Comparison of Biot–Savart’s Law and 3D FEM in the Study of Electromagnetic Forces Acting on End Winding

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Abstract – An induction motor operated with high voltage source generally generates high current in starting mode and has a long transient time after being started. This large and sustaining starting current causes the end windings of the stator to have excessive electromagnetic force. This force is the source of vibration and has a negative and serious influence on the insulation of end windings. Therefore, designing the end winding part with an appropriate support system is needed. To design the support ring enclosing the end windings, we analyze the distribution of electromagnetic force on the end windings by applying the Biot–Savart’s law and the 3D finite element method (FEM), and comparing two simulation methods. Finally, we verify the safety of the support structure of the end winding part using stress analysis, which is analyzed with the electromagnetic forces from the 3D FEM simulation.

Keywords: Biot–Savart’s law, End winding of stator, Electromagnetic force distribution, 3D finite element method

1. Introduction

Designing with high power density is currently needed for electrical rotating machines such as motors and generators. The rotating machine is designed with high fill factor of windings and compact size. In designing a high-power induction motor, the electromagnetic force acting on the end windings of a stator becomes increasingly severe because of high current density in a coil. There are also many problems related to the fault of the winding insulation due to stress and vibration, among other factors. The distribution of electromagnetic force on end windings should be considered and analyzed accurately before manufacturing.

To date, several approaches for calculating the distribution of the end winding force have been proposed [1]–[5]. There is a difference between the calculated force and the real force. In 1988, Williamson and Ellis presented a method for calculating the end winding force considering the rotor current distribution of induction motor using Biot–Savart’s law [6]. However, a limitation of Biot–Savart’s law is that it is applicable only to free space; that is, magnetic fields near the ferromagnetic material in the vicinity of the end winding have significant errors between the measured force and the calculated force. Khan, Buckley, and Brooks used quasi 3D finite element method (FEM) in 1989 [7]. However, quasi 3D FEM is very complicated to program and assessing the magnetic flux variation in the peripheral direction is difficult. Furthermore, considering the effect of a teeth structure of the core is impossible.

In the current study, we calculate the distribution of elec-

tromagnetic force acting on end windings of a high-power induction motor by using two calculation methods. One is Biot–Savart’s law and the other is 3D FEM. The latter can analyze end winding forces more precisely than the former. As an application of the force calculation results, we can conduct a stress analysis of the end windings by inputting the force distribution already calculated by 3D FEM. Finally, we design the support ring surrounding the end windings to prevent the fault of winding insulation by stress caused by electromagnetic forces. The proposed method can be widely applied to enhance the reliability of electrical machines in the future.

2. Analysis of End Winding Force

The high voltage induction motor generally generates a high starting current with considerably long transient time. The starting current causes end windings to have excessive electromagnetic force. Therefore, the calculation method for end winding forces is needed in the design stage.

2.1 Biot–Savart’s Law

Biot–Savart’s law is one of the electromagnetic force calculation methods between two conductors that are short, straight, and have a current. We can calculate the force distribution of end windings with any complex structure in free space. Free space means there is no magnetic material in the region.

In this paper, each coil of end windings is replaced by an equivalent filamentary conductor carrying the equal ampere-turn. The electromagnetic force between the filamen-

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tary conductors is also calculated. This method has an advantage because it is readily applicable to any structure of the end winding, any connection of the stator winding, and any current condition. This method is conceptually simple and relatively straightforward to use.

In Fig. 1, the elementary force that acts on the element a carrying current $i_a(t)$ due to the current $i_b(t)$ in the element b can be calculated by

$$\vec{F}_e(t) = i_a(t) i_b(t) \frac{1}{|\vec{r}|^3} \vec{a} \times \vec{b} (\times \vec{r}) 10^{-7} [N]. \quad (1)$$

In the above equation, $\vec{r}$ is the position vector from the center of the element b to the center of the element a . The electromagnetic force per unit length is (2), and the coefficient of the elementary magnetic force is (3).

$$\vec{f}_e(t) = \frac{\vec{F}_e(t)}{|\vec{a}|} = \vec{k}_e i_a(t) i_b(t) [N/cm]. \quad (2)$$

Note that k_e is the coefficient of the elementary magnetic force.

$$\vec{k}_e = \frac{0.1}{|\vec{a}| |\vec{r}|^3} [(\vec{a} \cdot \vec{r}) \vec{b} - (\vec{a} \cdot \vec{b}) \vec{r}]. \quad (3)$$

where

$\vec{a} = \vec{a}(m, n)$: m -th element of the n -th coil
 $\vec{b} = \vec{b}(\mu, \nu)$: μ -th element of the ν -th coil.

$$k_e = k_e(m, n, \mu, \nu). \quad (4)$$

The electromagnetic force on the element $a(m, n)$ is computed by vector addition of elementary forces between the element a and all the other winding elements. This

Fig. 1. Vector expression of two conductor segments

method has an advantage because it integrates the force-coefficients instead of the forces. For a more detailed explanation related to the calculation algorithm, refer to [8].

2.2 3D FEM Analysis

The 3D FEM is capable of representing a complex geometry accurately and giving a more exact solution for the electromagnetic field even in the vicinity of the end core region, including slots and tooth structure.

Table 1. Specifications of the Analysis Model

Item	Unit	Specification
Power	kW	900
Rated Voltage	V	6,600
Number of Pole		4
Connection Type		Y
Coil Pitch		10
Number of Stator Slot/Rotor Slot		48/36
Silicon Steel Grade		S23
Starting Current/Rated Current	A	623/95
Diameter of Support Ring	mm	10
Material of Support Ring		Polyglass

The specification of the analysis model with a square-type armature winding is shown in Table 1. The starting current of the motor is 6.56 times as high as the rated current, and takes 8 s from start to rated speed. The end winding force due to the large starting current affects the insulation material of end winding [9].

The end winding model with core and requirements for the pre-processor for 3D FEM is shown in Fig. 2. To save the calculating time, one pole model with half periodic boundary conditions (magnetic field intensity H at master boundary plane and $-H$ at slave boundary plane) is used. In this case, the number of slots per pole per phase is four. The end core, rotor bar, and end ring are included in the model for the purpose of magnetic flux considering the end core and the end ring current. For simple analysis, we assume that the sinusoidal current is flowing in the stator winding. The end winding force at the start is calculated at the time when the starting current of the phase A is $\sqrt{2}$ times of the rms value, whereas the end winding force of phases B and C is the half and negative value of phase A current, respectively. In this model, the number of tetrahedral elements amounts to 234,563.

In order to calculate the distribution of an end winding force, we segment a winding with 39 parts as shown in Fig. 3. We calculate the force caused by the current of each segment and the magnetic flux density at each segment. We consider the distribution of the end ring current by 2D transient FE analysis as well as three-phase current in a stator winding. After calculating the force on each part of a winding, the distribution result of an end winding is obtained. Therefore, 12 repetitive calculations generate the whole electromagnetic force distribution at the entire end windings.

Fig. 2. 3D FEM analysis model with one pole and boundary condition.

Fig. 4. Comparison of force calculation results by applying Biot-Savart's law and 3D FEM.

(a) Reference winding (x-y plane)

(b) The coil segment number (x-z plane)

Fig. 3. Definition of the reference winding for force calculation

3. Comparison of End Winding Forces

To compare the calculation results obtained by applying Biot-Savart's law with those by 3D FEM, we select the 4th winding of A phase which is in the vicinity of the 1st winding of B phase as a sample. The maximum electromagnetic force occurs at the 4th winding of the A phase. The results obtained by applying 3D FEM in Fig. 4 show that segment numbers 1 and 2 of the upper layer coil and segment num-

(a) x-z plane

(b) x-y plane

(c) x-y-z space

Fig. 5. The results of the magnetic field of the end winding region

bers 38 and 39 of the lower layer coil, which are both near the core in Fig. 3, have larger forces than the others due to their high flux density at the core. However, it is impossible to consider the effect of ferromagnetic material by applying Biot–Savart’s law. Therefore, the two results have a high level of discrepancy. In conclusion, although the electromagnetic force by Biot–Savart’s law is somewhat significant at the region of air, this force cannot be used at the region adjacent to the end core.

Fig. 5 shows the analysis of the magnetic field, including the leakage flux by 3D FEM. In this figure, we can verify that the leakage flux is generated high at the straight part of end winding in the vicinity of the core, at A phase windings which have maximum current, and at the lower part instead

(a) The electromagnetic forces of the A phase group coils

(b) The electromagnetic forces of the B phase group coils

(c) The electromagnetic forces of the C phase group coils

Fig. 6. The electromagnetic forces distribution of end windings

of the upper part.

Similar to the distribution of the magnetic flux, the electromagnetic force at the straight part, lower layer, and maximum current carrying the conductor is the largest, as shown in Fig. 6. There is a difference in force distribution even on the same phase windings having the same current. Fig. 7 shows the force vector on an x-y domain of the straight and involute part of winding a pole. All the forces act in a radial outward direction. Therefore, the support ring surrounding the end windings is needed to prevent divergence of end windings.

(a) Straight part of windings

(b) Involute part of windings

Fig. 7. Vector distribution of electromagnetic force on three phase windings

4. Reliability of the End Winding Insulation based on the Support Ring Design

The stress analysis of an end winding with a supporting ring is conducted by inputting the distribution of electromagnetic force and comparing it with the yield stress of the support ring to ascertain the reliability of the support structure at the end winding of the induction motor. Fig. 8 shows the stress analysis result. We can verify the location of the maximum stress point. The insulation stability of end windings is also investigated by comparing the simulated maximum stress with the stress value at the insulation fault from a bending moment test. The properties and mechanical characteristics of the end winding part are in Table II.

The stress analysis results are listed in Table III. The maximum stress of end winding is 21.94 [MPa]. The maximum stress of a support ring is 3.8 [MPa]. As the maximum stress ratio is 0.318 ($=21.94/69$), the design of end winding is within the very reliable level in comparison with the yield stress.

Finally, we calculate the stress ratio according to the displacement of the support ring. Fig. 9 shows the analysis model with the support ring. We can observe that the stress ratio decreased as the position of the support ring became more distant from the core part, as shown in Table IV.

Fig. 8. The result of the stress analysis of end winding with the support ring

Table 2. Properties of Materials

Section part	Conductor	Support ring	Insulator
Material	Copper	Polyglass	Mica glass
Yield Stress (MPa)	69–350	-	-
Tensile Stress (MPa)	-	900	65

Table 3. Results of Stress Analysis

Section	Coil number	Radial Displacement (mm)	Stress (MPa)	Stress ratio
U phase Coil	1	0.071	10.13	0.147
	2	0.098	12.78	0.185
	3	0.106	17.89	0.259
	4	0.096	21.94	0.318
Support ring	-	0.03	3.8	0.055

Fig. 9. The position of the supporting ring (a is 20% of the involute part, b is 40%, c is 60%, and d is 80%)

Table 4. The stress ratio based on the positions of the support ring

Position	Displacement (mm)	Stress (MPa)	Stress ratio
None	0.144	23.89	0.346
20%	0.14	23.58	0.342
40%	0.125	22.77	0.33
60%	0.102	21.77	0.316
80%	0.076	20.86	0.302

5. Conclusion

In this paper, two kinds of calculation methods for the distribution of end winding force were presented. We calculated the magnetic field and electromagnetic force of the end winding part of the high-power induction motor by applying Biot–Savart’s law and 3D FEM. The distribution results of the electromagnetic forces using the two methods were compared, and 3D FEM was found to be a good method that precisely analyzes end winding forces. However, the calculation method using Biot–Savart’s law has some validity at the region apart from the core and end ring. We also conducted a stress analysis of end windings by inputting the force distribution calculated by 3D FEM for the electromagnetic field. Finally, the reliability of the support ring in reducing the stress of end windings and in preventing insulation fault was verified by comparing the calculated stress with the yield stress of the support ring material.

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