Original Paper

Analytical Study on Stall Stagnation Boundaries in Axial-Flow Compressor and Duct Systems

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Abstract

Stall stagnations in the system of axial-flow compressors and ducts occur in transition from deep surge conditions to decayed or converged stall conditions. The present study is concerned with the boundaries between the deep surges and the stagnation stalls on the basis of analytical results by a code on surge transients analysis and simulation. The fundamental acoustical-geometrical stagnation boundaries were made clear from examinations of the results on a variety of duct configurations coupled with a nine-stage compressor and a single stage fan. The boundary was found to be formed by three parts, i.e., B- and A-boundaries, and an intermediate zone. The B-boundary occurs for the suction-duct having a length of about a quarter of the wave-length of the first resonance in the case of very short and fat plenum-type delivery duct. On the other hand, the A-boundary occurs for the long and narrow duct- type delivery flow-path having a length about a fifth of the wavelength and relatively small sectional area in the case of short and narrow suction ducts. In addition to this, the reduced surge-cycle frequencies with respect to the duct lengths are observed to have respective limiting values at the stagnation boundaries. The reduced frequency for the B-boundary is related with a limiting value of the Greitzer's B parameter. The tendency and the characteristic features of the related flow behaviors in the neighborhood of the boundaries were also made clearer.

Keywords: Compressor, surge, stall, stall stagnation, unsteady flow, flow oscillation

1. Introduction

Stagnation stall phenomena have been paid attention to in recent years (for example, Ishizawa [1]). It is a situation in which a jet engine could be recovered by no means to its sound operating condition when the engine happens to stall during the flight by some unexpected causes, for example, sudden encounter with gust, hard ignition of after-burners, etc. It could be recovered only by turning off the engine and restarting it in the air. It is considered to be a situation affected by both the stall characteristics of high-pressure-ratio compressors and the flow-path features.

When a high-pressure-ratio compressor stalls, it tends to yield a deep surge, which shows a large flow-pressure loop ranging from the normal flow condition to a reversed flow condition as shown in Fig. 1(a). It repeats the process of emptying and filling of a large amount of air contained in nearly the whole flow-path. In such a situation, some means to take advantage of the large flow duration could bring about a chance of re-stabilization of the engine.

Fig. 1 Pressure-flow behaviors in the situation of deep surge and stagnation stall

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In some circumstances, however, the compressor happens to settle down onto a rotating-stall branch of the characteristics as shown in Fig. 1(b). Greitzer [2] called the situation as rotating stall. The situation is now often called stagnation stall, in which the engine could have fallen into an unrecoverable situation.

In this context, the stagnation stall is often called non-recoverable stall or hung stall. In contrast to that, the deep surge is called recoverable stall. The boundary between the deep surges and the stagnation stalls are named here stall stagnation boundary. Further information about the stall stagnation boundary and the behaviors, and the features of various phenomena near the boundary could contribute much to the reliability of the system.

In addition to that, the author has often felt that behaviors of the surges experienced in laboratory tests and on sites could sometimes be more or less different. It is unfavorable in the situation where the post-stall behaviors are becoming increasingly important. This study has been conducted with attention to the stall stagnation boundary and related phenomena, reflecting such experiences.

With respect to the stall stagnation phenomena, Greitzer [2, 3] conducted a basic study in 1976. He has simplified the flow path into three parts, i.e., the upstream suction duct and the compressor, the delivery plenum, and the exit-valve passage. In each part, equations of motion or mass continuity were formulated and solved numerically. The Greitzer's model, though very simple, explains well the behaviors of surges in compressor duct systems. The results have proved that the so-called B parameter, $B.F.$, is an essential dominating parameter in the surge behaviors.

$$B.F. = u_m / (4\pi f_0 L_c) \quad (1)$$

Here, u_m : mean peripheral speed of blades, L_c : length of the suction duct including the compressor, and f_0 : dominating frequency of the system, for which Greitzer [2] has employed the Helmholtz resonator frequency with respect to the delivery plenum.

$$f_0 = (a/2\pi) \sqrt{(A_c/V_p L_c)} \quad (2)$$

Here, a : speed of sound, A_c : sectional area of the suction duct including the compressor, and V_p : plenum volume. Finally, the following formula has been proposed as the Greitzer's B parameter:

$$B.F. = (u_m/2a) \sqrt{(V_p/A_c L_c)} \quad (3)$$

The results for changing $B.F.$ given by Greitzer [3] show that deep surges occur for large $B.F.s$ and stagnation stalls for small $B.F.s$. From the results of analyses and the experiments, He suggested the following values of $B.F.$ as the stall stagnation boundary:

$$B.F.=0.7-0.8 \quad (4)$$

The B parameter has been often employed for various purposes, for example, in the basic equations for active stall control of compressor systems.

Post-stall behaviors of compressors have been paid attention to and studied from the aspects of safety in recent jet engines and gas turbines. In the circumstances, the surge behaviors in the compressor-duct systems are also important. However, relatively few studies about the stall stagnation phenomena, its boundary, its features, etc. appear to have been conducted subsequently after the achievements by Greitzer [2, 3]. Further information is now desired about the phenomena.

The author studied about the stall stagnation boundaries and the related phenomena on the basis of analytical results on surge behaviors. The tool for the analysis is a one-dimensional code named SRGTRAN developed by the author, which treats the compressor and flow-path system as consisted of a number of control volumes (Yamaguchi [4-7]). This report outlines the results.

2. Outline of SRGTRAN

SRGTRAN is a code for Surge Transients Analysis and Simulation and the details can be found in Yamaguchi [4-7]. The compressor is approximated by control volumes for respective stages, in which one-dimensional equations of conservation of mass, momentum, and energy are applied and solved by use of two-stage Lax-Wendroff method (Ref.[8]) for each stage. The stage conditions are estimated for the steady-state stage performance characteristics, from which the steady-state axial forces and works of the stage are evaluated. Then they are converted to the respective unsteady values in consideration of time lags. The flow-path zones are divided into a sufficient number of small control volumes, in each of which the method of characteristics (Tanahashi [9]) is applied. The code treats only the axial velocity components with no considerations on the swirling components and rotating stall phenomena.

Time is expressed by the following equation;

$$t = k\Delta t \quad (5)$$

Here, k : time number, and Δt : time step. The time step is determined to satisfy the Courant-Friedrichs-Lewy condition (Ref. [10]).

SRGTRAN could express the system of a compressor and flow-paths by a practically reasonable distributed-parameter system, in which the details of the system could well be taken into account in the analysis.

In addition to that, it can take into account the effect of compressor pressure ratios.

Table1 Specifications of the compressors for the analysis and the reference conditions

	Comp19	Comp11
Design-condition		
Stages	9	1
RPM	11300	11300
Flow rate (m ³ /s)	11.5	11.5
Pressure ratio	3.82	1.17
Ref. values		
Tip and hub diameters	0.508 0.356	0.508 0.356
D_t, D_h (m)	(Stage 1)	(Stage 1)
Area A_{c2} (m ²)	0.04125	0.0975
Length L_{c^*} (m)	4.29	3.67
RPM for analysis (%design speed)	10,000 (88)	12,000—7000 (106—62%)
Stage characteristics	Identical for all stages	

Although it has not finished an experimental validation by checking with actual surge behaviors of high-pressure-ratio compressors owing to lack of available consistent data, some trial analyses have suggested that the results could suitably describe and explain surge behaviors in accordance with our experiences (Yamaguchi [6,17-19]).

3. Compressors for the Analyses

Two hypothetical axial flow compressors were employed for the analyses: Comp19 and Comp11. Main dimensions of them are given in **Table 1**. Comp19 has nine stages of constant design flow and pressure coefficients, designed for 11300 rpm and tip speed of 300m/s. The surge analyses were conducted at 10000rpm (88 % of the design rpm) where the first stage is most critical to stalling and at the same time, all other stages also are rather near stalling. Other rpms were not studied here because of the possible significant effects of mis-matchings among the stages on the surge behaviors, which should be examined in other places (for example, Yamaguchi [18, 19]). Comp11 is a single stage fan using only the first stage of Comp19. Surge analyses were conducted at 12000, 10000, and 7000 rpms (106, 88, and 62 % of the design rpm, respectively).

Figure 2 shows the assumed stage characteristics of pressure coefficient ψ_P and temperature coefficient ψ_T versus stage flow coefficient ϕ_t , which are normalized with respect to the tip peripheral speed u_t of the first stage rotor. It has a characteristic behavior of abrupt stalling and unstalling showing a stall hysteresis, which follows the local load curves proportional to the flow rate squared. For the present study, the same characteristics are applied to all stages for Comp19 and Comp11.

It requires a very wide range of flow coefficients from the turbine action zone to the reversed flow zone, information on which are very few, regrettably. The present characteristics has been constructed on the few data given in several literature, for example, Chamblebee, Davies, Jr., and Kimzey[11], Davies, Jr. and O'Brien[12], Gamache and Greitzer[13], Greitzer[3], etc. in addition to the existing data.

Constant values of stage pressure and temperature coefficients given in the far off-design zone in Fig. 2 are set for the convenience of easier convergence in the calculation procedure.

4. Flow-paths for the Analyses

Figure 3 shows schematically a simplified configuration of the system of a compressor and ducts for the analyses. Here, it is to be noticed that the word “delivery flow-path” is used in a generalized sense to express not only the plenum-type delivery duct but also a long and narrow delivery duct.

Representative sizes of the system are as follows; A_p and L_p : sectional area and length of the delivery flow-path, A_{c2} : exit annulus area of the compressor last stage as the representative area of the suction duct, $L_{c^{**}}$: length of the suction duct including the compressor. These sized have been empirically selected to fit relatively well the stall stagnation geometry data from examinations of the results. A_{c2} is considered at the same time to be the most important area dividing the flow-paths upstream and downstream. $L_{c^{**}}$ is the length of the suction flow-path upstream of the A_{c2} section. L_{co} is the length of the compressor bladed path, but it is not used here. The connecting length from the compressor exit to the delivery inlet is assumed here to be very short. If necessary, the connecting volume should be included in the volume of the delivery flow-path.

This flow-path model, though rather near the Greitzer’s simplified model (Greitzer [2]), can change the flow-path configurations of the delivery and the suction. Particularly, it can modify the delivery configuration in terms of A_p and L_p , in contrast to only the plenum volume V_p in the Greitzer’s model. The model could extend the range of research, by taking into account more

Fig. 2 Wide-range stage characteristics of Comp19 and Comp11

Fig.3 Schematic compressor and duct arrangement and important sizes

Fig. 4 Examples of the distributions of duct sectional areas for Comp19

practical situations of the details of the flow-path, if necessary.

Some examples of Comp19 flow-paths are shown in Fig. 4. The suction duct reduces its sectional area gradually up to the compressor inlet. The suction duct and the compressor were fixed the same throughout the present analyses. The delivery flow-paths are enlarged or reduced downstream of the compressor exit, forming a plenum chamber or a narrow duct. The delivery flow-path is terminated by an exit valve.

The present analyses used fourteen control volumes in the suction flow-path, nine control volumes for the compressor, i.e., one for each stage, and thirty-four control volumes for the delivery flow-path. A_{mod2} and X_{mod2} are scale factors for changing sizes of the sectional area and length of the delivery plenum, respectively, in relation with a reference configuration data.

Comp11 uses the sectional area of the first stage exit of Comp19 as A_{c2} and is connected to similarly changing delivery flow-paths.

The compressor operating condition is calculated by steady-state procedure until the initial stalling is reached. At the initial stalling time, the exit valve opening A_{vo} is determined, and the unsteady calculation procedure is started with the valve opening A_v being throttled gradually to the specified opening ratio A_v/A_{vo} .

5. Surge Behaviors for Changing Configurations of the Delivery Flow-path

Surge behaviors of Comp19 and Comp11 for changing configurations of delivery flow-paths, i.e., for changes in A_p and L_p , are shown in Figs. 5-8.

Figures 5 and 6 show time histories of pressure and surge loops of pressure vs. mass flow, respectively, at the exit of the compressor of Comp19 at 10000 rpm. Inserted figures near the center in Figs. 5 and 6 plot the calculation points of L_p vs. A_p .

Here, the fluid motion through the exit valve passage is given by the following equation (Yamaguchi [5]);

$$\Delta L_v \frac{\partial W_v}{\partial t} = A_v (p_{v1} - p_{v2}) - \left(\frac{\beta_v}{\alpha_v} \right) \frac{W_v^2}{\rho_{v1} A_v} \quad (6)$$

Here, W_v : mass flow through the valve passage, p_{v1} and p_{v2} : pressures upstream and downstream of the passage, respectively, ρ_{v1} : density of the fluid upstream of the valve inlet, α_v : steady-state flow coefficient of the valve, β_v : flow adjust in factor (expressed also as $adjVEX$), and ΔL_v : length of the valve-passage length assumed as follows;

$$\Delta L_v = \varepsilon_v \sqrt{A_v} \quad (7)$$

Here, ε_v : valve-passage length factor (expressed also as $fdelexV$).

The results in Figs. 5 and 6 are for the exit valve opening closed up to 50% of the stalling one ($A_v/A_{vo}=0.5$), and for the valve coefficients $\beta_v=2$ and $\varepsilon_v=2$. It is experienced in many preliminary results that these values A_v/A_{vo} , β_v and ε_v do not affect much the tendency of the surge behaviors.

As seen in these figures, both of reduction in the delivery flow-path length L_p for a sufficiently large constant sectional area A_p , and reduction in the area A_p for a sufficiently large constant length L_p , bring about changes from typical deep surge behaviors, through deep surges with superposed higher frequencies, and finally to decaying of surge or stagnation stall conditions. As far as Comp19 is concerned, most stagnation stalls settle down to lower pressure level with superposed oscillations of high frequency and small amplitude. For particular conditions of small A_p and very long L_p , the stagnation tends to settle immediately below the initial stalling point as seen in Fig. 6. It could possibly be affected by the frictional effects along the very long and narrow flow-paths.

Surge behaviors for Comp11 of low pressure are shown in Figs. 7 and 8. The results are for the exit valve opening kept equal to the stalling one ($A_v/A_{vo}=1$), and for the valve coefficients $\beta_v=5$ and $\varepsilon_v=2$. For sufficiently large A_p and L_p , oscillograms show ordinary deep surges of nearly sinusoidal ones. Decreasing A_p and L_p brings about successively double-loop surge behaviors, rather irregular ones, and finally decayed surges or stagnation stalls. In the last situation, the oscillations finally decay completely.

Based on these results, boundary lines could be drawn between the deep surge zone and the stagnation stall zone, as seen by the red chain lines in the inserted figures in Figs. 5-8. The author would like to name the lines "stall stagnation boundary". The boundary is seen to be formed by two parts of different tendency.

6. Geometrical Stall Stagnation Boundary

Figure 9 shows raw data of the geometrical stall stagnation boundaries in terms of the delivery flow-path sizes A_p and L_p for the same conditions of the suction flow-path and the compressor. These data have been obtained from analytical results for the valve conditions, $\beta_v=1$, $\varepsilon_v=1$, and $A_v/A_{vo}=0.9$ and 0.5 , similarly to those in Figs. 5-8. The explanatory note, for example, 9_10000_0.9, means Comp19 at 10000rpm and the exit valve throttled to the opening ratio $A_v/A_{vo}=0.9$. The way of symbol explanation is followed in all the figures below. The influence of the differences in the throttled valve areas on the stagnation boundary is shown. In the case of Comp11, the stagnation boundaries are seen to be slightly different in the zone of smaller L_p for the three different rotational speeds.

As seen in Fig. 9, the geometrical boundaries are located far apart between Comp19 and Comp11. Both boundaries are formed by two or three parts of different tendency, of which one is a negative-sloped line and the other is a line of roughly constant A_p .

Fig. 5 Surge pressure behaviors for Comp19 at 10000rpm, depending on the changing plenum dimensions for a fixed suction duct configuration

Comp19 10000rpm
Vcrat=-0.5
AdjVEX=2 FdelexV=2

Av/Avo=0.5,
 $\beta v=2$, $\epsilon v=2$

Deep Surge

Stagnation Stall

Plenum Length L_p (m)

Plenum Sectional Area A_p (m**2)

PTdelv(1/2)

Wdelv(1/2)

(kg/s) (Wdelv(1/2))

Amod2=-0.02 Xmod2=5

Amod2=-0.02 Xmod2=1

Amod2=-0.024 Xmod2=5

Amod2=0.1 Xmod2=0.13

Amod2=-0.0437 Xmod2=1

Amod2=-0.045 Xmod2=0.6

Amod2=0 Xmod2=0.3

Amod2=0.1 Xmod2=0.1

Comp19 10000rpm
Vcrat=-0.5
Amod2=0.6 Xmod2=0.0125

In the zone of larger sectional area of the delivery flow-path A_p , as suggested by a two-dot chain line in Fig. 9, the boundary could be expressed very roughly by,

$$L_p \propto 1/A_p \quad (8)$$

or in terms of delivery flow-path volume,

$$V_p = A_p L_p \sim \text{constant} \quad (8a)$$

The author would like to name it B-Boundary, since the tendency is related with Greitzer's B parameter as is described later.

In the zone of smaller sectional area A_p and longer length L_p of the delivery flow-path, the boundary appears to be given by roughly constant values of A_p , which are relatively small in the order of magnitude of the compressor exit sectional area A_{c2} . The author would like to name it A-Boundary.

Stagnation boundaries normalized by the typical sizes (refer Table 1 and Fig. 3) are shown in Fig. 10.

Effective delivery area-pressure ratio:

$$APR = (A_p / A_{c2}) PR \quad (9)$$

$$PR = p_{pl} / p_{cl} \quad (10)$$

Length ratio:

$$L_p / L_C^{**} \quad (11)$$

Here, PR : stalling pressure ratio, p_{pl} : delivery pressure at stalling, p_{cl} : suction pressure at stalling. These are obtained at the steady-state stalling point. The effective delivery area-pressure ratio, Eq. (9), was employed empirically from the view of pressure force balances, which could express the boundary tendency to some extent.

Thus the large difference seen in Fig. 9 has diminished and the boundaries have become gathered fairly together between Comp19 and Comp11. At the A-boundary, APR is observed to be given roughly by the following value.

$$APR \sim 2.5 \quad (12)$$

These results suggest the effect of stalling pressure levels on the surge behaviors, which should be taken into considerations for higher pressure ratios.

7. Surge-Related Frequencies

Frequencies related to surges and related phenomena were surveyed in order to investigate the surge frequencies at the stall stagnation boundaries. Figure 11 shows some examples of the results for Comp19 at 10000rpm with delivery flow-paths of changing lengths L_p at some fixed sectional areas A_p . The frequencies were read from analytical surge oscillograms. Solid marks and hollow marks are, respectively, surge-cycle frequencies $fs0$ and superposed higher-frequencies $fs1$ on the surge oscillations. The higher frequencies used to be observed in the neighborhood of the stall stagnation boundaries, as seen in examples in Figs. 5 and 7. With a decrease in the delivery flow-path length L_p , surge-cycle frequencies $fs0$ tend to increase and vanish somewhere, which means the occurrence of stagnation. Then only the higher frequency remains. The frequencies observed at and after the stagnation condition are located on the extension of the curves of the higher frequencies $fs1$ having been superposed on the surge waves.

Figure 11 includes also estimated frequencies $f1$ and $f2$ of the first mode and the second mode of the resonance in the whole duct system, and the resonance frequencies $f0$ of the Helmholtz resonator about the approximated delivery plenum. The surge-cycle frequencies $fs0$ are near the Helmholtz resonator frequencies, and the higher frequencies $fs1$ in the stagnation stalls are near the duct resonance frequencies $f1$. Similar

Fig. 9 Raw data of geometrical stall-stagnation boundaries in terms of delivery flow-path size data for Comp19 and Comp11 for given suction flow-path sizes

Fig.10 Normalized geometrical stall-stagnation boundaries for Comp19 and Comp11

Fig.11 Some examples and tendency of surge frequencies in Comp19-duct systems affected by changes in lengths L_p of delivery flow-path for given areas A_p

tendencies are observed also for changing sectional area A_p for fixed lengths L_p of the delivery flow-paths. Similar tendency is observed also for Comp11.

Here, f_0 is a plenum-resonance frequency estimated according to Eq. (2), where the system is divided into two parts of the suction duct including the compressor as an inlet pipe and the delivery duct as a plenum, and the throat area is regarded as the compressor exit area A_{c2} . On the other hand, f_1 and f_2 are duct-resonance frequencies estimated on the basis of acoustical equations about the duct system. It is considered that the accuracy of f_1 and f_2 is sufficiently high, but that f_0 could have some weakness in the arbitrariness in approximating the Helmholtz resonator.

On the basis of the observations on the flow conditions in the previous section and on the frequency behaviors in this section, the deep surge is considered to be a low-frequency process of repeated evacuation and filling of a vast quantity of air over nearly the whole flow-path, which is a self-excited oscillation near the plenum resonance condition. Stall stagnation is a transition from the deep surge condition to the decaying duct-mode resonance condition, in which the local flow oscillations have to be restricted to small amplitudes.

The $fs1$ data at the stagnation boundaries are plotted versus $f1$ in Fig. 12. The frequencies $fs1$ are seen to be near the frequencies $f1$ of the duct-mode resonance estimated independently. Some points seen above the $f1$ line could possibly be higher mode ones. The duct mode resonance frequency $f1$ could reasonably be employed as the characteristic frequency or the reference frequency in the surge phenomena.

Figure 13 plots the surge-cycle frequencies $fs0$ at the stagnation boundaries versus the reference frequencies $f1$. The upper bound of the frequencies $fs0$ is seen to be given by the reference frequencies $f1$, but large scatterings of the data and deviations from $f1$ are seen in some zone. The same data are given in another expression in Fig. 14, in which relative surge-cycle frequencies ($fs0/f1$) are plotted versus the following parameter of effective plenum volume ratio:

$$RVP = \frac{A_p L_p}{A_{c2} L_c} PR \quad (13)$$

The parameter RVP is derived from the effective area-pressure ratio APR (Eq.(9)) multiplied by the length ratio ($L_p/L_{c^{**}}$)(Eq.(11)). It could be considered very approximately as a mass ratio between the flow-paths of the delivery and the suction.

The relative surge-cycle frequencies ($fs0/f1$) at the stagnation boundaries are seen to be near unity for RVP of either sufficiently large values or sufficiently small ones, which could suggest limiting situations of the A- and B-boundary zones. But some of them tend to decrease much for RVP between about 0.7 and 7 named zone of interaction in Fig. 14. The tendency is significant for Comp19 having higher stalling pressure ratios and for the higher tip speed of Comp11, which suggests the effects of stalling pressure ratios. The large scatterings in $fs0$ seen in Fig.13 could be attributed to this phenomenon.

8. Normalized Stall Stagnation Boundary

From the geometrical data and the frequency data hitherto obtained on the stall stagnation boundary, some non-dimensional parameters are derived for further investigations as follows:

Fig.12 Situations of higher frequencies $fs1$ superposed on the surge cycles in relation with the corresponding reference frequencies $f1$ at the stall stagnation boundary zone

Fig.13 Situations of surge cycle frequencies $fs0$ in relation with the corresponding reference frequencies $f1$ at the stall stagnation boundary

Fig.14 Behaviors of relative surge-cycle frequencies ($fs0/f1$) at the stall stagnation boundaries in relation with the effective relative volumes of delivery flow-path

The wavelength of the duct-mode first-resonance frequency f_1 is used to normalize the lengths of the flow-paths. The reference wavelength is given by using the speed of sound a ;

$$\lambda = a/f_1 \quad (14)$$

The flow-path lengths are normalized as follows;

Relative delivery length:

$$RLP = L_p/\lambda \quad (15)$$

Relative length of the suction duct-compressor flow-path:

$$RLC = L_c^{**}/\lambda \quad (16)$$

The empirical parameter found in Section 5 could be important also.

Effective delivery area-pressure ratio:

$$APR = (A_p/A_{c2})PR \quad (9a)$$

The acoustical-geometrical stagnation boundaries are shown in Fig. 15 in terms of these normalized parameters. The data give the fundamental conditions for the boundary. The top zone and the bottom zone are concerned with APR and RLC , respectively. Both are the requirements for the stall stagnation.

Black data and colored data in Fig. 15 are concerned with Comp19 and Comp11, respectively. The global tendency of the boundaries becomes clear, though some deviations are significant between Comp19 and Comp11 mainly in the intermediate zone.

The stagnation boundary is expressed for respective zones as follows;

$$\begin{aligned} \text{B-boundary: } RLC &\sim 0.25 \text{ and} \\ APR &\sim 1.1/\sqrt{(RLP)} \\ \text{for sufficiently small } RLP \end{aligned} \quad (17)$$

$$\begin{aligned} \text{A-boundary: } RLP &\sim 0.2 \text{ and} \\ APR &\sim 2.5 \\ \text{for sufficiently small } RLC \end{aligned} \quad (18)$$

Qualitative sketches of the flow-path configurations for near the B-boundary and the A-boundary are given in Figs. 16 and 17, respectively.

In the yellow-colored zones enclosed by the stagnation boundary lines occur deep surges, in which the point of coordinate (RLP, RLC) should lie in the lower yellow zone and, at the same time, the point of (RLP, APR) should lie in the upper yellow zone. If both or either of the points (RLP, RLC) and (RLP, APR) lie outside of the corresponding yellow zones, stagnation stalls could occur.

Qualitatively, stagnation stalls occur when the length of either the suction duct-compressor or the delivery flow-path is longer than roughly $(\lambda/4 \text{ or } \lambda/5)$, or when the delivery duct sectional area is relatively small. In the situation, the air mass could hardly move as a one-body in the responsible part of the flow-path, which tends to suppress the occurrence of strong oscillations of Helmholtz resonator type observed in the deep surges. A quarter or a fifth of the reference wave-length might be a reasonable measure of reference to understand the acoustical phenomena.

Figure 18 shows the stall stagnation boundaries, same as those in Fig. 15, versus effective delivery volume ratio RVP . It is easy to overview the global behaviors of the related parameters. The boundaries could be classified roughly as follows;

$$\begin{aligned} \text{B-boundary} & RVP < \sim 0.7 \\ \text{Intermediate zone of stagnation boundary} & \sim 0.7 < RVP < \sim 7 \\ \text{A-boundary} & RVP > \sim 7 \end{aligned} \quad (19)$$

The behaviors of the stagnation boundary in the intermediate zone in Figs. 15 and 18 appear to be reflected by comparable influences of both of the suction flow-path and the delivery flow-path, and particularly the effect of the levels of stalling pressure ratios.

Fig. 15 Acoustical-geometrical conditions for the stall-stagnation boundary in terms of flow-path lengths relative to the fundamental wavelength, and the effective delivery area ratios between the delivery and the compressor-duct. The yellow-colored zones indicate the required conditions for occurrence of deep surges.

9. Reduced Frequencies about Surge Cycles

Attentions are paid to the following surge-cycle reduced frequencies to know the situations from the point of view to the fluid oscillation nature at the stall stagnation boundaries.

With respect to the delivery flow-path length:

$$f_{RP} = \frac{L_p f_{S0}}{u_t} \quad (20)$$

With respect to the length of the suction flow-path and the compressor:

$$fRC = \frac{L_C^{**} f_{s0}}{u_t} \quad (21)$$

These reduced frequencies indicate a measure of repeated surge cycles during the time required by the flow passing through the respective flow-path.

In relation with the above, the B parameter $B.F.$ of Greitzer[2] is proportional to a reciprocal of a reduced frequency similar to the above fRC , which is related through Eq. (1), as follows;

$$fRC^* = \frac{L_C^{**} f_0^*}{u_t} = \frac{u_m / u_t}{4\pi B.F.} \quad (22)$$

Here, fRC^* : reduced frequency corresponding to $B.F.$, um : mean blade peripheral speed of the compressor, and $f\theta^*$: Helmholtz resonator frequency. Greitzer [2] employs the Helmholtz resonator frequency as the representative one. If $f\theta^*$ is replaced by the surge-cycle frequency $fs\theta$, then fRC^* is equal to fRC .

Figure 19 shows the behaviors of the surge-cycle reduced frequencies at the stall stagnations against the effective delivery volume-pressure ratio. It shows following tendency in the respective zones;

the B-boundary zone
 $f_{RC} \sim 0.2-0.4$
 f_{RP} : very small

(23)

For the A-boundary zone

$$f_{RP} \sim 0.2-0.45 \quad (24)$$

From these observations, it could be said that some phenomena related with the suction duct are dominating the B-boundary zone and other phenomena related with the delivery flow-path are dominating the A-boundary zone.

In addition to that, the red-colored horizontal chain-line means fRC^* corresponding to Greitzer's B parameter $B.F.$ of 0.8, which uses f_0 in place of the Helmholtz resonator frequency in the original Greitzer's formula, Eq. (3).

For the present compressors,

$$u_m/u_t = 0.85.$$

From these conditions,

$$fRC^* = 0.085 \quad (25)$$

Thus, the limiting B parameter is seen to be roughly of the same order of magnitude as the fRC in the intermediate zone.

In the intermediate zone, both reduced frequencies tend to drop abruptly and become of comparable order of magnitude. Similar changes in the situations and complicated behaviors have been observed also in Fig. 15 and Fig. 18, which suggest interactive effects from phenomena coexisting in both the suction part and the delivery part of the flow-path.

Fig. 16 Qualitative sketch of very short and fat plenum-type delivery duct near B-type stall stagnation boundary

Fig. 17 Qualitative sketch of long and narrow delivery duct near A-type stall stagnation boundary

Fig.18 Geometrical parameters at the stall stagnation boundaries against effective flow-path volume ratios between the delivery and the suction-duct and compressor

The reduced frequencies are related with the relative lengths RLC and RLP , tip Mach numbers M_t , and relative surge-cycle frequencies (fs_0/f_1) as follows;

$$M_t = u_t / a \quad (26)$$

$$fRP = \frac{L_p}{\lambda} \frac{f_{s0}}{f_1} \frac{a}{u_t} = RLP \frac{f_{s0}}{f_1} \frac{1}{M_t} \quad (27)$$

$$fRC = \frac{L_c}{\lambda} \frac{f_{s0}}{f_1} \frac{a}{u_t} = RLC \frac{f_{s0}}{f_1} \frac{1}{M_t} \quad (28)$$

The unsmooth behaviors seen in Fig. 19 could be attributed to those in the relative surge-cycle frequencies (fs_0/f_1) which have been given in Fig. 14.

Figure 20 shows the behaviors of the surge-cycle reduced frequencies in terms of fRC versus fRP . This is corresponding to the behaviors of the relative lengths shown in the bottom figure in Fig. 15. Both reduced frequencies at the stagnation boundaries are seen to be changing in a correlated manner.

For reference, reduced frequencies $fRPA$ and $fRCA$ are estimated very roughly, where the representative speeds in Eqs. (20) and (21) are replaced by axial velocities at stalling in the respective parts of the flow-paths.

$$fRPA = \frac{L_p f_{s0}}{v_{aP}} \quad (29)$$

$$fRCA = \frac{L_c f_{s0}}{v_{aC}} \quad (30)$$

Here, v_{aP} and v_{aC} are the axial velocities at stalling in the delivery flow-path and suction flow-path, respectively.

Order of magnitude evaluations, the details of which are neglected here, give roughly 0.5-2.8 of $fRPA$ for the A-boundary, and 1-4 of $fRCA$ for the B-boundary. The magnitude around unity suggests that the stagnation could have occurred when the surge-cycle period becomes comparable to the time required for the flow to pass through the length of the responsible flow-path.

Thus, the geometrical stall-stagnation boundaries are given fundamentally by Figs. 15 and 18, and the dynamic conditions are given by reduced frequencies in Figs. 19 and 20. Both are connected together by Eqs. (27) and (28).

The present investigation has given both the fundamental geometrical conditions and the dynamic conditions for the stall stagnations for the stall stagnation boundary. However, more study about changing compressor speeds would be required, since the present study includes relatively few such cases.

10. Flow Behaviors at B- and A-Boundaries

Now, what differences could be observed in the flow behaviors between at the B-boundary and at the A-boundary?

Contour maps of pressure and mass flow are shown in Figs. 21 and 22 for the B-boundary and the A-boundary, respectively. Each subfigure shows axial location numbers as the abscissa and time number k as the ordinate upward. The axial location numbers of the representative flow elements and control volumes are given in Table 2.

Figure 21 shows the changing situations in approaching from a deep surge condition (i), near the B-Boundary (ii), and to the B-type stagnation (iii) by decreasing the length L_p of the delivery flow-path with a large sectional area A_p kept constant.

Although the pressure oscillation patterns are seen to change little, the amplitudes of mass flow oscillations tend to decrease in

Fig. 19 Geometrical parameters at stall stagnation boundaries against effective flow-path volume ratios between the delivery and the suction-duct and compressor

Fig. 20 Behaviors of both surge reduced frequencies at stall stagnation boundaries changing in relation with each other

Near B-Boundary

Pressure Contours

Mass Flow Contours

(a) Pressure Contours

(b) Mass Flow Contours

A_p kept constant at $A_{mod2}=0.6$.

L_p is reduced from top to bottom. $X_{md2}=0.035, 0.015$, and 0.003

Fig. 21 Surge pressure and mass flow behaviors of Comp19 at 10000rpm for conditions near B-Boundary, when the delivery duct lengths L_p are changed with the area A_p kept constant.

Near A-Boundary

(i) Deep Surge ($A_{mod2}=0/X_{mod2}=5$)

(ii) Near A-Stagnation ($A_{mod2}=-0.041/X_{mod2}=5$)

(iii) Very Near A-Stagnation ($A_{mod2}=-0.0425/X_{mod2}=5$)

(a) Pressure Contours

(b) Mass Flow Contours

L_p kept constant at $X_{mod2}=5$.

A_p is reduced from top to bottom. $A_{mod2}=0, -0.041$, and -0.0425

Fig.22 Surge pressure and mass flow behaviors of Comp19 at 10000rpm for conditions near A-Boundary, when the delivery duct areas A_p are changed with the length L_p kept constant

Table 2 Location numbers of the representative flow elements (Abcissa in Figs. 21 and 22)

1	Open inlet of the flow path	15	Inlet to the first stage of the compressor,
25	Exit from the compressor last stage	37	Plenum inlet
55	Plenum exit	59	Exit of the flow path terminated by a valve

Table 3 Typical flow-path geometrical data for Comp19 at 10000 rpm in Figs.21 and 22

Boundary	Duct specification		Suction Duct		Delivery Duct (plenum)		Situation
	A_{mod2}	X_{mod2}	$Ac2$ (m ²)	Lc^{**} (m)	Ap (m ²)	Lp (m)	
Near B	0.6	0.035	0.04125	3.67	0.3165	0.35	Deep surge
	0.6	0.015	0.04125	3.67	0.3165	0.15	Near B-stagnation
	0.6	0.003	0.04125	3.67	0.3165	0.03	B-stagnation
Near A	0	5	0.04125	3.67	0.0229	47.74	Deep surge
	-0.041	5	0.04125	3.67	0.0224	47.74	Near A-stagnation
	-0.0425	5	0.04125	3.67	0.0218	47.74	Very near A-stagnation

the delivery plenum, resulting finally in the B-type stagnation.

Figure 22 shows the changing situations in approaching from a deep surge condition (i) to very near the A-type stagnation (iii) by decreasing the sectional area Ap with a large length Lp kept constant. Although the oscillation patterns are seen to change little among them, the oscillation amplitudes of both pressure and mass flow are seen to be suppressed in the delivery duct, resulting finally in the stagnation of A-type.

The phenomena are considered as follows; Near the B-boundary, the lengths of the delivery flow-path Lp could be significantly short compared with that of the suction duct Lc^{**} , which would cause the mass flow oscillation to occur mainly in the suction-duct. At the same time, in the situation, the hypothetical Helmholtz resonator would not function well. Thus the stall stagnation of B-type could occur.

On the other hand, near the A-boundary, Lp could be very long compared with Lc^{**} , which would cause the mass flow oscillations to occur mainly within the delivery flow-path, where, because of the long length and the narrow area of the duct, oscillation patterns had to become distributed ones like a sinusoidal one along the x-axis. It would suppress the mass movement as a near-one-body which could have easily occurred in the Helmholtz type oscillation of deep surges. Thus the stall stagnation of A-type could occur.

Examples of typical sizes of the flow-paths for the cases in Figs. 21 and 22 are given in Table 3 for reference. Data on the suction duct geometry are given in Table 1.

It is interesting to note here that the A-boundary cases given in Fig. 22 and Table 3 have values of Ap very small compared with that of $Ac2$. The situation results from Eq. (18) for the A-boundary.

$$(Ap/Ac2) \sim 2.5/PR \quad (18a)$$

From the above relation, for PR larger than 2.5,

$$(Ap/Ac2) < 1$$

In ordinary practical applications, $(Ap/Ac2)$ is usually larger than unity. Therefore, in high pressure-ratio compressors, because of the resulting large values of area-pressure ratio APR , the stagnation depends only on the RLP condition in the A-boundary and on the combined conditions of RLC and RLP in the intermediate zone.

11. Experiences about the Stall Stagnation Boundaries

Representative geometrical parameters of several fans from the author's experiences and other sources are plotted in Fig. 23 which is an expanded view of Fig. 15. The small-sized points and solid lines are the stall stagnation boundaries given in Fig. 15. The black data and the colored data are for Comp19 and Comp11, respectively. Deep surges could occur in the yellow-colored zones bounded by the solid lines. At a value of RLP , a vertical chain line connects a pair of large-sized marks, whose points at the top and the bottom give values of APR and RLC , respectively. Hollow circle marks and solid triangle marks mean fans accompanied by deep surges, and those of no deep surge, respectively.

Fig. 23 Some experiences of deep surges and stagnation stalls checked with the acoustical-geometrical stall-stagnation boundary in Fig. 15

When both points are existing in the corresponding yellow-colored zones, the fan has surely experienced deep surges. When both or either of the points lie outside of the zones, the fan has shown stagnation stalls.

These fans have been designed for quite different aerodynamic design parameters and geometries. The geometrical stagnation boundaries given in Fig. 15 appear to be valid to some extent in such extended situations also. More data of such kind are wanted for further information of the stagnation.

Here, the data sources are, F1: a low-pressure fan test facility (Ref.[14]), F2: a high-load fan test facility (Ref.[15]), F3: low-pressure fan test facility (Katto[16]). They are single stage ones of axial inlet type, whose stage characteristics, geometries and sizes, tip speeds, flow-paths of the suction and the delivery are different from one another and from those of Comp11. C1 shows analytical results about speed-changing conditions of a 9-stage compressor similar to Comp19 but having different ducting conditions (Refs.[18, 19]).

Laboratory experiments could often use flow-paths different from those in site. For example, in the case of F1 shown in Fig. 23, although a long and narrow delivery duct has not yielded deep surges (solid black triangle marks: $\blacktriangle$), a plenum-type delivery has yielded violent deep surges (hollow black circle marks: $\circ$). Thus the arrangement could often have significant effects on post-stall behaviors and near-surge phenomena. The above results suggest that it would be necessary to arrange suitably the flow-path conditions compatible with the site situations.

12. Greitzer's B Parameter

The stall stagnation boundaries analytically obtained above are converted to Greitzer's B parameters according to Eq. (3), and are shown in Fig. 24. Here, it requires attention that definitions of some of the related geometrical variables are different from the original formulation in that Ac_2 , Lc^{**} and $(ApLp)$ are employed as Ac , Lc , and Vp in Eq (3).

It is seen in Fig. 24 that, for sufficiently small values of the relative delivery lengths RLP corresponding to the B-boundary, the B parameters $B.F.$ are located in the range 0.08-0.2, which are of rather lower level compared with Greitzer's limiting value of 0.8 indicated by the red horizontal chain line. It is interesting that Comp19 of a higher pressure ratio shows nearly constant level of limiting $B.F.s$, while Comp11 of lower pressure ratio shows a tendency of the $B.F.s$ approaching 0.8 in the intermediate zone. A suitable choice of the average level of the B parameter $B.F.$ could allow rough prediction of the B-boundary and the intermediate zone. But, it is not valid about the A-boundary, whose tendency cannot be predicted by the B parameter only.

The B parameter is important as an essential parameter in describing the general surge behaviors as far as short and fat plenum type delivery flow-paths are concerned. It will continue to have its value, if carefully and suitably employed.

13. Conclusion

Stall stagnation boundaries and related phenomena were investigated on the basis of analytical results by use of a code for surge transients analysis and simulation. With respect to the stall stagnation boundaries for simplified conditions of flow-paths, following conclusions are obtained.

- (1) Fundamental stall stagnation boundary is found to be given by the following acoustical-geometrical conditions.
- (2) The boundary is formed by three parts; B-boundary, A-boundary and the intermediate zone, each having different features.
- (3) The B-boundary is determined by the relative length of the suction duct and compressor RLC ($=Lc^{**}/\lambda$) of about 0.25 and sufficiently large values of the effective plenum area ratio APR ($=[Ap/Ac_2]PR$) for sufficiently small relative lengths of the delivery flow-path RLP ($=Lp/\lambda$). Here, λ is the wavelength of the first resonance frequency of the duct-mode in the whole flow-path. The boundary is related closely with some limiting value of Greitzer's B parameter.
- (4) The A-boundary is determined by the relative length of the delivery flow-path RLP of about 0.2 and the effective plenum area-pressure ratio APR of about 2.5 for sufficiently small RLC ($=Lc^{**}/\lambda$).
- (5) Intermediate zone of the stall stagnation boundary could be affected from both of the suction duct phenomena and the delivery flow-path ones.
- (6) With respect to the relative volume-pressure ratio RVP ($=[ApLp/Ac_2Lc^{**}]PR$), the B-boundary and the A-boundary are in the ranges; $RVP < 0.7$ and $RVP > 7$, respectively. The intermediate zone is for $0.7 < RVP < 7$.
- (7) Tentatively, the B-type stagnation tends to occur in delivery flow-paths of very short and fat plenum-type, whereas the A-type tends to occur in delivery flow-paths of very long and narrow duct type.
- (8) With respect to the fluid-oscillation nature of the stall stagnation, it is necessary to pay attention also to the surge-cycle reduced frequencies: f_{RC} concerned with the suction flow-path and f_{RP} concerning the delivery flow-path. f_{RC} and f_{RP} dominate the B-boundary and the A-boundary, respectively. f_{RC} is nearly proportional to a reciprocal of Greitzer's B parameter. f_{RC} ranges from 0.2 to 0.4 for the B-boundary, and f_{RP} ranges from 0.2 to 0.45 for the A-boundary. In the intermediate zone, both of f_{RC} and f_{RP} appear to affect the stagnation boundary.

Fig.24 Stall stagnation boundaries expressed by B-parameters according to Greitzer's formulation, Eq. (3)

The above results have been derived only on analytical bases and on very simplified conditions of the compressors and the ducts. However, the author believes that, within the limited analytical situation, the results given here could be useful to understand the stall stagnation phenomena. Further investigations in the experimental fields will also be required for the confirmation.

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Nomenclature

Symbols			
A	Flow path sectional area (m^2)	p_{pl}	Pressure in the delivery flow-path (Pa)
a	Sound of speed (m/s)	PR	Stalling pressure ratio
Ac	Sectional area of suction duct and compressor in Greitzer's model (m^2)	ps	Static pressure (Pa)
$Ac2$	Sectional area of compressor exit (m^2)	RLC	Relative compressor-duct length normalized by the wavelength
A_{mod2}	Parameter specifying the sectional area of the delivery flow-path	RLP	Relative length of delivery flow-path normalized by the wavelength
Ap	Sectional area of the delivery flow-path (m^2)	RVP	Effective volume ratio of the delivery flow-path to the suction duct-compressor
APR	Ratio of product of area and pressure of the plenum to the compressor-duct	t	Time (s)
Av	Valve opening area (m^2)	u	Flow velocity, axial velocity (m/s)
Av_0	Valve opening area at stalling (m^2)	um	Blade mean speed of the compressor (m/s)
$B.F.$	Greitzer's B parameter	ut	Blade tip speed of the compressor (m/s)
f_0	Resonant frequency of a plenum (Hz)	Vp	Volume of delivery flow-path (m^3)
f_1	Duct-mode first resonance frequency (Hz)	W	Mass flow (kg/s)
f_2	Duct-mode second resonance frequency (Hz)	Wv	Valve mass flow (kg/s)
f_{RC}	Reduced surge frequency given by the compressor -duct length and the compressor tip speed	x	Axial coordinate of flow passage (m)
f_{RCA}	Reduced surge frequency given by the length and the axial velocity of the compressor-duct length	X_{mod2}	Parameter specifying the length of the delivery flow-path
f_{RP}	Reduced surge frequency given by the delivery flow-path length and compressor tip speed	αv	Steady valve flow coefficient
f_{RPA}	Reduced surge frequency given by the length and the axial velocity of the delivery flow-path	βv	Adjusting factor for the valve mass flow
f_{s0}	Surge-cycle frequency (Hz)	εv	Adjusting factor for the valve passage length
f_{s1}	Frequency in the stagnation stall (Hz)	ΔL_v	Valve passage length (m)
i	Number showing a control volume or the section in the duct	Δt	Time increment, time step of analysis (s)
k	Number showing time	λ	Wavelength of the duct first-mode resonant frequency (m)
L_c	Axial length of the suction-duct and compressor (m)	ρ	Flow density (kg/m^3)
L_{co}	Axial length of the compressor (m)	ϕ_t	Stage flow coefficient
L_{c**}	Axial length of the compressor-duct flow-path (m)	ψ_P	Stage pressure coefficient
L_p	Length of the delivery flow-path (m)	ψ_T	Stage temperature-rise coefficient
p_0	Pressure upstream of the compressor (Pa)		
Suffices			
c	Suction flow-path including the compressor	v	Exit valve
p	Delivery flow-path or plenum	0	Surge cycle, or plenum mode
s	Surge	1	The first duct-mode resonance in the flow-path system
$s0$	Surge-cycle frequency	2	The second duct mode resonance in the flow-path system
$s1$	Higher frequency in surge		Exit of the compressor

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