THE q -ANALOGUE OF TWISTED LERCH TYPE EULER ZETA FUNCTIONS

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ABSTRACT. q -Volkenborn integrals ([8]) and fermionic invariant q -integrals ([12]) are introduced by T. Kim. By using these integrals, Euler q -zeta functions are introduced by T. Kim ([18]). Then, by using the Euler q -zeta functions, S.-H. Rim, S. J. Lee, E. J. Moon, and J. H. Jin ([25]) studied q -Genocchi zeta functions. And also Y. H. Kim, W. Kim, and C. S. Ryoo ([7]) investigated twisted q -zeta functions and their applications.

In this paper, we consider the q -analogue of twisted Lerch type Euler zeta functions defined by

$$\zeta_{E,q,\varepsilon}(s) = [2]_q \sum_{n=0}^{\infty} \frac{(-1)^n \varepsilon^n q^{sn}}{[n]_q}$$

where $0 < q < 1$, $\Re(s) > 1$, $\varepsilon \in T_p$, which are compared with Euler q -zeta functions in the reference ([18]). Furthermore, we give the q -extensions of the above twisted Lerch type Euler zeta functions at negative integers which interpolate twisted q -Euler polynomials.

1. Introduction

Let p be a fixed odd prime number. Throughout this paper, $\mathbb{Z}_p$, $\mathbb{Q}_p$ and $\mathbb{C}_p$ will, respectively, the ring of p -adic rational integers, the field of p -adic rational numbers and the p -adic completion of the algebraic closure of $\mathbb{Q}_p$. The p -adic absolute value in $\mathbb{C}_p$ is normalized so that $|p|_p = \frac{1}{p}$. When one talks of q -extension, q is variously considered as an indeterminate, a complex number $q \in \mathbb{C}$ or a p -adic number $q \in \mathbb{C}_p$. If $q \in \mathbb{C}$, one normally assumes $|q| < 1$. If $q \in \mathbb{C}_p$, one normally assumes $|1 - q|_p < p^{\frac{1}{1-p}}$ so that $q^x = \exp(x \log q)$ for each $x \in \mathbb{Z}_p$. We use the notation

$$(1.1) \quad [x]_q = \frac{1 - q^x}{1 - q} \quad \text{and} \quad [x]_{-q} = \frac{1 - (-q)^x}{1 + q} \quad \text{cf. [2-32]}$$

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for all $x \in \mathbb{Z}_p$. For a fixed odd positive integer d with $(p, d) = 1$, set

$$\begin{aligned} X &= X_d = \varprojlim_n \mathbb{Z}/dp^n\mathbb{Z}, \quad X_1 = \mathbb{Z}_p, \\ X^* &= \bigcup_{\substack{0 \leq a < dp \\ (a, p) = 1}} (a + dp\mathbb{Z}_p), \\ a + dp^n\mathbb{Z}_p &= \{x \in X \mid x \equiv a \pmod{dp^n}\}, \end{aligned}$$

where $a \in \mathbb{Z}$ lies in $0 \leq a < dp^n$. For any $n \in \mathbb{N}$,

$$(1.2) \quad \mu_q(a + dp^n\mathbb{Z}_p) = \frac{q^a}{[dp^n]_q}$$

is known to be a distribution on $\mathbb{Z}_p$, cf. [2-32].

We say that f is a uniformly differentiable function at a point $a \in \mathbb{Z}_p$ and denote this property by $f \in UD(\mathbb{Z}_p)$, if the difference quotients

$$(1.3) \quad F_f(x, y) = \frac{f(x) - f(y)}{x - y}$$

have a limit $l = f'(a)$ as $(x, y) \rightarrow (a, a)$, cf. [2-32].

The bosonic p -adic q -integral of a function $f \in UD(\mathbb{Z}_p)$ was defined by

$$(1.4) \quad I_q(f) = \int_{\mathbb{Z}_p} f(x) d\mu_q(x) = \lim_{n \rightarrow \infty} \frac{1}{[p^n]_q} \sum_{x=0}^{p^n-1} f(x) q^x.$$

In the sense of fermionic, let us define the fermionic p -adic q -integral as

$$(1.5) \quad I_{-q}(f) = \int_{\mathbb{Z}_p} f(x) d\mu_{-q}(x) = \lim_{n \rightarrow \infty} \frac{1}{[p^n]_{-q}} \sum_{x=0}^{p^n-1} f(x) (-q)^x,$$

(see, [2, 7, 11, 12, 18, 25, 28, 32]).

We note that q -Volkenborn integrals ([8]) and fermionic invariant q -integrals ([12]) are introduced by T. Kim. By using these integrals, Euler q -zeta functions are introduced by T. Kim ([18]). Then, by using the Euler q -zeta functions, S.-H. Rim, S. J. Lee, E. J. Moon, and J. H. Jin ([25]) studied q -Genocchi zeta functions. And also Y. H. Kim, W. Kim, and C. S. Ryoo ([7]) investigated twisted q -zeta functions and their applications.

In Section 2, we define the twisted q -Euler polynomials and numbers of higher order by using the fermionic p -adic q -integral. Let $\mathbb{N}$ be the set of positive integers. Let $T_p = \bigcup_{n \geq 1} C_{p^n} = \lim_{n \rightarrow \infty} C_{p^n} = C_{p^\infty}$ be the locally constant space, where $C_{p^n} = \{\varepsilon \mid \varepsilon^{p^n} = 1\}$ is the cyclic group of order p^n . In Section 3, we consider the q -analogue of twisted Lerch type Euler zeta functions defined by

$$\zeta_{E, q, \varepsilon}(s) = [2]_q \sum_{n=0}^{\infty} \frac{(-1)^n \varepsilon^n q^{sn}}{[n]_q},$$

where $0 < q < 1$, $\Re(s) > 1$, $\varepsilon \in T_p$, which are compared with Euler q -zeta functions in T. Kim ([18]). And we also give the q -extensions of the above

twisted Lerch type Euler zeta functions at negative integers which interpolate twisted q -Euler polynomials.

2. The twisted q -Euler numbers and polynomials

For $m, k \in \mathbb{N}$ and $\varepsilon \in \mathbb{T}_p$, by using the fermionic p -adic q -integral, the twisted q -Euler polynomials $E_{m,q,\varepsilon}^{(-m,k)}$ of order k in the variable x in $\mathbb{C}_p$ are defined by

$$(2.1) \quad E_{m,q,w}^{(-m,k)}(x) = \int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} [x + x_1 + \cdots + x_k]_q^m \varepsilon^{x_1 + \cdots + x_k} q^{-x_1(m+1) - \cdots - x_k(m+k)} d\mu_{-q}(x_1) \cdots d\mu_{-q}(x_k),$$

(see [4, 5, 20, 26, 27, 28, 31, 29]). From (2.1) with $x = 0$, we can define the twisted q -Euler numbers of order k as follows:

$$(2.2) \quad E_{m,q,w}^{(-m,k)} = E_{m,q,w}^{(-m,k)}(0).$$

From (2.1), we can derive

$$\begin{aligned} E_{m,q,\varepsilon}^{(-m,k)} &= \lim_{n \rightarrow \infty} \frac{1}{[p^n]_q^k} \sum_{x_1=0}^{p^n-1} \cdots \sum_{x_k=0}^{p^n-1} [x_1 + \cdots + x_k]_q^m \\ &\quad \times (-1)^{x_1 + \cdots + x_k} \varepsilon^{x_1 + \cdots + x_k} q^{-x_1 m - \cdots - x_k(m+k-1)} \\ &= \frac{[2]_q^k}{(1-q)^m} \sum_{i=0}^m \binom{m}{i} (-1)^i \frac{1}{(1 + \varepsilon q^{i-m}) \cdots (1 + \varepsilon q^{i-m-k+1})} \\ &= \frac{[2]_q^k}{(1-q)^m} \sum_{i=0}^m \binom{m}{i} (-1)^i \sum_{l=0}^{\infty} \binom{k+l-1}{l}_q (-1)^l \varepsilon^l q^{(i-m-k+1)l} \\ &= [2]_q^k \sum_{l=0}^{\infty} \binom{k+l-1}{l}_q (-1)^{(1-m-k)l} \varepsilon^l (1-q)^m \sum_{i=0}^m \binom{m}{i} (-1)^i q^{il}. \end{aligned}$$

Note that $(1-q)^m \sum_{i=0}^m \binom{m}{i} (-1)^i q^{il} = \frac{(1-q^l)^m}{(1-q)^m} = [l]_q^m$. Thus, we obtain the following theorem.

Theorem 1. For $m, k \in \mathbb{N}$ and $\varepsilon \in T_p$, we have

$$(2.3) \quad E_{m,q,\varepsilon}^{(-m,k)} = [2]_q^k \sum_{l=0}^{\infty} \binom{k+l-1}{l}_q (-1)^l \varepsilon^l q^{(1-m-k)l} [l]_q^m,$$

where $\binom{z}{w} = \frac{z!}{w!(z-w)!}$ and $\binom{z}{w}_q = \frac{[z]_q!}{[w]_q! [z-w]_q!}$ for all $z, w \in \mathbb{N}$.

Note that $\lim_{q \rightarrow 1} E_{m,q,1}^{(-m,k)} = E_m^{(k)}$ where the $E_m^{(k)}$ are ordinary Euler numbers of order k , which are defined by

$$(2.4) \quad \left(\frac{2}{e^t + 1} \right)^k = \sum_{n=0}^{\infty} E_n^{(k)} \frac{t^n}{n!}.$$

By (2.1) with $k = 1$, it is easy to see that

$$\begin{aligned}
 E_{m,q,\varepsilon}^{(-m,1)} &= \int_{\mathbb{Z}_p} [x + x_1]_q^m \varepsilon^{x_1} q^{-x_1(m+1)} d\mu_{-q}(x_1) \\
 &= \int_{\mathbb{Z}_p} ([x]_q + q^x [x_1]_q)^m \varepsilon^{x_1} q^{-x_1(m+1)} d\mu_{-q}(x_1) \\
 &= \sum_{i=0}^m \binom{m}{i} q^{ix} [x]_q^{m-i} \int_{\mathbb{Z}_p} [x_1]_q^i \varepsilon^{x_1} q^{-x_1(m+1)} d\mu_{-q}(x_1) \\
 (2.5) \quad &= \sum_{i=0}^m \binom{m}{i} q^{ix} [x]_q^{m-i} E_{i,q,\varepsilon}^{(-m,1)}.
 \end{aligned}$$

Note that by (2.3) with $k = 1$, we have

$$(2.6) \quad E_{m,q,\varepsilon}^{(-m,1)} = [2]_q \sum_{l=0}^{\infty} (-1)^l q^{-ml} \varepsilon^l [l]_q^m.$$

Thus we obtain the following theorem.

Theorem 2. For $m, k \in \mathbb{N}$ and $\varepsilon \in T_p$, we have

$$(2.7) \quad E_{m,q,\varepsilon}^{(-m,1)}(x) = [2]_q \sum_{l=0}^{\infty} \sum_{i=0}^m \binom{m}{i} (-1)^l q^{-ml+ix} \varepsilon^l [x]_q^{m-i} [l]_q^m.$$

3. The twisted Lerch type Euler q -zeta functions

In this section, we define twisted Lerch type Euler q -zeta function as follows:

$$(3.1) \quad \zeta_{E,q,\varepsilon}(s) = [2]_q \sum_{n=0}^{\infty} \frac{(-1)^n \varepsilon^n q^{sn}}{[n]_q},$$

where $0 < q < 1$, $s \in \mathbb{C}$ with $\Re(s) > 1$, and $\varepsilon \in T_p$. Then we give the value $\zeta_{q,E,\varepsilon}(-m)$ for all $m \in \mathbb{N}$, which is an answer if the following problem: Are there twisted q -Euler polynomials which can be viewed as interpolating of $\zeta_{E,q,\varepsilon}(s)$ at negative integers and construct a new complex q -analogue of twisted Lerch type Euler zeta function.

From (2.1) with $k = 1$, it is easy to see that

$$\begin{aligned}
 E_{n,q,\varepsilon}^{(-n,1)} &= \int_{\mathbb{Z}_p} [x + x_1]_q^n \varepsilon^{x_1} q^{-x_1(m+1)} d\mu_{-q}(x_1) \\
 (3.2) \quad &= \frac{[2]_q}{[2]_{q^d}} [d]_q^n \sum_{i=0}^{d-1} (-1)^i \varepsilon^i q^{-ni} \int_{\mathbb{Z}_p} \varepsilon^{dt} q^{(m+1)dt} \left[\frac{x+i}{d} + t \right]_{q^d}^n \mu_{-q^d}(t),
 \end{aligned}$$

where d, n are positive integers with $d \equiv 1 \pmod{2}$. Note that

$$E_{n,q^d,\varepsilon^d}^{(-n,1)} \left(\frac{x+i}{d} \right) = \int_{\mathbb{Z}_p} \varepsilon^{dt} q^{(m+1)dt} \left[\frac{x+i}{d} + t \right]_{q^d}^n \mu_{-q^d}(t).$$

Thus, by (3.2), we obtain the following property of twisted q -Euler polynomials:

Theorem 3. For $d, n \in \mathbb{N}$ with $d \equiv 1 \pmod{2}$ and $\varepsilon \in T_p$, we have

$$(3.3) \quad E_{n,q,\varepsilon}^{(-n,1)}(x) = \frac{[2]_q}{[2]_{q^d}} [d]_q^n \sum_{i=0}^{d-1} (-1)^i \varepsilon^i q^{-ni} E_{n,q^d,\varepsilon^d}^{(-n,1)}\left(\frac{x+i}{d}\right).$$

From (2.1) with $k = 1$, we can derive

$$\begin{aligned} E_{m,q,w}^{(-m,1)}(x) &= \int_{\mathbb{Z}_p} q^{-y(m+1)} \varepsilon^y [y+x]_q^n d\mu_{-q}(y) \\ &= \lim_{N \rightarrow \infty} \frac{1}{[p^N]_{-q}} \sum_{y=0}^{p^N-1} q^{-ny} \varepsilon^y [y+x]_q^n (-1)^y \\ &= \frac{1}{(1-q)^n} \sum_{l=0}^n \binom{n}{l} (-1)^l q^{xl} \lim_{N \rightarrow \infty} \frac{1+q}{1+q^{p^N}} \sum_{y=0}^{p^N-1} (q^{-n} \varepsilon q^l)^y (-1)^y \\ &= \frac{[2]_q}{(1-q)^n} \sum_{l=0}^n \binom{n}{l} (-1)^l q^{xl} \left(\frac{1}{1+q^{-n} \varepsilon q^l} \right) \\ &= \frac{[2]_q}{(1-q)^n} \sum_{l=0}^n \binom{n}{l} (-1)^l q^{xl} \sum_{m=0}^{\infty} (-1)^m (q^{-n} \varepsilon q^l)^m \\ (3.4) \quad &= [2]_q \sum_{m=0}^{\infty} (-1)^m q^{-nm} \varepsilon^m \frac{1}{(1-q)^n} \sum_{l=0}^n \binom{n}{l} (-1)^l q^{(m+x)l}. \end{aligned}$$

Note that

$$[m+x]_q^m = \frac{1}{(1-q)^n} \sum_{l=0}^n \binom{n}{l} (-1)^l q^{(m+x)l}.$$

Thus we obtain the following another property of twisted q -Euler polynomials:

Theorem 4. For $n \in \mathbb{N}$ and $\varepsilon \in T_p$, we have

$$(3.5) \quad E_{n,q,\varepsilon}^{(-n,1)}(x) = [2]_q \sum_{m=0}^{\infty} (-1)^m q^{-nm} \varepsilon^m [m+x]_q^n.$$

Let $F_{q,\varepsilon}(t, x)$ be the generating function of twisted q -Euler polynomials $E_{k,q,\varepsilon}^{(-k,1)}(x)$ as follows:

$$(3.6) \quad F_{q,\varepsilon}(t, x) = \sum_{k=0}^{\infty} E_{k,q,\varepsilon}^{(-k,1)}(x) \frac{t^k}{k!}.$$

From (3.5) and (3.6), we can find the explicit series of the generating function $F_{q,\varepsilon}(t, x)$.

$$\begin{aligned}
 F_{q,\varepsilon}(t, x) &= \sum_{k=0}^{\infty} E_{k,q,\varepsilon}^{(-k,1)}(x) \frac{t^k}{k!} \\
 &= \sum_{k=0}^{\infty} \left([2]_q \sum_{m=0}^{\infty} (-1)^m q^{-km} \varepsilon^m [m+x]_q^k \right) \frac{t^k}{k!} \\
 &= [2]_q \sum_{m=0}^{\infty} (-1)^m \varepsilon^m \sum_{k=0}^{\infty} q^{-km} [m+x]_q^k \frac{t^k}{k!} \\
 (3.7) \qquad &= [2]_q \sum_{m=0}^{\infty} (-1)^m \varepsilon^m e^{q^{-m}[m+x]_q t}.
 \end{aligned}$$

Finally, we assume that $q \in \mathbb{R}$ with $0 < q < 1$ and $\varepsilon \in T_p$. From (3.1) with $x = 0$, we can see that the twisted Lerch type q -Euler zeta function is interpolating of twisted q -Euler numbers as follows:

Theorem 5. For $m \in \mathbb{N}$ and $\varepsilon \in T_p$, we have

$$(3.8) \qquad \zeta_{q,E,\varepsilon}(-m) = E_{m,q,\varepsilon}^{(-m,1)}.$$

From Theorem 4, we can define the q -extension of the twisted Lerch Euler zeta function as follows: For $s \in \mathbb{C}$ with $\Re(s) > 1$ and $\varepsilon \in T_p$,

$$(3.9) \qquad \zeta_{q,E,\varepsilon}(s, x) = [2]_q \sum_{n=0}^{\infty} \frac{(-1)^n \varepsilon^n q^{sn}}{[n+x]_q^s}.$$

Note that $\zeta_{q,E,\varepsilon}(s, x)$ is an analytic continuation in the whole complex plane. By (3.9) and Theorem 4, we can see that the q -extension of the twisted Lerch type Euler zeta function is interpolating of twisted q -Euler polynomials as follows:

Theorem 6. For $m \in \mathbb{N}$ and $\varepsilon \in T_p$, we have

$$\zeta_{q,E,\varepsilon}(-m, x) = E_{m,q,\varepsilon}^{(-m,1)}(x).$$

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