Dispatching Vehicles Considering Multi-lifts of Quay Cranes

Vu Duc Nguyen

Department of Industrial Engineering, Pusan National University
30 Jangjeon-dong, Geumjeong-gu, Busan 609-735, South Korea
Tel: +82-51-510-2419, Fax: +82-51-512-7603, E-mail: duc.ise@pusan.ac.kr

Kap Hwan Kim[†]

Department of Industrial Engineering, Pusan National University
30 Jangjeon-dong, Geumjeong-gu, Busan 609-735, South Korea
Tel: +82-51-510-2419, Fax: +82-51-512-7603, E-mail: kapkim@pusan.ac.kr

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Abstract. To improve the ship operation in automated container terminals, it is important to schedule different types of handling equipment to operate synchronously. For example, a vehicle with container receiving and lifting capabilities is used to transport containers from a storage yard to a vessel and vice versa, while a triple quay crane (QC) can handle up to three 40-ft containers simultaneously. This paper discusses the manner in which vehicles should be assigned to containers to support such multi-lifts of QCs by using information about the locations and times of deliveries. A mixed-integer programming model is introduced to optimally assign delivery tasks to vehicles. This model considers the constraint imposed by the limited buffer space under each QC. A procedure for converting buffer-space constraints into time window constraints and a heuristic algorithm for overcoming the excessive computational time required for solving the mathematical model are suggested. A numerical experiment is conducted to compare the objective values and computational times of the heuristic algorithm with those of the optimizing method to evaluate the performance of the heuristic algorithm.

Keywords: Vehicles, Container Terminal, Dispatching, Time Window, Multi-lifts

1. INTRODUCTION

The transport of containers via seaport container terminals has rapidly increased during the last decade. An increasing number of countries have recognized the resulting challenges and opportunities for investment. In particular, the governments of Hong Kong, Singapore, Korea, and China have invested large amounts of money to increase the capacities of their container terminals. Therefore, the competition between seaports has become much more severe.

One of the most important issues facing container terminals is how to reduce the time that is needed for ship operations. The ship operation process consists of unloading and loading operations, as well as the transport of containers between vessels and storage blocks. In this paper, we consider an automated port container terminal in which three main types of equipment, i.e.,

QCs, vehicles, and yard cranes (YCs), are used for ship operations. Nowadays, the development of handling equipment in container terminals is improving the ship operation. This paper considers the multi-lifts of QCs, which are capable of transferring up to six 20-ft containers or three 40-ft containers. The triple lift of a QC is shown in Figure 1. Recently, some modern container terminals have applied the multi-lifts of QCs to improve the ship operation. However, with such multi-lifts, since an increasing number of containers will be transferred by QCs, it is necessary to deal with the problem of how to distribute and supply containers to eliminate delays and make the QC operation efficient.

During an unloading operation, up to three 40-ft containers are picked up by a triple QC and put down in the buffer space under the QC in an apron. Then, vehicles pick up and deliver the containers to the storage yard. In the yard, the vehicle releases a container in the

[†] : Corresponding Author

buffer space at a transfer point (TP) of the yard. A YC picks up and stacks it in an empty slot in a bay. A loading operation is performed in the reverse order of the unloading process. Figure 2 illustrates the layout of an automated port container terminal that has buffer space under the QCs and TPs in an apron and yard, respectively.

In the field of manufacturing scheduling (Mori-kawa and Takahashi, 2009), the dispatching of vehicles has been treated as an important issue. In previous researches on vehicle dispatching methods, it was assumed that pickup calls were issued randomly and that the sequence of the calls could not be known in advance. Thus, the dispatching decision was made after a pickup call was issued or when a vehicle became free from a previous delivery task (Egbelu, 1987; Bilge and Ulusoy, 1995; Kim *et al.*, 1999; Lim *et al.*, 2003; Grunow *et al.*, 2004, 2006; Briskorn *et al.*, 2006; Correa *et al.*, 2007). However, in practice, before a vessel arrives at a port, a work schedule for the QCs is created on the basis of a stowage plan, which is used as a guideline for unloading and loading operations. Then, a sequence list of unloading and loading operations for individual containers is made by using the work schedule. When the vessel actually arrives at its berth, ship operations are carried out in the same order as specified in the sequence list. Thus, it can be said that the delivery tasks of vehicles are known in advance and that their sequence is also predetermined. Bish *et al.* (2005) introduced a vehicle-dispatching problem in which a detailed sequence specifying the order of the containers that are to be discharged/loaded onto the ship is given. They easily developed implementable heuristic algorithms and identified both the absolute and asymptotic worst-case performance ratios of these heuristics.

In this paper, dispatching can be defined as the assignment of vehicles to delivery tasks. Specific buffer spaces in aprons and yards are considered, within which QCs, YCs, and vehicles can release containers. We will assume that a QC performs triple lifts in which three 40-ft containers are handled simultaneously. QCs or YCs can pick up (put down) containers directly from (onto) a vehicle. However, because a vehicle is also capable of lifting a container from the ground by itself, a QC can put three 40-ft containers in the buffer space if sufficient buffer space is available (at least 3 empty buffer spaces are required). Therefore, in relation to releasing containers in the buffer space, a vehicle or QC must wait when there is insufficient buffer space. The number of containers that can be transferred by QCs capable of performing triple lifts is much larger than that transferred by QCs capable of performing single operations. Therefore, in order to make ship operations efficient, it is essential to determine the manner in which vehicles must be dispatched to support QC operations to minimize the delay times of QCs, as well as the travel times of vehicles.

Together with the development of equipment techniques, we introduce more efficient and functional hand-

ling equipment such as automated lifting vehicles (ALVs) with the capacity of picking up containers from the floor. However, a few previous studies have examined ALVs. Through a simulation study, Van Der Meer (2000) evaluated various dispatching rules, including rules using pre-arrival information, for ALVs in container terminals. Vis and Harika (2004) and Yang *et al.* (2004) compared the performance of two types of automated vehicles, namely, AGVs and ALVs, by a simulation study.

Figure 1. QC with triple lift capability in container terminal.

Figure 2. Layout of an automated container terminal.

Recently, Nguyen and Kim (2009, 2010) developed a mathematical formulation of the dispatching problem for ALVs. They suggested a heuristic algorithm and compared the solutions of this heuristic algorithm with optimal solutions. Unlike their study, this study addre-

sses the vehicle-dispatching problem considering the multi-lifts of QCs.

This paper is organized as follows. The ship operations with multi-lifts of QCs in automated container terminals are introduced in section 2. Section 3 suggests a mixed integer programming model for vehicle dispatching that considers the multi-lifts of QCs. Section 4 proposes a procedure for converting buffer constraints into time window constraints and a heuristic algorithm for solving the dispatching problem that uses these time window constraints. Section 5 shows a numerical experiment to compare the heuristic algorithm with an optimizing method and evaluate the performance of the heuristic algorithm. Finally, section 6 concludes this study and discusses further studies.

2. SHIP OPERATION WITH MULTI-LIFTS OF QUAY CRANES

Before a vessel arrives at a port container terminal, a sequence list of the unloading and loading operations for individual containers is prepared. The stowing plan is devised to allow tandem and triple lifts. When the vessel actually arrives, ship operations are usually performed on the basis of the sequence list of loading and discharging.

At the quayside, an unloading operation cycle by a QC ends with the release of containers onto the buffer space at the QC in an apron, while a loading operation cycle begins with the pickup of containers from the buffer space at the QC in an apron. In this study, the QC operations are triple lifts, as shown in Figure 3c. In this figure, there are six possible combinations of triple lifts, with up to three 40-ft containers or six 20-ft containers. For tandem lifts, two 40-ft containers or four 20-ft containers can be lifted simultaneously Figure 3b. While single lifts can only handle one 40-ft container or two 20-ft containers Figure 3a. The ability of tandem or triple QCs to handle a larger number of containers than single QCs is expected to enhance ship operations in container terminals.

Figure 3. Types of operations of QCs.

Table 1 shows an example of a sequence list for applying QC 1 to ship operations, to perform different types of QC operations. As shown in column 3, QC 1 is used for tandem and triple lifts. Table 2 explains the progress of the operations performed by QC 1 as it follows the sequence list shown in Table 1.

Table 2 lists the events and their notations in the ship operations of QC 1. Table 2 assumes that the operation of QC 1 is not delayed because of the late arrival of vehicles or full occupancy of the buffer space. With

Table 1. Example of a sequence list of different types of operation of QC 01.

QC 01						
Task sequence	Type ⁺	No. of 40-ft containers	Ship location [†]	Yard location [‡]	Operation cycle time	Earliest event time
1	L	3	06/03/04 06/04/04 06/05/0	A/21/3/2 A/11/1/1 B/05/3/4	120	0
2	U	2	10/03/10 10/04/10	B/17/4/2 B/21/7/3	110	230
3	L	2	18/05/06 18/06/06	A/09/4/2 B/07/4/1	120	230
4	U	3	26/04/08 26/05/08 26/06/08	A/21/3/1 B/13/8/1 B/15/6/4	110	460
...	...	...	...	...	...	...

Notes: ⁺L-loading; U-unloading; [†]ship bay no./row no./tier no.; [‡]yard block no./yard-bay no./row no./tier no.

Table 2. Events of the QC 1 ship operation.

Event sequence	Time	Notation	Description
1	0	q_1^1	As the 1 st task, QC 1 picks up three containers from the buffer at QC 1 (assume that three containers were ready in the buffer at QC 1).
	120		QC 1 releases the three containers of the 1 st task in the ship's bay, and picks up the two containers of the 2 nd task from the ship's bay.
2 3	230	q_2^1 q_3^1	QC 1 releases the two containers of the 2 nd task into the buffer (at this time, sufficient buffer space must be available) and picks up the two containers of the 3 rd task from the buffer at QC 1 (assume that two containers were ready in the buffer at QC 1).
	350		QC 1 releases the two containers of the 3 rd task in the ship's bay, and picks up the three containers of the 4 th task from the ship's bay.
4	460	q_4^1	QC 1 releases the three containers of the 4 th task into the buffer at QC 1 (at this time, sufficient buffer space must be available).

regard to the loading operation of a QC, which is to be performed without delay, the assigned vehicles must put down the loading containers into a specific buffer space at the QC in an apron before the QC can begin to pick them up. For an unloading operation, the buffer space must be available for the inbound containers at the QC apron or the assigned vehicles must be ready to receive the inbound containers. It should be noted that because the QC operations considered in this study are multi-lifts, a number of containers must be ready in the buffer space for a loading operation by a QC and sufficient buffer space must be available for an unloading operation (at least 2 slots for tandem lifts and 3 slots for triple lifts are required).

3. MODEL FORMULATION

The dispatching problem for ALVs was formulated as a mixed integer programming (MIP) model by Nguyen and Kim (2009). However, this study addresses the vehicle-dispatching problem considering the multi-lifts of QCs. To formulate the mathematical model, some notations from Nguyen and Kim (2009) are summarized, and additional notations are defined in this section.

Let q_i^k be an event representing the moment that QC k transfers the containers of the i^{th} operation (of QC k). When the i^{th} operation of QC k is a loading operation, event q_i^k corresponds to the pickup of the containers of the i^{th} operation of QC k from the buffer at QC k . When the i^{th} operation of QC k is an unloading operation, it corresponds to the release of the containers of the i^{th} operation by QC k into the buffer at QC k . The event time of q_i^k is denoted by y_i^k , which is a decision variable.

Let a_{ie}^k also be an event representing the moment that a vehicle transfers the e^{th} container of the i^{th} operation of QC k . When the i^{th} operation of QC k is a loading operation, event a_{ie}^k corresponds to the release of the e^{th} container of the i^{th} operation by vehicle into the buffer at QC k . When the i^{th} operation of QC k is an unloading operation, it corresponds to the pickup of the e^{th} container of the i^{th} operation by a vehicle from the buffer at QC k . The event time of a_{ie}^k is denoted by z_{ie}^k .

Let s_i^k be the earliest possible event time of q_i^k .

The additional notations are summarized as follows:

- a_{0j}^O = The starting event of vehicle $j, j \in V$.
- a_{0j}^F = The stopping event of vehicle $j, j \in V$.
- m_k = The number of operations for QC k .
- n_i = The number of 40-ft containers of operation $i, n_i = 1, \dots, 3$.
- $l(a_{ie}^k)$ = The location where event a_{ie}^k occurs. $l(a_{0j}^O)$ represents the initial location of vehicle j . $l(a_{ie}^k)$

represents the position where the e^{th} container of the i^{th} operation of QC k is to be transferred. $l(a_{0j}^F)$ represents the location where a vehicle completes its final delivery task.

t_{kie}^{diff} = The pure travel time from $l(a_{ie}^k)$ to $l(a_{if}^l)$.

c_{kie}^{diff} = The time required for a vehicle to be ready for a_{if}^l after it experiences a_{ie}^k . For example, if both a_{ie}^k and a_{if}^l are related to loading operations, then the starting event for evaluating c_{kie}^{diff} is the release of the e^{th} container of the i^{th} operation by a vehicle. c_{kie}^{diff} includes the travel time from an apron to the location of the next container (the j^{th} container of the j^{th} operation of QC l) in the marshalling yard, the pickup time of the container by vehicle, and the travel time of the vehicle to QC l .

b_k = The capacity of the buffer space under QC k .

M = A large positive number.

$u_i^k = \begin{cases} n_i, & t \geq y_i^k, \\ 0, & t < y_i^k, \end{cases}$ for $i \in U^k \cup L^k$ and $k \in K$.

$u_{ie}^k = \begin{cases} 1, & t \geq z_{ie}^k, \\ 0, & t < z_{ie}^k, \end{cases}$ for $i \in U^k \cup L^k, k \in K$ and $e = 1, \dots, n_i$.

V = The set of vehicles.

K = The set of QCs.

S = The set of a_{0j}^O s, $j \in V$.

D = The set of a_{0j}^F s, $j \in V$.

L^k = The set of loading tasks, for QC k .

U^k = The set of unloading tasks, for QC k .

T = The set of a_{ie}^k s, for $i = 1, \dots, m_k; e = 1, \dots, n_i$; and $k \in K$.

The problem in this paper is addressed as a static scheduling problem, where vehicles must be assigned to complete all the tasks in a known set under a limited buffer capacity. A feasible dispatching decision is made as a one-to-one assignment between all the events in $S \cup T$ and those in $D \cup T$.

Let $K' = \{O\} \cup K$, $K'' = \{F\} \cup K$, and x_{kie}^{diff} be a decision variable that becomes 1 if a_{ie}^k is assigned to a_{if}^l , for $k \in K'$ and $l \in K''$. For k and $l \in K$, the assignment of a_{ie}^k to a_{if}^l implies that the vehicle, which have just delivered the e^{th} container of the i^{th} operation of QC k , is scheduled to deliver the f^{th} container of the j^{th} operation of QC l .

Let α be the travel cost per unit time of a vehicle, and β be the penalty cost per unit time for a delay in the completion time. It is assumed that $\alpha < \beta$. Further, let m_O and m_F be equal to $|V|$. Then, the dispatching problem can be formulated as follows.

Minimize

$$\alpha \sum_{k \in K'} \sum_{i=1}^{m_k} \sum_{e=1}^{n_i} \sum_{j=1}^{m_k} \sum_{f=1}^{n_j} x_{kie}^{liff} + \beta \sum_{k \in K} (y_{m_k}^k - s_{m_k}^k) \quad (1)$$

Subject to

$$\sum_{l \in K'} \sum_{j=1}^{m_l} x_{kie}^{liff} = 1; \forall k \in K'; \quad (2)$$

$$i = 1, \dots, m_k; e = 1, \dots, n_j$$

$$\sum_{k \in K'} \sum_{i=1}^{m_k} x_{kie}^{liff} = 1; \forall l \in K''; \quad (3)$$

$$j = 1, \dots, m_l; f = 1, \dots, n_j$$

$$z_{lff}^l - (z_{ie}^k + c_{kie}^{liff}) \geq M(x_{kie}^{liff} - 1); \quad (4)$$

$$\forall k \in K'; l \in K; i = 1, \dots, m_k; j = 1, \dots, m_l; e = 1, \dots, n_i; f = 1, \dots, n_j$$

$$y_{i+1}^k - y_i^k \geq s_{i+1}^k - s_i^k; \forall k \in K; i = 1, \dots, m_k - 1 \quad (5)$$

$$y_i^k \geq s_i^k; \forall k \in K'; i = 1, \dots, m_k \quad (6)$$

$$y_i^k \leq \min_e \{z_{ie}^k\}; \forall k \in K'; i \in U^k; e = 1, \dots, n_i \quad (7)$$

$$y_i^k \geq \max_e \{z_{ie}^k\}; \forall k \in K'; i \in L^k; e = 1, \dots, n_i \quad (8)$$

$$\sum_{i \in U^k} u_i^{kt} - \sum_{i \in L^k} u_i^{kt} + \sum_{i \in L^k} v_{ie}^{kt} - \sum_{i \in U^k} v_{ie}^{kt} \leq b_k; \quad (9)$$

$$\forall k \in K; t \in \left\{ y_i^k, \forall i \in U^k, z_{ie}^k, \forall i \in L^k, e = 1, \dots, n_i \right\}$$

$$t - y_i^k < M \cdot u_i^{kt}; \forall k \in K; i \in U^k; t \in \{y_i^k\} \quad (10)$$

$$t - y_i^k > M \cdot (u_i^{kt} - n_i); \forall k \in K; i \in L^k; t \in \{y_i^k\} \quad (11)$$

$$t - z_{ie}^k < M \cdot v_{ie}^{kt}; \forall k \in K; i \in L^k; e = 1, \dots, n_i; t \in \{z_{ie}^k\} \quad (12)$$

$$t - z_{ie}^k > M \cdot (v_{ie}^{kt} - 1); \forall k \in K; i \in U^k; e = 1, \dots, n_i; t \in \{z_{ie}^k\} \quad (13)$$

$$u_i^{kt} = 0 \text{ or } n_i; v_{ie}^{kt} = 0 \text{ or } 1; \quad (14)$$

$$\forall k \in K; i = 1, \dots, m_k; e = 1, \dots, n_i; t \in \{y_i^k, z_{ie}^k\}$$

$$x_{kie}^{liff} = 0 \text{ or } 1; \forall k \in K'; l \in K''; \quad (15)$$

$$i = 1, \dots, m_k; j = 1, \dots, m_l; e = 1, \dots, n_i; f = 1, \dots, n_j$$

Note: $s_i^0 = 0$, for $\forall i \in V$.

From the objective function, the penalty cost per unit time is considered for a delay in the completion time. In many practical cases, early arrivals of vehicles may speed up the ship operation. Especially, in case that the handling times of QCs are uncertain, there may be some possibility that the next cycle can start earlier when the delivery of the container for the previous cycle is finished earlier. However, this paper assumes that the QC handling operation is constant and the objective is to minimize the delay times of QCs. Thus, when vehicles arrive at the transfer positions earlier than their due times, the vehicles have to wait the arrivals of the spreaders of the quay cranes. Therefore, there is no incentive for the earlier arrivals of vehicles. Objective function (1) is used to minimize the total travel time of the vehicles and the total delays in QC operations. Because $\alpha \ll \beta$, the sum of the delays in the QC operations will be

minimized first. For the same value of total delay, the total travel distance of the vehicles will be minimized. Constraints (2) and (3) force a one-to-one assignment between all the events in $S \cup T$ and those in $D \cup T$. Constraint (4) implies that two events that are served consecutively by the same vehicle must be set apart by at least the time required by a vehicle for traveling and transferring between the two events. Constraint (5) implies that two events that are served by the same QC must be set apart by at least the time required for the QC to perform all the movements between the two events. Constraint (6) implies that the actual event time is always greater than or equal to the earliest possible event time. Constraints (7) and (8) show that the containers of the i^{th} operation in the buffer at QC k must be available for pickup by vehicles in an unloading operation and by the QC in a loading operation, respectively. Constraint (9) indicates that the number of containers in the buffer at QC k released or picked up by the QC or vehicles at time t cannot exceed the capacity of the buffer. Constraint (10) ensures that u_i^{kt} is equal to n_i when $t > y_i^k$, which implies that n_i containers of the i^{th} operation have already been released at time t by QC k into the buffer at QC k . Constraint (11) ensures that u_i^{kt} is equal to n_i when $t > y_i^k$, which implies that n_i containers of the i^{th} operation have already been picked up at time t by QC k from the buffer at QC k . Constraint (12) ensures that v_{ie}^{kt} is equal to 1 when $t > z_{ie}^k$, which implies that the e^{th} container of the i^{th} operation has already been released by a vehicle into the buffer at QC k . Constraint (13) ensures that v_{ie}^{kt} is equal to 1 when $t > z_{ie}^k$, which implies that the e^{th} container of the i^{th} operation has already been picked up by a vehicle at time t from the buffer at QC k . Constraints (14) and (15) ensure that the decision variables are restricted to binary values.

In the model formulation process, the decision variables x_{kie}^{liff} do not have an index for identifying a specific vehicle. However, a given set of x_{kie}^{liff} values can be interpreted as an assignment of delivery tasks to specific individual vehicles as follows. Consider the single QC example in Table 1, with four operation tasks (three containers in the 1st and 4th operation tasks and two containers in the 2nd and 3rd operation tasks) and two vehicles. Suppose that a solution ($x_{001}^{111} = x_{111}^{121} = x_{121}^{122} = x_{122}^{141} = x_{141}^{142} = x_{142}^{F01} = x_{002}^{112} = x_{112}^{113} = x_{113}^{131} = x_{131}^{132} = x_{132}^{143} = x_{143}^{F02} = 1$, and all the other $x_{kie}^{liff} = 0$) is obtained.

A feasible solution can be represented graphically as shown in Figure 4, in which each arrow corresponds to x_{kie}^{liff} with a value of 1. The set $S \cup T$ includes the starting events of vehicles (S) and events related to the transfer operations by vehicles (T). A vehicle that started traveling from its initial location or completed a delivery for a transfer operation can be assigned to another transfer operation (event) (T) or to its final destination (D). Therefore, nodes in $S \cup T$ can be considered as vehicle supply sources, and nodes in $D \cup T$ as vehicle demand

Figure 4. Graphical representation of a feasible solution.

sources. A feasible solution of the x_{kie}^{lf} variables is a one-to-one assignment from a node in $S \cup T$ to a node in $D \cup T$.

In Figure 4, the solid arrow shows an assignment for vehicle 1, while the dashed arrow shows an assignment for vehicle 2. Node a_{01}^O in the supply source is connected to node a_{11}^I in the demand source by a solid arrow, which implies that vehicle 1 starts traveling empty to the pickup position of the task related to the 1st container of the 1st operation of QC 1 and performs the loading and travel for the task. Then, node a_{11}^I in the supply source is connected to node a_{21}^I in the demand source, which implies that after vehicle 1 completes the task related to the 1st container of the 1st operation, it performs tasks related to the two containers of the 2nd operation. Then, it goes on to perform the tasks related to the 1st and 2nd containers of the 4th operation, which implies that nodes a_{22}^I and a_{41}^I in the supply source are connected to nodes a_{41}^I and a_{42}^I in the demand source, respectively. Finally, node a_{42}^I in the supply source is connected to node a_{01}^F in the demand source, which can be interpreted as vehicle 1 ceases to travel after it completes all the assigned tasks. In a similar way, the solution for vehicle 2 can be interpreted as follows. Vehicle 2 delivers the 2nd and 3rd containers of the 1st operation of QC 1, and then the two containers of the 3rd operation, followed by the 3rd container of the 4th operation of QC 1, and finally stops traveling.

Formulations (1)-(15) are for a scheduling problem with precedence constraints and buffer constraints, which is similar to the multiple traveling salesmen problem with precedence constraints and time windows. Both these problems must be NP-hard. Thus, a heuristic algorithm is suggested in the following section to solve this problem.

4. A HEURISTIC ALGORITHM

As presented in the previous section, formulations (1)-(15) are addressed as a scheduling problem with precedence and buffer constraints. The heuristic algo-

rithm proposed in this section consists of two steps. First, a procedure for converting the buffer constraints in the formulation into time window constraints is presented in section 4.1. Then, a heuristic algorithm for solving this scheduling problem with precedence and window constraints is suggested in section 4.2.

4.1 Procedure for Converting buffer Constraints into time window Constraints

A general procedure for converting buffer constraints into time window constraints in a formulation is presented in this section. At the initial stage, we suppose that the y_i^k variables are equal to the s_i^k variables for $i = 1, \dots, m_k$ and $k = 1, \dots, |K|$ and that the events are sequenced in the increasing order of y_i , for $k = 1, \dots, |K|$. We will denote the j^{th} event in the sequence as “event (j)” and the event time of event (i) as y_i for operations of a QC.

The main idea for determining the time window is to calculate the latest allowable pickup time of the inbound container for an unloading task from the buffer space and the earliest allowable release time of the outbound container for a loading task on the buffer space by a vehicle. A time axis is divided into multiple periods, each of which corresponds to a time interval between event times. Detailed explanations can be found in Nguyen and Kim (2009). However, some notations and modifications are needed to deal with the dispatching problem when considering multi-lifts by QCs.

Let b be the capacity of the buffer and m be the number of tasks for the QC. Denote i as the index of the period in which period i corresponds to the time interval between event $i-1$ and i . Furthermore, n_i is the number of containers added at period i . Let q_i be the number of containers in the inventory at period i when the inventory plans up to the preceding event are considered. Let W_i and W_{ij} be the time window for all of the containers of task i and for container j of task i , respectively. Denote $L(W_i)$ and $U(W_i)$ as the lower and upper bounds of time window W_i , and let $L(W_{ij})$ and $U(W_{ij})$ be the lower and upper bounds of time window W_{ij} , respectively.

		L1(3*)	U2(3)	U3(1)	L4(2)	U5(3)	U6(2)	L7(3)	U8(3)	U9(1)	L10(2)	
Period \ Inventory		1	2	3	4	5	6	7	8	9	10	11
5												
4												
3		L1										
2		L1										
1		L1										

Note: * Number of 40ft-containers.

(a) Time window of the first task

	L1(3)	U2(3)	U3(1)	L4(2)	U5(3)	U6(2)	L7(3)	U8(3)	U9(1)	L10(2)	
<div>Period Inventory</div>	1	2	3	4	5	6	7	8	9	10	11
5											
4											
3	L1		U2								
2	L1		U2	U2							
1	L1		U2	U2	U2						

L4' L7' L10'

(b) Time window of the first two tasks

		L1(3)	U2(3)	U3(1)	L4(2)	U5(1)	U6(2)	L7(3)	U8(3)	U9(1)	L10(2)	
Period \ Inventory		1	2	3	4	5	6	7	8	9	10	11
5												
4												
3		L1		U2	U3							
2		L1		U2	U2	U3						
1		L1		U2	U2	U2	U3					

L4' L7' L10'

(c) Time window of the first three tasks

		L1(3)	U2(3)	U3(1)	L4(2)	U5(1)	U6(2)	L7(3)	U8(3)	U9(1)	L10(2)	
Period \ Inventory		1	2	3	4	5	6	7	8	9	10	11
5		L4		L4	L4							
4		L4		L4	L4							
3		L1		U2	U3							
2		L1	L4	U2	U2	U3						
1		L1	L4	U2	U2	U2	U3					

(d) Time window of the first four tasks

		L1(3)	U2(3)	U3(1)	L4(2)	U5(1)	U6(2)	L7(3)	U8(3)	U9(3)	L10(2)	
Period Inventory		1	2	3	4	5	6	7	8	9	10	11
5		L4		L4	L4	L7	L7	L7		L10	L10	
4		L4		L4	L4	L7	L7	L7	L10	L10	L10	
3		L1		U2	U3	L7	L7	L7	L10	U8	U9	U9
2		L1	L4	U2	U2	U3	U5	U6	U6	U8	U9	U9
1		L1	L4	U2	U2	U2	U3	U6	U6	U8	U9	U9

(e) Time window of all tasks

Time window Temporary inventory at period i

Figure 5. Converting buffer constraints into time window constraints.

• **Procedure for converting buffer constraints into time window constraints for a QC**

Step 0: (Initializing) Set $y_i = s_i$ and $W_i = \emptyset$, for $i = 1, \dots, m$. And set $q_i = 0$, for $i = 1, \dots, m$. Start $\xi = 0$.

Step 1: (Next task) $\xi = \xi + 1$. If $\xi > m$, then stop. Otherwise, if event ξ is a loading task, then go to Step 2.1, and if event ξ is an unloading task, then go to Step 2.2.

Step 2: (Determining time windows)

Step 2.1: (Extending the lower bound for the loading task)

Start $i = \xi$.

Repeat

$q_i = q_i + n_i$.

If $q_i \leq b$, then $W_\xi = W_\xi \cup [y_{i-1}, y_i]$ (add period i to the time window for event ξ).

Otherwise, let $c = (n_i + b - q_i)$ be the remaining inventories at period i , the time window of c containers of n_i , $W_{\xi e}$, is $W_{\xi e} = W_\xi \cup [y_{i-1}, y_i]$, $e = 1, \dots, c$.

$i = i - 1$.

Until $i = 0$.

Go to Step 1.

Step 2.2: (Extending the upper bound for the unloading task)

(Set tentative arrival times for loading containers)

Start $k = \xi + 1$.

Repeat

If event k is a loading task, then set the tentative lower bound of the time window for event k to be $s = (\xi + k) / 2$. (For example, if $s = 3$, then the tentative lower bound is set to be the end of period 3).

$k = k + 1$.

Until $k > m$.

Start $i = \xi + 1$.

Repeat

r is the change in the number of containers at event time i .

$p = q_i + r$.

If $p \leq b$, then $W_\xi = W_\xi \cup [y_i, y_{i+1}]$.

Otherwise, let $c = (n_i + b - p)$ be the remaining inventories at period i , the time window of c containers of n_i , $W_{\xi e}$, is $W_{\xi e} = W_\xi \cup [y_i, y_{i+1}]$, $e = 1, \dots, c$.

$i = i + 1$.

Until $i = m$.

Go to Step 1.

The above procedure is illustrated in Figure 5 by using an example of ten loading and unloading operations, with a buffer capacity of five. For the first task, L1 (loading task), we extend the time window by using Step 2.1 in the procedure. Then, it is easy to obtain a time window of L1, $W_1 = [0, y_1]$ (as shown in Figure 5a). For the second task, U2 (unloading task), we extend the time window by using Step 2.2 as follows: first, we estimate the period numbers corresponding to the lower

bounds of the time windows for the loading tasks succeeding U2, and we denote them with an apostrophe ($'$). For example, we consider L4 ($\xi = 2, k = 4, s = 3$), the tentative lower bound of the time window for L4, which is represented by L4', to be 3, indicating the end of period 3. In a similar way, the tentative lower bounds for L7' and L10' are 4.5 and 6, respectively (Figure 5b). Then, we start to extend the upper bound of the time window for U2 one period at a time. For period 3, because L4' is located at the beginning of period 3, the inventory is five, which includes three inbound containers for U2 and two outbound containers for L4. For period 4, one inbound container for U3 may arrive at the buffer; therefore, the inventory level may exceed six if the upper bound for U2 is extended to period 4. Thus, the time window for the first container of U2 becomes $W_{21} = [y_2, y_3]$. However, if one of the three inbound containers for U2 departs from the buffer, then the inventory level is five. Thus, the time window for the second container of U2 is $W_{22} = [y_2, y_4]$. For period 5, two outbound containers for L4 and one of the two inbound containers for U2 may depart from the buffer, and three outbound containers for L7 may arrive at the buffer; therefore, the inventory level remains five. For period 6, three inbound containers for U5 and two outbound containers for L10 may arrive at the buffer; therefore, the inventory level is ten. Thus, the time window for the third container of U2 becomes $W_{23} = [y_2, y_5]$.

For the third task, U3 (unloading task), the calculation is similar to that of U2. First, we estimate the tentative lower bounds for the loading tasks succeeding U3. For L4 ($\xi = 3, k = 4, s = 3.5$), the tentative lower bound of the time window for L4' is 3.5. In the same way, the values of s for L7' and L10' are 5 and 6.5, respectively (as shown in Figure 5c). Then, we start extending the time windows one period at a time. For period 4, the inventory becomes five: two from the arrival for U2, one for U3, and two for L4. For period 5, we have one departure of an inbound container for U2, two departures of outbound containers for L4, and three arrivals of outbound containers for L7; therefore, the inventory remains five. For period 6, one inbound container for U2 departs from the buffer and one inbound container for U5 arrives at the buffer, leaving the inventory at five. Thus, the upper bound of the time window for U3 can be extended to the end of period 6. For period 7, then, if we extend the upper bound of the time window for U3 by one more period, the inventory begins to exceed the limitation of five because of the additional arrivals for U6 and L10. Thus, the final time window of U3 becomes $W_3 = [y_3, y_6]$. For the fourth task, L4 (loading task), the calculation is similar to that of L1. The time window is extended in the left direction one period at a time as far as the buffer capacity allows. Thus, the time window of L4 becomes $W_4 = [0, y_4]$ (as shown in Figure 5d). This process is repeated until the time windows for all of the tasks are determined. The time windows for the remaining tasks are, as shown in Figure 5e, $W_5 = [y_5, y_6]$, $W_6 = [y_6, y_8]$, $W_7 = [y_4, y_7]$, $W_8 = [y_8, y_9]$, $W_9 = [y_9, \infty]$,

and $W_{10} = [y_7, y_{10}]$.

In this way, the constraints on the buffer for z_{ie}^k , (9)-(14), were converted into time window (W_i^k) constraints as follows.

$$z_{ie}^k \in W_{ie}^k = [y_i^k, y_v^k];$$

$$i < v; v \in L^k \cup U^k; i \in U^k; k \in K; e = 1, \dots, n_i \quad (16)$$

$$z_{ie}^k \in W_{ie}^k = [y_u^k, y_i^k];$$

$$u < i; u \in L^k \cup U^k; i \in L^k; k \in K; e = 1, \dots, n_i \quad (17)$$

4.2 Heuristic Algorithm

By applying the above procedure for converting buffer constraints, (9)-(14), into time window constraints, the formulation becomes a scheduling problem with precedence and time window constraints. This section proposes a heuristic algorithm for solving this formulation. To precisely describe the heuristic algorithm, some notations and modifications to Nguyen and Kim (2009) are required as follows: in the initial stage, we suppose that y_i^k variables equal to s_i^k variables $k = 1, \dots, |K|$, $i = 1, \dots, m_k$ are given. In addition, let z_{ie}^k s be set to $L(W_i^k)$. Then, events are sequenced in an increasing order of y_i^k , and the sequence of events (z_{ie}^k s) in T is set the same as that of the y_i^k variables.

Denote the f^{th} event in sequence T as event (f), the event time of event (e) as z_e for the operation of vehicles, the time required for a vehicle to be ready for event (f) after it experiences event (e) as c_{ef} (which corresponds to the notation of c_{kie}^{lif}), the pure travel time from the location of event (e) to the location of event (f) as t_{ef} , the decision variable for the assignment of event (e) to event (f) as x_{ef} , and the time window of z_e as W_e .

Let T_ξ be a subset of T that includes only the first ξ events in the sequence, and let L_ξ and U_ξ be subsets of T_ξ that include only loading and unloading tasks, respectively. Then, the constraint subset ξ of (2)-(4) and (16)-(17) can be written as follows:

$$\text{Constraint subset } \xi$$

$$\sum_{e \in S \cup T_\xi} x_{ef} = 1, \quad \forall f \in D \cup T_\xi \quad (18)$$

$$\sum_{f \in D \cup T_\xi} x_{ef} = 1, \quad \forall e \in S \cup T_\xi \quad (19)$$

$$z_f - (z_e + c_{ef}) \geq M(x_{ef} - 1), \quad \forall e \in S \cup T_\xi$$

$$\forall e \in S \cup T_\xi \quad \text{and } f \in T_\xi \quad (20)$$

$$z_e \in W_e, \quad \forall e \in L_\xi \cup U_\xi \quad (21)$$

$$x_{ef} = 0 \text{ or } 1, \quad \forall e \in S \cup T_\xi \quad \text{and } f \in D \cup T_\xi \quad (22)$$

Then, the following algorithm assigns vehicles to tasks.

• Heuristic solution procedure for solving the scheduling problem with precedence and time window constraints.

Step 0: Initializing. Set $y_i^k = s_i^k$, $i = 1, \dots, m_k$, $k \in K$. Also, evaluate the time window for each z_{ie}^k . Set $z_{ie}^k = L(W_{ie}^k)$, and set $z_{ie}^{kU} = U(W_{ie}^k)$ for all $i \in L^k \cup U^k$, $e = 1, \dots, n_i$, $k \in K$. Let $z_{0i}^0 = 0$, for all $i \in V$. Start $\xi = 0$.

Step 1: Next Task. $\xi = \xi + 1$. If $\xi > m$ (the total number of tasks in sequence T), then go to Step 5. Otherwise, sequence the events in the increasing order of y_i^k , and the sequence of events (z_{ie}^k s) in T is set the same as that of y_i^k variables, and then go to Step 2.

Step 2: Feasibility Check.

Step 2.1: Check the existence of a feasible solution to constraints (18)-(20). If there is a feasible solution, then go to Step 2.2. Otherwise, go to Step 3.1.

Step 2.2: Check whether the event time of event (ξ) for vehicles satisfies constraint (21). If it is satisfied, then go to Step 4. Otherwise, go to Step 3.2.

Step 3: Delaying Event Time.

Step 3.1: Let the event time of event (ξ) for vehicles be $z_{\lambda\mu}^\gamma (= z_\xi)$.

$$\text{Let } \pi_{e\xi}^\gamma = \min_{e \in S \cup T_{\xi-1}} [\max\{c_{e\xi} - (z_\xi - z_e), 0\}].$$

Then, $z_{\lambda\mu}^\gamma = z_{\lambda\mu}^\gamma + \pi_{e\xi}^\gamma$. Go to Step 2.1.

Step 3.2: Let the event time of event (ξ) for QC be y_λ^γ , and the event time of z_ξ^U be y_σ^γ .

Let $\pi_\xi = z_\xi - z_\xi^U$. Then, $y_r^\gamma = y_r^\gamma + \pi_\xi$, for $r \geq \sigma$ (Note $\sigma = \lambda$ for $\xi \in L_\xi$). Update $z_{rp}^\gamma = z_{rp}^\gamma + \pi_\xi$, for $r \geq \lambda$, $p = \mu+1, \dots, n_r$ (Note $z_{rp}^\gamma \neq 0$), and $z_{rp}^{U\gamma} = z_{rp}^{U\gamma} + \pi_\xi$, for $r \geq \lambda$, $p = \mu, \dots, n_r$. Go to Step 2.2.

Step 4: Time Window Adjustment.

Let $z_{\lambda\mu}^\gamma$ be an event time of $z_{\lambda\mu}^\gamma$ in the previous stage.

If $\xi \in L_\xi$ and the upper bound of a related preceding unloading task r , $p = 1, \dots, n_r$, $z_{rp}^{U\gamma} \in [z_{\lambda\mu}^\gamma, z_{\lambda\mu}^\gamma]$, then it must be extended to $z_{rp}^{U\gamma} = z_{\lambda\mu}^\gamma$. Go to Step 1.

If $\xi \in U_\xi$ and the lower bound of a related succeeding loading task r , $p = 1, \dots, n_r$, $z_{rp}^\gamma \in (z_{\lambda\mu}^\gamma, z_{\lambda\mu}^{U\gamma}]$, then it must be extended to $z_{rp}^\gamma = z_{\lambda\mu}^\gamma$. Go to Step 1.

Step 5: Task Assignment. Evaluate t_{ef} . Solve the assignment problem with the objective of minimizing the total travel distance subject to constraint subset $|T|$. Stop.

Feasibility Check: This step consists of two steps. In the first step, for given values of z_{ie}^k variables, the feasibility can be checked by solving a maximum cardinality matching problem in the bipartite graph (Evans and Minieka 1992). When the maximum cardinality is the same as $\{S \cup T\}$, it can be concluded that the constraint subset ζ of (18)-(20) has a feasible solution. Otherwise, it is infeasible. For a given feasible solution in Step 2.1, Step 2.2 checks whether z_{ie}^k does not exceed the upper bound of the time window.

Delaying Event Time: To satisfy the constraint subset of (18)-(20), one or more x_{kie}^{lf} must be allowed to become 1 by relaxing constraint (20). That is, the event time of event (ζ) is delayed by such an amount that at least one $x_{e\zeta}$, for $e < \zeta$, becomes 1, which is denoted as $\pi_{e\zeta}$. Further, in some cases, for satisfying the constraint subsets of (18)-(20), it may be necessary to increase z_{ie}^k beyond the upper bound of the time window (constraint subset (21)), in which the event time for QCs, y_r^r , must be increased, and the time windows must be updated simultaneously. In the process, when the current constraint subset becomes feasible, the iteration is stopped, and we proceed to the next stage.

Time Window Adjustment: After obtaining a feasible solution for constraint subset ζ of (18)-(22), the time window must be adjusted if possible. When a feasible solution is found, the event time $z_{\lambda u}^r$ of event (ζ) will be determined. Then, the time window of the event related to that of event (ζ) may be adjusted by extending its bound. For a loading event (ζ), the upper bound of the related preceding unloading tasks may be extended. For an unloading event (ζ), the lower bound of the related succeeding loading tasks may be extended.

An application of the heuristic algorithm to the example in Table 1, which includes QC 1, $b_1 = 5$, $m_1 = 4$, and two vehicles, is illustrated as follows. The event numbers, e and f , in Table 3 correspond to task numbers for QC 1.

Table 3. The time required for a vehicle to be ready for event (f) after it experiences event (e), cef, (in seconds).

To f	0*	1	2	3	4	5	6	7	8	9	10
From e		(a ₁₁ ¹)	(a ₁₂ ¹)	(a ₁₃ ¹)	(a ₂₁ ¹)	(a ₂₂ ¹)	(a ₃₁ ¹)	(a ₃₂ ¹)	(a ₄₁ ¹)	(a ₄₂ ¹)	(a ₄₃ ¹)
0	-	180	200	180	80	80	200	180	80	80	80
1	-	-	220	200	0	0	240	220	0	0	0
2	-	-	-	240	0	0	240	220	0	0	0
3	-	-	-	-	0	0	240	220	0	0	0
4	-	-	-	-	-	220	260	240	220	220	220
5	-	-	-	-	-	-	240	260	240	240	240
6	-	-	-	-	-	-	-	220	0	0	0
7	-	-	-	-	-	-	-	-	0	0	0
8	-	-	-	-	-	-	-	-	-	220	220
9	-	-	-	-	-	-	-	-	-	-	240

Note: * Initial starting event of vehicles.

Step 0: *Initializing.* Set $y_i^l = (0, 230, 230, 460)$. The time windows for the loading operations are $z_{ie}^l = 0$, $e = 1, \dots, 3$; and $0 \leq z_{3e}^l \leq 230$, $e = 1, 2$; while the time windows for the unloading operations are $230 \leq z_{2e}^l \leq \infty$, $e = 1, 2$; and $460 \leq z_{4e}^l \leq \infty$, $e = 1, \dots, 3$. Thus, $z_{ie}^l = (0, 0, 0, 230, 230, 0, 0, 460, 460, 460)$, and $z_{ie}^{lu} = (0, 0, 0, \infty, \infty, 230, 230, \infty, \infty, \infty)$, $i = 1, \dots, 4$, $e = 1, \dots, n_i$. Let $z_{0i}^o = 0$, $i = 1, 2$. Start $\zeta = 0$.

Step 1: *Next Task.* $\zeta = \zeta + 1 = 1$. Sequence the events in the increasing order of y_i^k , $y_i^l = (y_1^l, y_2^l, y_3^l, y_4^l)$, and the sequence of events (z_{ie}^k s) in T is $z_{ie}^l = (z_{11}^l, z_{12}^l, z_{13}^l, z_{21}^l, z_{22}^l, z_{31}^l, z_{32}^l, z_{41}^l, z_{42}^l, z_{43}^l)$.

Step 2: *Feasibility Check.*

Step 2.1: There is no feasible solution to constraints (18)-(20).

Step 3: *Delaying Event Time.*

Step 3.1: Calculate $\pi_{e,1} = \min \{180-(0-0); 180-(0-0)\} = 180 = \pi_{1,1}$. Thus, $z_{11}^l = 180$.

Step 2: *Feasibility Check.*

Step 2.1: At least, a feasible solution to constraints (18)-(20) can be found.

Step 2.2: Compare z_{11}^l with its upper bound. It does not satisfy constraint (21).

Step 3: *Delaying Event Time.*

Step 3.2. Let $\pi_1 = 180-0 = 180$. Then, $y_i^l = (180, 410, 410, 640)$, and update $z_{ie}^l = (180, 0, 0, 410, 410, 0, 0, 640, 640, 640)$, and $z_{ie}^{lu} = (180, 180, 180, \infty, \infty, 410, 410, \infty, \infty, \infty)$.

Step 2: *Feasibility Check.*

Step 2.2. Compare z_{11}^l with its upper bound. It satisfies constraint (21).

Step 4: *Time window Adjustment.* There is no preceding unloading task of task (ζ). Thus, there is no adjustment of time windows.

Step 1: *Next Task.* $\zeta = 2$. Sequence the events in the increasing order of y_i^k , $y_i^l = (y_1^l, y_2^l, y_3^l, y_4^l)$, and the sequence of events (z_{ie}^k s) in T is $z_{ie}^l = (z_{12}^l, z_{13}^l, z_{11}^l, z_{21}^l, z_{22}^l, z_{31}^l, z_{32}^l, z_{41}^l, z_{42}^l, z_{43}^l)$.

Step 2: *Feasibility Check.*

Step 2.1: There is no feasible solution to constraints (18)-(20).

Step 3: *Delaying Event Time*.

Step 3.1: Calculate $\pi_{e_2} = \min \{220-(0-180); 200-(0-0)\} = 200 = \pi_{e_2}$. Thus, $z_{12}^1 = 200$.

Step 2: *Feasibility Check*.

Step 2.1: At least, a feasible solution to constraints (18)-(20) can be found.

Step 2.2: Compare z_{12}^1 with its upper bound. It does not satisfy constraint (21).

Step 3: *Delaying Event Time*.

Step 3.2: Let $\pi_2 = 200-180 = 20$. Then, $y_i^1 = (200, 430, 430, 660)$, and update $z_{ie}^1 = (180, 200, 0, 430, 430, 0, 0, 660, 660, 660)$, and $z_{ie}^{1U} = (180, 200, 200, \infty, \infty, 430, 430, \infty, \infty, \infty)$.

Step 2: *Feasibility Check*.

Step 2.2: Compare z_{12}^1 with its upper bound. It satisfies constraint (21).

Step 4: *Time window Adjustment*. There is no preceding unloading task of task (ξ). Thus, there is no adjustment of time windows.

Step 1: *Next Task*. $\xi = 3$. Sequence the events in the increasing order of y_i^k , $y_i^1 = (y_1^1, y_2^1, y_3^1, y_4^1)$, and the sequence of events (z_{ie}^k s) in T is $z_{ie}^1 = (z_{13}^1, z_{11}^1, z_{12}^1, z_{21}^1, z_{22}^1, z_{31}^1, z_{32}^1, z_{41}^1, z_{42}^1, z_{43}^1)$.

Step 2: *Feasibility Check*.

Step 2.1: There is no feasible solution to constraints (18)-(20).

Step 3: *Delaying Event Time*.

Step 3.1: Calculate $\pi_{e_3} = \min \{200-(0-180); 240-(0-200)\} = 380 = \pi_{e_3}$. Thus, $z_{13}^1 = 380$.

Step 2: *Feasibility Check*.

Step 2.1: At least, a feasible solution to constraints (18)-(20) can be found.

Step 2.2: Compare z_{13}^1 with its upper bound. It does not satisfy constraint (21).

Step 3: *Delaying Event Time*.

Step 3.2: Let $\pi_3 = 380-200 = 180$. Then, $y_i^1 = (380, 610, 610, 840)$, and update $z_{ie}^1 = (180, 200, 380, 610, 610, 0, 0, 840, 840, 840)$, and $z_{ie}^{1U} = (180, 200, 380, \infty, \infty, 610, 610, \infty, \infty, \infty)$.

Step 2: *Feasibility Check*.

Step 2.2: Compare z_{13}^1 with its upper bound. It satisfies constraint (21).

Step 4: *Time window Adjustment*. There is no preceding unloading task of task (ξ). Thus, there is no ad-

justment of time windows.

Step 1: *Next Task*. $\xi = 4$. Sequence the events in the increasing order of y_i^k , $y_i^1 = (y_1^1, y_2^1, y_3^1, y_4^1)$, and the sequence of events (z_{ie}^k s) in T is $z_{ie}^1 = (z_{11}^1, z_{12}^1, z_{13}^1, z_{21}^1, z_{22}^1, z_{31}^1, z_{32}^1, z_{41}^1, z_{42}^1, z_{43}^1)$.

Step 2: *Feasibility Check*.

Step 2.1: At least, a feasible solution to constraints (18)-(20) can be found.

Step 2.2: Compare z_{21}^1 with its upper bound. It satisfies constraint (21).

Step 4: *Time window Adjustment*. There is a succeeding loading task of task (ξ). However, since its lower bound z_{31}^1 is less than z_{21}^1 , it cannot be extended.

Step 1: *Next Task*. $\xi = 5$. Sequence the events in the increasing order of y_i^k , $y_i^1 = (y_1^1, y_2^1, y_3^1, y_4^1)$, and the sequence of events (z_{ie}^k s) in T is $z_{ie}^1 = (z_{11}^1, z_{12}^1, z_{13}^1, z_{22}^1, z_{21}^1, z_{31}^1, z_{32}^1, z_{41}^1, z_{42}^1, z_{43}^1)$.

Step 2: *Feasibility Check*.

Step 2.1: At least, a feasible solution to constraints (18)-(20) can be found.

Step 2.2: Compare z_{22}^1 with its upper bound. It satisfies constraint (21).

Step 4: *Time window Adjustment*. There is a succeeding loading task of task (ξ). However, since its lower bound z_{31}^1 is less than z_{22}^1 , it cannot be extended.

This process is repeated until ξ is greater than the total number of containers (10). Finally, the solution obtained by the assignment problem is as follows: $x_{001}^{111} = x_{113}^{113} = x_{121}^{121} = x_{132}^{132} = x_{141}^{141} = x_{143}^{143} = x_{002}^{F01} = x_{002}^{112} = x_{112}^{122} = x_{122}^{131} = x_{131}^{142} = x_{142}^{F02} = 1$, and all the other, $x_{kie}^{diff} = 0$; and $y_i^1 = (380, 610, 850, 1080)$, $z_{ie}^1 = (180, 200, 380, 610, 610, 850, 850, 1080, 1080, 1300)$ for $i = 1, \dots, 4$; $e = 1, \dots, n_i$.

Figure 6 illustrates the solution procedure. The final solution can be explained as follows: from the initial location, vehicle 1 and vehicle 2 start to travel empty to the pickup positions for the 1st and 2nd containers of the 1st loading task, respectively, and perform the loaded travel for the tasks. After vehicle 1 moves the 1st container, it goes to move the 3rd container of the 1st loading task. Because the 1st loading task of QC 1 consists of three containers that are transferred by the triple lift, QC 1 must wait 380 s for the arrival of the containers at the buffer. Next, vehicle 1 waits 230 s for picking up the 1st container of the 2nd unloading task and delivers it to storage block. Then, vehicle 1 goes to the storage block of the 2nd container of the 3rd loading task to transfer the container to QC 1. After then, it waits 230 s for the arrival of the 1st container of the 4th unloading task, and goes to the storage block of its container. Finally, it moves to QC 1 for delivering the 3rd container of the 4th unloading task to its storage block. In a similar way, after

Figure 6. An illustration of the heuristic algorithm.

Table 4. Comparison between results by the heuristic algorithm and those by formulations (1)-(15).

No.	Problem	Objective Value			Computational Time (s)		
		Heuristic algorithm (A)	Optimal solution (B)	Ratio (=A/B)	Heuristic algorithm (C)	Optimal solution (D)	Ratio (= C/D)
1	Triple(3-1-4-3)*	3200	2540	1.260	0.39	60	0.0065
2	Triple(4-1-4-3)	2880	2280	1.263	0.578	58	0.0100
3	Triple(5-1-4-3)	1820	1820	1.000	0.391	53	0.0074
4	Triple(6-1-4-3)	1760	1760	1.000	0.265	21	0.0126
5	Triple(3-1-4-6)	3100	2540	1.220	0.453	372	0.0012
6	Triple(4-1-4-6)	2680	2280	1.175	0.469	95	0.0049
7	Triple(5-1-4-6)	1820	1820	1.000	0.407	58	0.0070
8	Triple(6-1-4-6)	1760	1760	1.000	0.406	21	0.0193
9	Triple(5-1-6-3)	3420	2740	1.248	0.547	1037	0.0005
10	Triple(6-1-6-3)	2660	2480	1.073	0.328	63	0.0052
11	Triple(7-1-6-3)	2400	2400	1.000	0.297	84	0.0035
12	Triple(8-1-6-3)	2320	2320	1.000	0.36	68	0.0053
13	Triple(9-1-6-3)	2240	2240	1.000	0.359	90	0.0040
14	Triple(5-1-6-6)	2700	2700	1.000	0.64	162	0.0040
15	Triple(6-1-6-6)	2660	2480	1.073	0.578	101	0.0057
16	Triple(7-1-6-6)	2400	2400	1.000	0.531	53	0.0100
17	Triple(8-1-6-6)	2320	2320	1.000	0.516	98	0.0053
18	Triple(9-1-6-6)	2240	2240	1.000	0.468	36	0.0130
19	Triple(6-1-8-3)	3560	3200	1.113	0.797	282	0.0028
20	Triple(7-1-8-3)	3120	3120	1.000	0.469	88	0.0053
21	Triple(8-1-8-3)	3040	3040	1.000	0.594	258	0.0023
22	Triple(9-1-8-3)	2960	2960	1.000	0.453	125	0.0036
23	Triple(6-1-8-6)	3560	3200	1.113	0.844	210	0.0040
24	Triple(7-1-8-6)	3120	3120	1.000	0.578	352	0.0016
25	Triple(8-1-8-6)	3040	3040	1.000	0.656	229	0.0029
26	Triple(9-1-8-6)	2960	2960	1.000	0.562	111	0.0051
27	Triple(7-1-10-3)	3840	3840	1.000	0.531	677	0.0008
28	Triple(8-1-10-3)	3760	3760	1.000	0.531	421	0.0013
29	Triple(9-1-10-3)	3680	3680	1.000	0.484	437	0.0011
30	Triple(10-1-10-3)	3600	3600	1.000	0.672	464	0.0014
31	Triple(11-1-10-3)	3520	3520	1.000	0.485	463	0.0010
32	Triple(12-1-10-3)	3440	3440	1.000	0.484	247	0.0020
33	Triple(7-1-10-6)	3840	3840	1.000	0.703	8997	0.0001
34	Triple(8-1-10-6)	3760	3760	1.000	0.563	479	0.0012
35	Triple(9-1-10-6)	3680	3680	1.000	0.547	296	0.0018
36	Triple(10-1-10-6)	3600	3600	1.000	0.609	466	0.0013
37	Triple(11-1-10-6)	3520	3520	1.000	0.656	348	0.0019
38	Triple(12-1-10-6)	3440	3440	1.000	0.625	200	0.0031
Average				1.040			
Min				1.000			
Max				1.263			

Note: * Number of vehicles per QC-Number of QCs-Number of operations per each QC-Buffer capacity.

vehicle 2 moves the 2nd container of the 1st loading task, it waits 410 s for the arrival of the 2nd container of the 2nd unloading task and delivers it to the storage block. Next, it goes to the storage block of the 1st container of the 3rd loading task and transfers the container to QC 1. Then, vehicle 2 waits 230 s for the arrival of the 2nd container of the 4th unloading task, and finally goes to the storage block of its container.

5. NUMERICAL EXPERIMENTS

This section discusses a numerical experiment to compare the performance of the heuristic algorithm with that of the optimizing method and analyzes the properties of the heuristic algorithm proposed in this study.

5.1 Comparing the Heuristic Algorithm and the Optimizing Method

A number of problems were generated to compare the optimal solutions obtained from the formulations (1)~(15) with those from the heuristic algorithm. In the generated problems, the total number of vehicles, the number of operations for each QC, and the buffer capacity ranged from 3 to 12, from 4 to 10, and from 3 to 6, respectively. Moreover, the number of containers per operation was three to represent triple lifts. The formulations (1)~(15) were solved by ILOG CPLEX® 10.0.0. The results of the generated problems are shown in Table 4. The ratios of the objective values obtained by the heuristic algorithm to those obtained by ILOG CPLEX® ranged from 1.000 to 1.263, with an average value of 1.040. The ratios of the computational times for the heuristic algorithm to those of ILOG CPLEX® ranged from 0.0001 to 0.0193, with an average value of 0.0045.

5.2 Sensitivity Analysis

In order to analyze and evaluate the effects of the number of vehicles and the buffer capacity on the sum of the QC delay times, total travel time for the vehicles, and completion time, various numerical experiments were conducted as follows: the number of QCs, the number of operations per QC, and the number of containers for each operation were 2, 20, and 3, respectively. The number of vehicles was varied from 8 to 14. The buffer capacities under the QCs in the apron were considered in the case of no buffer (buffer capacity of zero) and in the cases of buffers from 3 to 6. The performances of the experiments were analyzed in terms of the sum of the QC delay times, total travel time of the vehicles, and completion time. For each combination, ten problems were generated randomly. Therefore, the total number of problems solved was 350. The results of these numerical experiments are shown in Figure 11-15.

In addition, this study compared the QC productivity, the sum of the QC delay times, total travel time of

the vehicles, and the average time loss caused by the lateness of arrival of vehicles for each operation between different types of operations by QCs (single, tandem, and triple lift). For this purpose, it was assumed that there was no buffer space under the QCs in the apron. The results of this comparison are shown in Figure 7-10.

Figure 7 illustrates the comparison between different types of QC operations (single, tandem, and triple lift) of the QC productivity. It was shown that the QC productivity of tandem lift was best as the number of vehicles was small. While, the QC productivity of triple lift was best as the number of vehicles was large. When the number of vehicles increased in this study, the QC productivity of single lift did not change and that of tandem lift increased slightly. However, the QC productivity of triple lift increased rapidly as the number of vehicles increased. When the number of vehicles (12 as shown in Figure 7) was sufficient to support tandem and triple lifts by QCs, the ratios of the QC productivity of tandem and triple lift to that of single lift were mostly twice and three times, respectively.

Figure 7. Comparison of QC productivity among various of QC operations.

Figure 8. Comparison of the sum of the QC delay times among various types of QC operations.

The comparison between different types of QC operations of the sum of the QC delay times is presented in Figure 8. The sum of the QC delay times of single lift

was best because the number of containers handled by single lifts was a half and one third in comparison with those by tandem and triple lifts. When the number of vehicles increased, the sum of the QC delay times of tandem lift reduced slightly, while the sum of the QC delay times of triple lift reduced rapidly. When the Number of vehicles reached 12 (as shown in Figure 8), the reductions were negligible. Similarly, the comparisons of total travel time of vehicles are shown in Figure 9. Total travel time of vehicles of single lift by QCs was best. However, total travel time of vehicles of single, tandem, and triple lift by QCs did not change as the number of vehicles increased.

Figure 9. Comparison of total travel time of Vehicles among various types of QC operations.

Figure 10. Comparison of the average time loss among various types of QC operations.

For the time loss caused by the lateness of arrival of vehicles, Figure 10 shows comparisons between two different types of QC operations for the average time loss. It was shown that the average time loss for tandem lifts by QCs was better than that for triple lifts by QCs. For example, when the number of vehicles was four, the average time loss for the tandem and the triple lifts of QCs were 19.50 (s) and 34.55 (s), respectively. However, the average time loss for both cases decreased rapidly with an increase in the number of vehicles. When the number of vehicles reached a value of nine (as shown in

Figure 10), the reductions were negligible, indicating that the number of vehicles was sufficient to support tandem and triple lifts by QCs.

The results from Figure 11 show the changes in the sum of the QC delay times for various buffer capacity values and different numbers of vehicles. For each buffer capacity value, the sum of the QC delay times decreased rapidly with an increase in the number of vehicles. When the number of vehicles exceeded a certain value (12 vehicles in Figure 11), the reductions were negligible, which was supported by a statistical test with a significance level of 5%. This implies that the number of vehicles (12) was sufficient to support the triple lifts of the QCs. For changes in the buffer capacity, the sum of the QC delay times was reduced by increasing the buffer capacity. The case with no buffer was the worst. However, when the buffer capacity and the number of vehicles reached 5 and 12, respectively, the sum of the QC delay times was not affected significantly. Similarly, the changes in the completion time are shown in Figure 12. When the buffer capacity was larger than three, the completion time was slightly reduced with an increase in the number of vehicles. When the buffer capacity reached six, the completion time showed a considerable reduction in comparison with the others.

Figure 11. Effects of the number of vehicles on the sum of the QC delay times.

Figure 12. Effects of the number of vehicles on the Completion time.

Figure 13. Effects of the number of vehicles on total travel time of vehicles.

Figure 14. Improvements in the sum of the delay times for cases with 3 to 6 buffers compared with that with no buffer.

Figure 15. Improvements in the completion time for cases with 3 to 6 buffers compared with that with no buffer.

Figure 13 shows a comparison of the total travel times of the vehicles. Similarly, the total travel time of the vehicles decreased rapidly with an increase in the number of vehicles. However, the cases with smaller buffer sizes showed shorter travel distances than cases

with larger buffer sizes. It can be concluded that the reduction in the objective of the delay time of the QCs was obtained at the cost of the other objective of the travel distance of the vehicles.

Figures 14 and 15 illustrate the differences in the sum of the QC delay times and completion times between the cases with buffer capacities from 3 to 6 and the case with no buffer. These differences increased with an increase in the buffer capacity. For example, when the number of vehicles was 12, the differences in the delay times and completion times ranged from 7.8% to 68.0% and from 2.3% to 33.0%, respectively, as the buffer capacity changed from 3 to 6. This can be explained as an improvement between an ALV system with buffers and an AGV system with no buffer.

6. CONCLUSIONS AND FURTHER STUDIES

This study discussed a method for dispatching vehicles to support efficient multi-lifts by QCs, especially tandem and triple lifts. The problem was introduced as a scheduling problem with precedence and buffer constraints. A mixed integer programming model was proposed for the vehicle scheduling problem considering the multi-lifts of QCs. Then, a procedure for converting buffer constraints into time window constraints and a heuristic algorithm for solving the scheduling problem with precedence constraints and time window constraints were suggested. A number of numerical experiments were conducted to compare and analyze the proposed heuristic algorithm. First, several problems were generated to compare the optimal solution from the mathematical formulations with those from the heuristic algorithm. The results of this experiment showed that the ratio of the objective values obtained from the heuristic algorithm to the optimal objective values obtained from the ILOG CPLEX® was between 1.000 and 1.263, with an average value of 1.040. The ratio of the computational time for the heuristic algorithm to that of ILOG CPLEX® ranged from 0.0001 to 0.0193, with an average value of 0.0045. Therefore, it could be concluded that the heuristic algorithm showed good performance.

From the comparisons between different types of QC operations (single, tandem, and triple lift), it can be said that the QC productivity of tandem lift was best as the number of vehicles was small. While, the QC productivity of triple lift was best as the number of vehicles was large. For the time loss caused by the lateness of arrival of vehicles, the average time loss for tandem lifts by QCs was better than that for triple lifts by QCs. The average time loss for both cases decreased with an increase in the number of vehicles.

According to the sensitive analyses, both the sum of the QC delay times and the completion time decreased with an increase in the number of vehicles and the buffer capacity. However, the effect of the buffer size was significant when the number of vehicles was

small, while the effect decreased with an increase in the number of vehicles. The performance of the ALV system with buffers was better than that of the AGV system with no buffer. For the total travel time of the vehicles, when the number of vehicles increased, the total travel time of the vehicles reduced. However, the cases with smaller buffer sizes showed shorter travel distances than cases with larger buffer sizes.

This study only evaluated and analyzed the heuristic algorithm as a dispatching method for vehicles in a static environment. Thus, a simulation study will be necessary to evaluate the performance of the algorithm proposed in this study in a dynamic environment. Moreover, the integrated scheduling problem for vehicle travel, YC operations, and QC operations may be addressed as a further extension of this study.

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