Investigation of Lateral-Torsional Buckling Strength Trends in Monosymmetric Stepped I-Beams Subjected to Several Loading Conditions

Surla, Albert S.* and Park, Jong Sup**

Abstract

It has been proven in recent studies that for monosymmetric I-section beams, in considering bending moment diagrams caused by any combination of applied end moments and transverse loads acting at the shear centre, the lowest critical lateral torsional buckling moment does not necessarily correspond to uniform bending. This finding is different from the intuitive expectation that researchers have that for lateral torsional buckling of thin walled beams, the lowest critical lateral torsional buckling moment always corresponds to a uniform bending moment diagram. To determine the applicability of the findings stated, considering stepped beams, this study will be focusing on the comparison of the lateral torsional buckling strength trends in monosymmetric I-beams having doubly stepped and compact cross section. Several loading conditions will be applied to see the effect of different moment diagrams having different inflection points on the lateral torsional buckling strength of stepped beams. The study will be made using the finite element program, ABAQUS. The study will investigate stepped beams having monosymmetric ratios ranging from 0.5 to 0.9. These ratios correspond to varying bottom flange width while keeping the top flange width unchanged. Unbraced length to height ratio of the beam to be analyzed is 20. Finally, the findings for this study will be shown using illustrative figures and conclusions will be made.

Key words : Lateral torsional buckling, Monosymmetric, Stepped beams

요 지

일축대칭 I형 단면보의 횡-비틀림 좌굴강도를 고려하는 데 있어서 강도기준 하중조건이 순수휨 작용시가 아님을 보여주는 연구가 제시된 바 있다. 이러한 연구결과는 오랫동안 적용해온 순수휨 작용시 가장 작은 횡-비틀림 좌굴강도를 보인다는 접근과는 다른 방법이다. 이에 본 연구에서는 제안된 새로운 경향을 확인하고 일축대칭 스텝보를 적용한 경우에도 같은 경향을 보이는 지 확인하고자 한다. 본 논문에는 다양한 하중조건을 고려하고 있으며 단면은 조밀단면을 적용하였다. 연구에 사용된 유한요소해석 프로그램은 ABAQUS이며, 스텝보에 적용한 일축대칭도는 0.5~0.9이다. 또한 본 연구에 고려한 해석모델의 L_b/h (지간/보 높이)비율은 20이다. 본 해석연구를 통하여 얻어진 새로운 경향과 결론을 자세하게 본 논문에 기술하였다.

핵심용어 : 횡-비틀림 좌굴, 일축대칭, 스텝보, 유한요소해석

1. Introduction

Over the years, as technology progresses, and more programs and equations are developed, the work of structural engineers and researchers have become faster and more efficient. However, these developments are countered by the continuous increase in population all over the world especially in major cities. With this, engineers and researchers are finding different ways to make their work even faster without sacrificing the efficiency of their works' results. One way of doing that is by

implementing "intuitive assumptions" when conducting several analyses. Intuitive is defined as agreeing with what is known or understood without any proof or evidence. In relation to structural engineering, one assumption that is usually made is that when lateral torsional buckling failure occurs in thin walled beams, the lowest critical moment always corresponds to the loading condition of equal end moments. This assumption can be observed in the universally accepted design assumption that replacing the real bending moment diagram by a uniform one would result to the beam having a lesser and

*Master Student, Department of Civil Engineering, Sangmyung University(E-mail: albert_surla24@yahoo.com)

**Corresponding Author: Member, Associate Professor, Department of Civil Engineering, Sangmyung University
(Tel: +82-41-550-5314, Fax : +82-41-550-5313, E-mail: jonpark@smu.ac.kr)

more conservative lateral torsional buckling capacity. In a study made by Camotim *et al.* (2012), they concluded that “amongst all bending diagrams caused by any combination of applied end moments and transverse loads acting at the shear center, the lowest critical moment does not necessarily correspond to uniform bending”. Their study focused on the analysis of monosymmetric prismatic straight beams. Thus, the inspiration for this study.

This study aims to investigate if the same conclusion, that the least critical moment does not necessarily correspond to uniform bending, can be applied to stepped beams. Stepped beams are beams having an increase in cross section either at the ends or at midspan depending on where the maximum moment is reached. Stepped beams are achieved by either changing the size of the cross section for hot rolled beams, adding cover plates to the beam flanges or by changing the dimensions for built up sections. Stepped beams are a good alternative to prismatic beams because they can be as efficient but be more economical. Different studies have been made about the lateral torsional buckling strength of stepped beams. Some of which are made by Gelera (2012), Park (2013) and Surla (2013). However, the previous studies available did not clearly mention the trends in the buckling strength of stepped beams as the monosymmetric ratio changes. Monosymmetric ratio or the degree of monosymmetry, ρ , is defined as the ratio of the second moment of area of the top flange about the y-axis (I_{yc}) to the second moment of area of the whole beam (I_y). For this study, stepped beams having a degree of monosymmetry of 0.5 to 0.9 will be analyzed to see the trends in beams having varying tension flange dimensions. Stepped beam parameters taken from geometry of real bridges will also be used to see the effect of these parameters to the lateral torsional buckling capacity of the beam. The stepped beams analyzed for this study are doubly stepped beams. This means that the increase in cross section are situated at both ends of the beam. The unbraced length, L_b , of 17.78 meters, which is under elastic range, is used and the beam selected is determined to be compact. For the results of this study, they are illustrated by using graphs to better see the trends as the parameters are varied. The results and conclusions of this investigation can serve as an additional information to help compare the differences and similarities in stepped beam and prismatic beam behaviors.

2. Model Parameters

The section used for this study is W36x160. The degree of monosymmetry, ρ is varied from 0.5 to 0.9. Below are the cross sections of the beams.

Fig. 1. Cross Section with Varying ρ .

Table 1. Other Material Properties

Property	Value
Total Area, A_t ($\rho=0.5$)	21,576
Mod. of Elasticity, E ($\rho=all$)	210,000 MPa
Mod. of Rigidity, G ($\rho=all$)	80,850 MPa
Yield Stress, F_y ($\rho=all$)	280 MPa
$L_p \approx$ ($\rho=all$)	2.36 m.
$L_r \approx$ ($\rho=all$)	9.15 m.

Table 2. Section Compactness

Flange	
λ	6.00
λ_p	10.41
λ_r	27.39
Web	
λ	52.34
λ_p	96.35
λ_r	156.10

λ , λ_p , λ_r and are factors that determine whether the section can be classified as compact, noncompact or slender.

As mentioned in AISC (2010) when $\lambda_r > \lambda_p$ the section can be classified as compact. It can be observed that for the section used, $\lambda_r > \lambda_p$ for both flange and web. So, the beam is classified as having both compact web and compact flanges.

In determining the effect of steps in beams. The stepped beam factors α , β and γ are used. The definition of the factors are illustrated below.

Fig. 2. Relative Factors Illustration.

Table 3. Stepped Beam Parameters

Case	ρ	α	β	γ
C1	0.5 - 0.9 (27 cases for each ratio with a 0.1 interval)	0.167	1	1.2
C2				1.4
C3				1.8
C4			1.2	1
C5				1.4
C6				1.8
C7			1.4	1
C8				1.4
C9				1.8
C10		0.25	1	1.2
C11				1.4
C12				1.8
C13			1.2	1
C14				1.4
C15				1.8
C16			1.4	1
C17				1.4
C18				1.8
C19		0.333	1	1.2
C20				1.4
C21				1.8
C22			1.2	1
C23				1.4
C24				1.8
C25			1.4	1
C26				1.4
C27				1.8

In Fig. 2, it can be observed that α corresponds to the ratio defining the relative length: β and γ corresponds to the ratio defining the relative width and thickness of the large and small cross sections respectively. To further see the effects of these factors, different parameters are selected. The parameters are introduced in a study by Park (2003) and are taken from the geometry of real bridges.

As seen on the table above, there are 27 cases to be analyzed for each monosymmetric ratio, ρ . Since there are 5 ρ to be analyzed, there will be a total of 135 cases for each loading condition for stepped beams. For prismatic beams there will only be

Fig. 3. Loading for Prismatic Beam Analysis.

Fig. 4. Loading for Stepped Beam Analysis.

5 cases for each loading condition. The loading conditions for this study are shown in Figs. 3 and 4.

Shown in Figs. 3 and 4 are the loading conditions to be used to determine the lateral torsional buckling strength of both prismatic and stepped beams. “LC” stands for loading condition and “LCS” stands for loading condition for stepped beams. The first one is equal end moments: followed by unequal end moments, uniformly distributed load, point load at midspan and two point loads at third points respectively.

3. Finite Element Modeling

The finite element program, ABAQUS (2009), is used to perform the numerical analysis of the lateral torsional buckling capacities of both monosymmetric prismatic and monosymmetric stepped I-beams. Beams with varying degree of monosymmetry are considered. The degree of monosymmetry, ρ , used is from 0.5-0.9 with a 0.1 interval. Beams having the degree of monosymmetry of 0.5 are doubly symmetric beams while beams having degree of monosymmetry that is greater than 0.5 are beams that have wider compression flanges than tension flanges. Due to its capability to provide enough degrees of freedom to clearly model the buckling deformations of the beam the standard large-strain, four-noded shell element, S4R, is chosen to model the beams. S4R elements are conventional shell elements that has several hourglass modes that may propagate over the mesh. It is also a robust, general-purpose element that is suitable for wide range of applications. To choose the optimum mesh for the models that are analyzed, a convergence test is held. Choosing the optimum mesh model will ensure that accurate results are achieved in as little time as possible. Simply supported boundary conditions are used for all the beams.

Fig. 5. Simply Supported Conditions: (a) Roller, (b) Hinge.

Fig. 6. Pure Bending Application.

There are 25 cases analyzed for the prismatic beams and there are 675 cases analyzed for the stepped beams. The modeling are shown below in the figures.

Shown in Figs. 7 and 8 are the typical models for ABAQUS. It can be seen that both the prismatic beam and stepped beam experienced a global buckling mode. This is important in ana-

(a) Initial State: Prismatic Beam

(b) Buckled State: Prismatic Beam

Fig. 7. ABAQUS Model (Prismatic Beam).

(a) Initial State: Stepped Beam

(b) Buckled State: Stepped Beam

Fig. 8. ABAQUS Model (Stepped Beam).

(a) Buckled State: $\alpha = 0.167$

(b) Buckled State: $\alpha = 0.25$

(c) Buckled State: $\alpha = 0.333$

Fig. 9. Stepped Beam with Varying α .

lyzing the behavior of the beams because it proves that the buckling failure that would occur is that of lateral torsional buckling. Shown in Fig. 9 are the buckling modes for stepped beam with varying stepped beam factor α .

It can be easily observed that as α increases the stepped length also increases.

4. Analysis Results

4.1 Prismatic Beam Results

To be able to have more data for comparison, and to clearly see the effects of monosymmetry on the lateral torsional buckling of the section being considered, the prismatic configuration of the section selected is also analyzed. Table 4 shows the results obtained using finite element analysis. Fig. 10 shows the illustrations of the finite element results of the prismatic beam.

Two illustrations for the lateral torsional buckling capacity of the prismatic beam analyzed are shown in Fig. 10. The first illustration, Fig. 10(a), utilizes the points in order to see the difference between the strengths clearly. With a $\rho=0.5$, the expected behavior is observed. That is, that the least critical moment is due to equal end moments or LC1. However, as ρ increases, it can be seen that at $\rho=0.7$, the buckling strength for LC3 and LC5 are almost the same as that of LC1. Then, for $\rho=0.8$ and $\rho=0.9$ the least critical moment corresponds to LC5 which has moments of 521.05 kN.m and 476.66 kN.m respectively followed by LC3 which has moments of 545.45 kN.m and 501.18 kN.m respectively and LC1 which has moments of 549.13 kN.m and 526.76 kN.m respectively. In Fig. 10(b), by connecting the points, the trends are clearly seen and it can be observed that the slope of LC3, LC4 and LC5 are higher than the slope of LC1 and LC2 which means that the change in strength is faster for LC3, LC4 and LC5 than that of LC1 and LC2. The actual change in strength for each interval are shown in Table 5. The table shows that LC1 and LC2 has a change in

Table 4. Analysis Results (Prismatic Beam)

Load Case	$L_b/h = 20$				
	Degree of Monosymmetry, ρ				
	0.5	0.6	0.7	0.8	0.9
LC1	617.19	591.58	569.7	549.13	526.76
LC2	813.54	779.91	751.22	724.12	696.64
LC3	689.07	639.25	590.4	545.45	501.18
LC4	839.95	760.77	695.15	636.62	579.87
LC5	675.17	615.73	565.95	521.05	476.66

Values at the right hand side of LC's are the lateral torsional buckling strength of the beam in kN.m.

(a) LTB Strength, $L_b/h=20$ (Points)

(b) LTB Strength, $L_b/h=20$ (Lines)

Fig. 10. Results for LTB of Prismatic Beam.

Table 5. Change in Strength, %

Load Case	Degree of Monosymmetry Interval			
	0.5-0.6	0.6-0.7	0.7-0.8	0.8-0.9
LC1	4.27%	3.77%	3.68%	4.15%
LC2	4.22%	3.75%	3.68%	4.16%
LC3	8.80%	8.01%	7.85%	8.46%
LC4	9.89%	9.01%	8.79%	9.33%
LC5	9.21%	8.42%	8.26%	8.90%

strength of about 3~5% while LC3, LC4 and LC5 has a change in strength ranging from 7~10%.

4.2 Stepped Beam Results

For the stepped beam analyses made, a total of 5 loading conditions, 5 degrees of monosymmetry and 27 cases of stepped beam parameters are observed. This means that a total of 675 doubly stepped beam models with an unbraced length to height ratio of 20 are made. Tables 6, 7, 8, 9, and 10 show results obtained using finite element analysis for each ratio considered.

For the illustration of results, three cases are selected. Each one has different α . The cases selected would have significant difference in their lateral torsional buckling strengths to clearly illustrate the effect of steps. The illustrations can then be used to see the difference in the trends as the monosymmetric ratio increases. Figs. 11, 12, 13 show the selected illustrations of the results obtained using finite element analysis:

4.2.1 Stepped Beam Results Analysis

In Tables 6, 7, 8, 9 and 10, the results are tabulated and are arranged based on the stepped beam cases enumerated in Table 3. By analyzing the results, it is evident that all throughout, the loading case LCS2 and LCS4, which are unequal end moments and point load at midspan respectively, correspond to the highest lateral torsional buckling moment. It is comparable to the results showed in Fig. 10 which are for prismatic beam. For the illustrations showed in Figs. 11, 12 and 13, it can be seen that as the stepped parameters increase, the strength increases. Getting into details with this results Fig. 11, which illustrates C2, showed a maximum strength of 1013 kN.m. Fig. 12, which illustrates C12, showed a maximum strength of 1325 kN.m. and Fig. 13, which illustrates C27, showed a maximum strength of 1849 kN.m. Thus, clearly showing the effect of steps on the beams. Meanwhile, looking at the trends of the lateral torsional buckling strength for the stepped beams subjected to LCS1, LCS3 and LCS5, it can be observed that as the stepped beam parameter increases, the cause of the least criti-

Table 6. Analysis Results (Stepped Beam)

Degree of Monosymmetry, $\rho=0.5$					
Case	LCS1	LCS2	LCS3	LCS4	LCS5
C1	685	887	790	933	760
C2	809	974	761	1013	828
C3	913	1142	999	1167	960
C4	673	841	747	882	717
C5	790	1021	896	1052	861
C6	939	1197	1041	1215	1001
C7	827	868	767	904	736
C8	821	1060	926	1085	890
C9	969	1242	1077	1255	1035
C10	809	927	820	965	788
C11	852	1058	924	1082	887
C12	1086	1325	1136	1319	1091
C13	728	879	768	901	736
C14	920	1146	983	1144	942
C15	1173	1427	1209	1397	1160
C16	790	930	801	936	768
C17	957	1214	1031	1196	988
C18	1204	1510	1271	1464	1219
C19	747	965	853	1000	821
C20	864	1141	994	1156	957
C21	1265	1516	1295	1485	1248
C22	759	965	814	942	782
C23	1037	1303	1103	1261	1063
C24	1438	1706	1424	1613	1374
C25	926	1037	878	1003	845
C26	1117	1423	1186	1343	1143
C27	1555	1849	1527	1718	1473

C1-C27 - Stepped beam parameters (Table 3)

LCS1-LCS5 - Loading Conditions (Fig. 4)

*values are in kN.m.

Table 7. Degree of Monosymmetry, $\rho=0.6$

Case	LCS1	LCS2	LCS3	LCS4	LCS5
C1	659	853	762	901	733
C2	777	937	827	975	796
C3	878	1098	953	1118	917
C4	647	809	723	854	694
C5	760	982	872	1011	838
C6	902	1151	991	1161	954
C7	795	835	741	875	712
C8	789	1019	887	1043	853
C9	932	1194	1024	1198	986
C10	777	891	789	930	758
C11	819	1018	884	1038	850
C12	1044	1274	1077	1256	1035
C13	700	845	743	872	712
C14	884	1102	939	1096	901
C15	1128	1372	1144	1329	1098
C16	760	894	774	905	742
C17	920	1168	984	1145	943
C18	1157	1452	1201	1391	1152
C19	718	928	820	962	789
C20	831	1097	949	1106	914
C21	1217	1458	1224	1410	1179
C22	730	928	788	912	758
C23	997	1253	1053	1207	1014
C24	1383	1641	1345	1530	1296
C25	890	997	850	972	817
C26	1074	1369	1131	1284	1089
C27	1495	1778	1440	1627	1388

*values are in kN.m.

Table 8. Degree of Monosymmetry, $\rho=0.7$

Case	LCS1	LCS2	LCS3	LCS4	LCS5
C1	632	818	733	868	706
C2	746	899	793	938	763
C3	843	1054	906	1068	873
C4	621	776	698	825	671
C5	729	942	848	971	816
C6	866	1104	941	1108	907
C7	763	801	715	845	688
C8	758	978	847	1000	816
C9	894	1146	971	1142	936
C10	746	855	758	895	729
C11	786	977	844	995	812
C12	1003	1223	1018	1194	979
C13	672	811	717	843	688
C14	849	1057	895	1049	859
C15	1082	1317	1079	1260	1037
C16	729	858	747	875	716
C17	883	1121	936	1094	898
C18	1111	1394	1130	1317	1086
C19	689	891	787	925	757
C20	798	1053	904	1057	871
C21	1168	1399	1152	1334	1110
C22	701	891	762	883	733
C23	957	1203	1003	1152	965
C24	1327	1575	1265	1446	1219
C25	855	957	821	940	790
C26	1031	1314	1075	1225	1036
C27	1436	1707	1352	1537	1304

*values are in kN.m.

Table 9. Degree of Monosymmetry, $\rho=0.8$

Case	LCS1	LCS2	LCS3	LCS4	LCS5
C1	609	787	704	833	677
C2	718	865	759	898	731
C3	811	1014	861	1017	831
C4	598	746	671	792	645
C5	702	906	799	929	781
C6	833	1062	893	1054	861
C7	735	770	687	811	661
C8	729	941	809	956	779
C9	861	1102	920	1085	885
C10	718	822	726	857	698
C11	757	940	805	950	774
C12	965	1176	963	1133	926
C13	647	780	689	809	661
C14	817	1017	853	1000	818
C15	1042	1267	1019	1194	979
C16	702	825	718	839	688
C17	850	1078	891	1043	854
C18	1069	1341	1065	1246	1023
C19	663	857	753	884	723
C20	768	1013	860	1006	827
C21	1124	1345	1086	1261	1044
C22	674	857	733	847	704
C23	921	1157	954	1098	917
C24	1277	1515	1192	1368	1147
C25	822	920	789	903	759
C26	992	1264	1022	1167	983
C27	1382	1642	1272	1452	1225

*values are in kN.m.

cal bending moment started to change from LCS1 to LCS3 and LCS5. In Fig. 11 the behavior is almost similar to the behavior of the prismatic beam in which the least critical moment is due to the uniform bending moment diagram. Also, as the monosymmetric ratio increases, the cause of the least critical bending moment started to shift to LCS3 and LCS5. Moreover, in Fig. 13, it is quite evident that even for a monosymmetric ratio of 0.5, the least critical moment does not correspond to the pure bending loading condition. This is the cause of the stepped section being larger and covering a greater of the percentage of the unbraced length. It can be seen that at $\rho=0.9$ the pure bending condition has a lateral torsional buckling moment of 1327 kN.m. which is almost equivalent to the value of LTB moment with loading condition of LS4 or concentrated load at midspan, which is 1367 kN.m and that the least critical bending moment are LCS5 and LCS3 which have values of 1146 kN.m and 1192 kN.m respectively. Hence, it can be said that the effects of steps are greatly realized and certain trends are observed.

Table 10. Degree of Monosymmetry, $\rho=0.9$

Case	LCS1	LCS2	LCS3	LC4	LC5
C1	585	755	674	797	648
C2	690	830	724	858	698
C3	780	973	816	967	788
C4	574	716	644	759	618
C5	674	870	749	887	745
C6	801	1019	844	1000	815
C7	706	739	659	778	634
C8	701	903	770	912	743
C9	827	1058	868	1027	833
C10	690	789	693	819	666
C11	727	902	765	904	736
C12	9277	1129	907	1072	873
C13	622	749	661	775	634
C14	785	976	810	952	778
C15	1001	1216	958	1128	921
C16	674	792	688	804	660
C17	816	1035	845	992	811
C18	1027	1287	999	1174	961
C19	637	822	718	843	689
C20	737	972	816	956	784
C21	1080	1291	1019	1189	979
C22	648	822	703	812	675
C23	885	1110	904	1043	869
C24	1227	1454	1118	1289	1075
C25	790	883	757	865	728
C26	953	1213	968	1108	931
C27	1327	1576	1192	1367	1146

*values are in kN.m.

(a) $L_b/h=20$, $\alpha=0.167$ $\beta=1.0$ & $\gamma=1.4$ (Points)

(b) $L_b/h=20$, $\alpha=0.167$ $\beta=1.0$ & $\gamma=1.4$ (Lines)

Fig. 11. Results for LTB of Stepped Beam.

Fig. 12. Results for LTB of Stepped Beam.

Fig. 13. Results for LTB of Stepped Beam.

5. Conclusions

Taking inspiration from the study made by Camotim *et al.* (2012), the lateral torsional buckling strength trends of monosymmetric stepped I-beams are investigated. In order to do this, different analyses are performed using the finite element program ABAQUS. First, to clearly understand the behavior of the section selected and to find out if the results are comparable to

the previous studies made, the prismatic configuration of the section is modeled. Five different monosymmetric ratios are used and the beams are subjected to five different loading conditions. Next, the stepped beams are modeled. Same conditions for the prismatic beam are used but in order to see the effects of steps, the stepped parameters α , β , and γ are introduced. Varying α , β , and γ resulted into 27 cases for stepped beam parameters.

After tabulating and analyzing all the results, certain conclusions are made. First, it can be concluded that the least critical moment does not always correspond to pure bending moment diagram. May it be prismatic beams or stepped beams. For the prismatic beams, as ρ reaches 0.9 the change in the least critical bending moment cause are evident enough in the illustrations. For stepped beams having minor changes in comparison with the original configurations of the beam (see Fig. 11), the trends are almost similar to the trends observed using prismatic beam (see Fig. 10). The increase in strength are observed but the lateral torsional buckling strength trends remain the same or there is an insignificant difference. However, as the stepped beam parameters increase the lateral torsional buckling strength change becomes more evident and the trends are seen to have quite a significant difference compared to the trends in the prismatic section. By observing, it can be seen that even at $\rho=0.5$ the least critical moment is not due to pure bending moment diagram anymore. And as ρ reaches 0.9 the strength due to pure bending is almost the same as that of the strength due to the point load at midspan. Moreover, in fig. 13, the strength of the stepped beam almost doubled in comparison with its prismatic section configuration. As the monosymmetric ratio increases, the trends are better seen and this is a critical issue especially when designing beams for buildings and other structures. Thus, it can be concluded that the degree of monosymmetry has a great effect on the lateral torsional buckling capacity of the stepped beams and that the intuitive assumption is verified to be not true for this study. However, improvements and further study can be made which can include monosymmetric ratios of less than 0.5 and other loading and boundary conditions can also be observed. Finally, it can be said that as more programs become available, most of intuitive assumptions will be verified and this will result to safer and more theoretically based design methods and assumptions.

Acknowledgement

This work is supported by National Research grant and funded by the Korean Ministry of Education, Science and Technology (NRF -2010-0023275).

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Received ■ May 28, 2014

Revised ■ May 29, 2014

Accepted ■ July 3, 2014