between wires where the boundary-layer thickness is thinner than in the low velocity (upwash) regions that occur downstream of each perturbation wires. These results agree with those of Peerhossaini and Bahri [22] who carried out the experiment for G_0 ranging from 2.0 to 7.7 at $U_\infty = 2.0$ m/s, wire spacing of 30 mm, and $R = 0.65$ m concave surface.

As shown in Fig. 4(d), three different regions can be identified as the 1st, 2nd, and 3rd regions based on the difference in disturbance growth rate. Figure 5 shows the growth rate of the Görtler vortices in term of the maximum disturbance amplitude as defined by Eqn. (3). The smaller growth is initiated at $x = 500$ mm where the inflection point is first observed in the boundary-layer velocity profile at upwash region (Fig. 7(a)). The formation of the inflection point in the velocity profile is an indicator that can be used to predict the onset of non-linear region. Hence, referring to Fig. 4(d), the region that is qualitatively depicted as the 2nd region is considered as the non-linear region in which the primary disturbance grows at a smaller rate than in the linear or the 1st region.

Fig. 4 Mean velocity (U/U_∞) contours in x-z plane on concave surface of $R = 2$ m at $U_\infty = 3$ m/s: (a) at $y = 0.3\delta$, (b) at $y = 0.5\delta$, (c) at $y = 0.75\delta$ and (d) at $y = \delta$, for 15.0 mm wire spacing.

Fig. 5 Streamwise distribution of maximum disturbance amplitude showing the linear and non-linear (exponential) growth of vortices in the boundary layer flow on some concave surfaces of different radius of curvature.

In the 1st region, the width of the low speed streaks increases slowly until $x \approx 500$ mm, which is attributed to the linear growth of Görtler vortices. It is followed by the 2nd region in which the width of the low speed streaks increases more rapidly. The difference in the width growth rate of the low speed streaks could be due to the transformation from the wavy shape into the horseshoe vortices that propagate downstream to form the mushroom-like structures. This transformation is observed to take place at $x \approx 500$ mm, as shown in Fig. 3(c). The width of the low speed streaks that is invariant in the 3rd region indicates the finite amplitude of the disturbances. From the phenomena presented in Figs. 3(a)–(k), it is found that the 3rd region coincides with the region in which the mushroom-like structures are dominant in the boundary-layer flow. This region is followed by the meandering of the vortices, that is, the side way oscillation of the low speed streaks which takes place at x of around 800 mm, as depicted in Figs. 4(a)–(d). This phenomenon indicates the presence of the secondary instability mode called sinuous mode that is believed to lead the flow to turbulence [27].

The streamwise disturbance amplitude that is defined as:

$$\kappa_u(\eta) = \frac{\{U_D(\eta) - U_U(\eta)\}}{2U_\infty} \quad (3)$$

where U_D is mean velocity at downwash, U_U mean velocity at upwash and $\eta = y \sqrt{U_\infty / x \nu}$ represents the dimensionless coordinate normal to the wall, was used by Finnis and Brown [28] to determine the maximum disturbance amplitude ($\kappa_{u,max}$) at every streamwise (x) location. Figure 5 compares the streamwise variations of $\kappa_{u,max}$ of the present results for $R = 1$ m, of Mitsudharmadi *et al.* [25] for $R = 2$ m, and of Finnis and Brown [29] for $R = 4$ m. Please note that Finnis and Brown's results [29] are for naturally developing Görtler vortices in the linear region at $U_\infty = 7.5$ and 10 m/s, while the present results and those of Mitsudharmadi *et al.* [25] are for pre-set most amplified wavelength Görtler vortices. The slopes of straight lines in the semi-log chart, as shown in Fig. 5, correspond to the constant values of vortex growth rate β assumed in the normal-mode analysis [28].

Comparing with the phenomenon of $R = 2$ m depicted in Fig 4(d), it is shown that $\kappa_{u,max}$ in the 2nd region increases exponentially with growth rate lower than that for the 1st region. This could be attributed to the occurrence of the 3rd region in which the finite amplitude of the disturbance is reached and the flow is dominated by the mushroom-like structures. This region is followed by the region at which $\kappa_{u,max}$ decreases exponentially that shows the saturation of the amplitude disturbance after the finite amplitude of the disturbance is reached. This agrees with Schmid and Henningson [30] who stated that once the disturbance has reached a finite amplitude at which the boundary-layer flow is dominated by the mushroom-like structures, it often saturates and transforms the flow into a new possibly steady state in which secondary instabilities can grow. However, the spectrum analysis of the fluctuating component u' shows that the secondary instabilities arise prior to the non-linear saturation as exhibited by the formation of the peak in the spectrum with a band between 50 to 200 Hz at the streamwise location where the amplitude of the disturbance is still developing.

As shown in Fig. 5, the linear region for $R = 1$ m, $R = 2$ m, and $R = 4$ m respectively range from 150–350 mm, 200–500 mm, and 356–980 mm. The linear region is shorter for shorter (stronger) radius of curvature. The slopes obtained from the figure (which corresponds to the vortex growth rate β) are respectively 7.42 m^{-1} , 4.74 m^{-1} , and 3.71 m^{-1} for $R = 1$ m, 2 m, 4 m at $U_\infty = 7.5$ m/s and 3.78 m^{-1} for $R = 4$ m at $U_\infty = 10$ m/s. Based on Finnis and Brown's [29] results, there is almost no significant effect of free-stream velocity to the vortex growth rate as long as the radius of curvature is constant. However, the vortex growth rate increases significantly for shorter radius of curvatures.

Fig. 6 Distribution of maximum disturbance amplitude against $G_\theta^{4/3}$ on some concave surfaces of different radius of curvature.

The non-linear development of Görtler vortices for concave surfaces of $R = 1$ m and 2 m ranges from 350-850 mm and 500-805 mm, respectively. A smaller radius of curvature has a longer non-linear region than a larger one. However, the streamwise locations where the flow becomes turbulence are about the same for both cases, that is, at about $x \approx 850$ mm. The slope of the line that corresponds to the vortex growth rate for non-linear region is 0.59 for $R = 1$ m, and 0.74 for $R = 2$ m.

From the definition of Görtler number G_θ as given by Eqn. (1) and the expression of boundary layer momentum thickness from Blasius solution ($\theta = 0.664\sqrt{vx/U_\infty}$), it can be shown that $x = G_\theta^{4/3}$. Replacing the streamwise position x in the Fig. 5 with $G_\theta^{4/3}$ gives an interesting plot as shown in Fig. 6. All data points of $\kappa_{u,max}$ for pre-set wavelength Görtler vortices seem to lie on a single line. Similarly, all data points of $\kappa_{u,max}$ for naturally developing Görtler vortices of Finnis and Brown [29] also lie on another single line. Finnis and Brown [29] mentioned that it was not clear whether this was a coincidence or whether the vortex maximum amplitudes would also lie on the same line. Based on the present results, it is believed that all data points of $\kappa_{u,max}$ obtained from the same experimental set-up will lie on a single line, regardless of the values of U_∞ and R .

It is interesting to note from Fig. 6 that the onset of linear and non-linear regions for both $R = 1$ m and 2 m occur at about the same Görtler number G_θ of respectively about 2.5 and 4.8. The maximum disturbance amplitude $\kappa_{u,max}$ at the onset of linear and non-linear regions is also about the same, that is, about 0.058 and 0.246, respectively. The maximum disturbance amplitude before meandering of secondary instability is about 0.32 regardless of the concave surface radius of curvature. It agrees with Winoto and Crane [21] who reported that this value is in the range of 0.3 to 0.4.

Fig. 7 Mean velocity (U/U_∞) profiles across boundary layers on concave surface of $R = 2$ m for $U_\infty = 3$ m/s at some streamwise (x) locations.

Fig. 8 Boundary layer flow on concave surface of $R = 2$ m at $U_\infty = 3$ m/s: (a) Spanwise distributions of U/U_∞ at $y/\delta = 0.5$ and $x = 500, 600, 700, 745, 805$ and 852 mm, (b) U/U_∞ contour at $x = 805$ mm ($G_\theta = 6.871$).

Another indicator to locate of the start of non-linear region is the formation of an inflection point in the boundary-layer velocity profile at upwash region. The horseshoe vortices as the secondary instability of Görtler vortices are known to be triggered by the high shear layer formed near the edge of the boundary-layer [8]. This high shear layer is a consequence of the formation of the inflection point in the velocity profile. The plots of U/U_∞ against η show that the inflection point is initially detected at $x = 500$ mm (Fig. 7(a)). Accordingly, the non-linear growth of the vortices in the present study for $R = 2$ m is likely to start at x of around 500 mm as stated earlier.

Referring to Fig. 5, the larger amplitude of the disturbances in the non-linear region will lead to point of inflection in the upwash velocity profiles as shown in Fig. 7(a). This agrees well with the results of Wortmann [19], Aihara and Koyama [31], and Swearingen and Blackwelder [15].

Fig. 9 Distribution of turbulent intensity (Tu) across boundary layers on concave surface of $R = 2$ m for $U_\infty = 3$ m/s at some streamwise (x) locations.

Fig. 10 Mean velocity (U/U_∞) contours in x-z plane on concave surface of $R = 2$ m at $U_\infty = 3$ m/s: (a) at $y = 0.3\delta$, (b) at $y = 0.5\delta$, (c) at $y = 0.75\delta$ and (d) at $y = \delta$, for 30.0 mm wire spacing.

Fig. 11 Mean velocity (U/U_∞) contours in x-z plane on concave surface of $R = 2$ m for $U_\infty = 3$ m/s: (a) at $y = 0.3\delta$, (b) at $y = 0.5\delta$, (c) at $y = 0.75\delta$ and (d) at $y = \delta$, for 7.5 mm wire spacing.

Associated with the non-linear growth region of Görtler vortices, the spanwise variation of the streamwise velocity becomes flattened at the high velocity (downwash) region, and narrow and sharp at the low velocity (upwash) region [31] as shown in Fig. 8(a) at $x = 500$ mm – 852 mm at a distance from the wall y of about 0.5δ (where δ is the Blasius boundary-layer thickness). These phenomena give rise to the formation of the mushroom-like structures observed in the velocity contours. Figure 8(b) shows these structures at $x = 805$ mm, where the dash lines indicates the direction of the fluid particles movement to form the streamwise counter-rotating vortices.

The formation of high shear layer near the boundary-layer edge causes the formation of the second peak at that region, as can be seen in the turbulence intensity profile at the upwash region which was first observed at $x = 500$ mm, as shown in Fig. 9(a) [25]. The formation of high shear layer initiates the development of horseshoe vortices as secondary instability [8]. As these

Fig. 12 Mean velocity (U/U_∞) contours in y - z plane showing the development of Görtler vortices in boundary layer flow on concave surface of $R = 2$ m for $U_\infty = 3$ m/s and wire spacing of 30.0 mm.

horseshoe vortices propagate downstream and breakdown, the turbulence spreads out in the boundary-layer resulting in the decay of the second peak near the boundary-layer edge until the turbulence near the wall becomes dominant in the flow. The decay process of the second peak is related to the breakdown of vortex structure in the boundary-layer flow (Fig. 9(a)).

In the linear region (for $x < 500$ mm on the test surface of $R = 2$ m), the initial developments of the Görtler vortices for wire spacings of 7.5 mm and 30.0 mm are similar to those reported by Mitsudharmadi *et al.* [25] and [32] for wire spacing of 15.0 mm. A spanwise modulated streamwise velocity U was observed with the wavelength equal to the spanwise spacing of the wires as shown by the formation of the low speed streaks behind each wire in Figs. 10 and 11, where the mean velocity contours in the x - z plane at 4 different y locations of $y/\delta = 0.3, 0.5, 0.75$ and 1.0 (where δ is the Blasius boundary layer thickness) are presented for both wire spacing, respectively.

Figures 10(a)-(d), clearly show the splitting of vortices that is manifested in the formation of low speed streaks in the space between the wires at the y locations ranging from $y/\delta = 0.3$ to $y/\delta = 1.0$. These low speed streaks occur in the non-linear region of Görtler vortex flow where the formation of the horseshoe vortices are clearly depicted in the mean velocity contours in the y - z plane (Fig. 12). The formation of the low speed streaks in the spacing between the wires as depicted in Fig. 10 coincides with the formation of the secondary peaks, which evolve downstream to form the secondary mushroom-like structures as shown in Figs. 12(d)-(h).

The merging process is not as visually clear as the splitting. As shown in Figs. 11(a)-(d), the merging of the low speed streaks is evident only at z of about -25 mm and 27 mm for y of 0.3δ to 0.5δ . The mean streamwise velocity contours in the y - z plane are presented in Fig. 13 to show the merging of the vortices, which could be attributed to the coalescence of equal sign vorticity regions as also observed by Bottaro and Zebib [33] in their computational study of Görtler vortices promoted by ribletted surfaces. This phenomenon results in the formation of the coherent structure of larger wavelength that can be amplified. It also gives rise to the increase of disturbance energy which can be attributed to the occurrence of the linear instability of the vortices with respect to spanwise perturbation (Eckhaus instability) as described by Guo and Finlay [34] and Bottaro [35].

Figure 13 also shows that as streamwise distance x increases downstream, there is a shift of the mushroom-like structures locations, especially for those located far from the center. It is evident that for the measurements close to the leading edge, the mushroom-like structures are formed behind the vertical wires. Further downstream, the structures at the right and left sides are shifting towards the center, while the structure at the center remains at its position. This irregular behavior is also suspected due to the occurrence of the Eckhaus instability.

Fig. 13 Mean velocity (U/U_∞) contours in y - z plane showing the development of Görtler vortices in boundary layer flow on concave surface of $R = 2$ m for $U_\infty = 3$ m/s and wire spacing of 7.5 mm.

Fig. 14 Comparison of mean velocity (U/U_∞) contours at $x = 700$ mm ($G_\theta = 6.87$) and $U_\infty = 3.0$ m/s for wire spacings of (a) 7.5 mm, (b) 15.0 mm, and (c) 30.0 mm.

Fig. 15 Görtler vortex stability diagram based on Blasius mean flow comparing some experimental results.

In the Görtler vortex stability diagram of G_θ versus $\alpha\theta$ of Smith [6] shown in Fig. 15, the experimental data for the vortices of $\lambda = 15$ mm are well inside the unstable region in the diagram (where $\alpha\theta$ is called dimensionless wave number and $\alpha = 2\pi/\lambda$). The non-dimensional wavelength Λ defined by Eq. (2) represents a family of straight lines on the stability diagram, which can be used to predict the wavelength of naturally developing Görtler vortices. The lines $\Lambda = 88$ and 732 in the diagram represent the development of pre-set wavelength Görtler vortices for wire spacing of 7.5 mm and 30 mm respectively at $U_\infty = 3$ m/s. The experimental results show that in the linear region, these pre-set vortex wavelengths are maintained. The merging and splitting of the vortices start after the linear region, and the vortex developments according to the lines $\Lambda = 88$ and 732 will then “switch” to the line $\Lambda = 249$. These confirm that $\lambda = 15$ mm (which corresponds to $\Lambda = 249$) is the most amplified vortex wavelength that will be maintained downstream prior to turbulence for laminar boundary layer on concave surface of $R = 2$ m at $U_\infty = 3$ m/s, as reported earlier by Mitsudharmadi *et al* [25].

4. Conclusions

From hot-wire anemometer data, the flow field of streamwise counter-rotating Görtler vortices with pre-set wavelength has been visualized prior to turbulence. The vortices were amplified until $G_\theta = G_{\theta tr}$ was reached where the start of boundary-layer transition occurred (see Mitsudharmadi *et al.* [25], for the details). This study of pre-set wavelength Görtler vortices has provided more information on the development stages of such vortices. The pre-set vortex wavelength is found to be equal to the spanwise spacing of the vertical wires along the streamwise direction until the vortices breakdown prior to turbulence when the dimensionless wavelength parameter was set in the range of 200 to 270 .

The mean streamwise velocity contours for 15 mm wire spacing on the x - z and y - z planes show that the spanwise modulation of streamwise velocity, which indicates the vortex wavelength, is equal to the spanwise spacing of the wires. The mean velocity contours in the y - z plane shows the non-linear region of the vortices where the boundary-layer is dominated by the mushroom-like structures that have reached finite amplitude. This region is followed by the meandering of the vortices which indicates the presence of sinuous mode secondary instability that rapidly leads the flow to turbulence.

The streamwise variations of maximum disturbance amplitude κ_{umax} for different concave surface radii curvature show that shorter radius of curvature results in higher growth rate in the linear region and lower growth rate in the non-linear region. If κ_{umax} is plotted against $G_\theta^{4/3}$, all the data from the same experimental set-up follow a single line, and the onset of linear (Görtler instability) and non-linear regions for pre-set wavelength Görtler vortices occur nearly at the same points.

The formation of the second peak near the boundary-layer edge in the turbulence intensity profiles at the upwash region shows the formation of high shear layer near the boundary-layer edge. As the turbulence spreads out in the boundary-layer, the second peak near the edge of the boundary-layer decays, which coincides with the breakdown of the vortex structure in the boundary-layer.

Study on the effect of wire spacing to the formation of pre-set wavelength Görtler vortices shows the splitting and merging in the non-linear region of Görtler vortex flow. This may explain the merging of the naturally developed Görtler vortices observed by Inagaki and Aihara [36], and demonstrates the occurrence of the linear instability of the vortices with respect to spanwise perturbation (Eckhaus instability) as numerically investigated by Guo and Finlay [34, 37].

Nomenclature

d	Vertical wire diameter [m]	α	Wave number ($\equiv 2\pi/\lambda$) [m^{-1}]
G_θ	Görtler number	$\alpha\theta$	Dimensionless wave number
R	Radius of curvature [m]	β	Vortex growth rate [m^{-1}]
Re_d	Wire Reynolds number ($\equiv U_\infty d/\nu$)	δ	Blasius boundary layer thickness [m]
Tu	Turbulence intensity	η	Dimensionless normal coordinate ($\equiv y(U_\infty/x\nu)^{1/2}$)
U	Streamwise mean velocity [m/s]	κ_{umax}	Maximum disturbance amplitude
U_∞	Free-stream velocity [m/s]	λ	Vortex wavelength [m]
U_U	Streamwise mean velocity at upwash [m/s]	Λ	Non-dimensional wavelength parameter
U_D	Streamwise mean velocity at downwash [m/s]	ν	Fluid kinematic viscosity [m^2/s]
x	Streamwise coordinate [m]	θ	Blasius boundary layer momentum thickness [m]
y	Normal coordinate [m]		
z	Spanwise coordinate [m]		

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