Disturbance-Observer-Based Robust H_∞ Switching Tracking Control for Near Space Interceptor

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Abstract

A novel robust H_∞ switching tracking control design method with disturbance observer is proposed for the near space interceptor (NSI) with aerodynamic fins and reaction jets. Initially, the flight envelop of the NSI is divided into small subregions, and a slow-fast loop polytopic linear parameter varying (LPV) model is proposed, to approximate the nonlinear dynamic of the NSI, based on the Jacobian linearization and Tensor-Product (T-P) model transformation approach. A disturbance observer is then constructed, to estimate the modeled disturbance. Subsequently, based on the descriptor system method, a robust switching controller is developed, to ensure that the closed-loop descriptor system is stable with a desired H_∞ disturbance attenuation level. Furthermore, the outcome of the proposed switching tracking control problem is formulated as a set of linear matrix inequalities (LMIs). Finally, simulation results demonstrate the effectiveness of the proposed design method.

Key words: Linear parameter varying (LPV), Disturbance observer, Switching tracking control, Near space interceptor (NSI)

1. Introduction

Near space is defined as the atmospheric region of about 20~100 km above the earth's surface, which has not been used by airplanes, due to the low-density atmosphere at that altitude [1]. Rapid development of near space air-breathing hypersonic vehicles (NSHVs) has made a quick response and global strike capabilities more practical for Air Force missions [2]. To deal with the threat of the NSHV, the near space interceptor (NSI) is an effective kind of defense scheme. The configuration scheme combined with aerodynamic fins and reaction jets can effectively improve the response speed of the guidance command, and achieve the objective of hitting an NSHV target.

In the past few years, many methods have been reported for the flight control system design of a blended control interceptor, such as optimal control [3], feedback linearization

[4], sliding mode control [5], and finite time control [6], [7]. Due to the complex flight environment and aerodynamic parameter variation of the NSI, a single model is not enough to accurately represent the NSI system dynamic, and the control design methods [3-6] based on this model cannot then obtain satisfactory control performance. Recently, linear parameter varying (LPV) control technology has been used to deal with systems that exhibit parameter dependent dynamics [8]. Some new results have also been reported on switching control, to cope with complex systems [9]. Therefore, switching LPV control is a natural choice for the NSI system.

On the other hand, the model uncertainty and external disturbance with the flight environment change will degrade the closed-loop control performance of the NSI system, while most of the above methods [3-7] have not directly taken into account the anti-disturbance performance, or dealt with the system disturbances, at the price of sacrificing normal

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control performance. For example, H_∞ control provides a passive feedback regulation method to effectively attenuate the effect of a system disturbance, and only rejects the disturbance under a prescribed disturbance attenuation level, rather than completely removing the influence of disturbances from the system [10]. Sliding mode control [11] has strong robustness against system disturbance and parameter uncertainty, by designing higher control gain, but there still exists some chattering.

An effective anti-disturbance control strategy is the disturbance observer-based control (DOBC) scheme [12–13], which has been applied in some engineering areas, e.g., NSHV [14]. Initially, a baseline controller is first designed, under the assumptions that there are no disturbances; and then disturbance compensation is added to counteract the influence of disturbances, which are estimated by a properly designed disturbance observer. Therefore, the robustness of the closed-loop system is obtained without sacrificing normal control performance, and the effect of the disturbance can be completely removed from the system, as long as the disturbance can be estimated by the designed disturbance observer.

Motivated by the above observations, this paper will deal with the problem of robust H_∞ switching tracking control for the NSI, based on a disturbance observer. Based on the Jacobian linearization and Tensor-Product (T-P) model transformation approach, a slow-fast loop polytopic LPV model is first employed, to approximate the nonlinear dynamic of the NSI; and a disturbance observer is constructed, to estimate the modeled disturbance. Then, a descriptor augmented system is obtained, by combining the system dynamics of the NSI, the command tracking error, and the estimated error of the modeled disturbances. Subsequently, a robust switching controller is developed, such that the closed-loop augmented system is stable, and a desired H_∞ disturbance attenuation level is also obtained. Finally, simulation results on the NSI are given, to show the effectiveness of the proposed method.

Briefly, the rest of this paper is organized as follows. Section 2 introduces a slow-fast loop polytopic LPV model for the NSI. Section 3 presents a slow-fast loop robust H_∞ switching tracking control design method, in terms of LMIs. Section 4 gives the simulation results for the NSI. Finally, a brief conclusion is drawn in Section 5.

2. The NSI model description

The NSI with aerodynamic fins and reaction jets is adopted to investigate the problem of robust H_∞ switching tracking control design. The structure diagram of this NSI is shown

in Fig. 1. Define the equivalent control surface deflections of the lateral thrust in the pitch and yaw direction, as follows:

$$\delta_{sy} = F_y / F_{s\max}, \quad \delta_{sz} = F_z / F_{s\max} \quad (1)$$

where, F_y and F_z denote the lateral thrusts in the pitch and yaw direction, and $F_{s\max}$ is the maximum steady thrust of the reaction jets.

Then, the dynamic equations of the NSI can be described as follows:

$$\begin{cases} \dot{\mathbf{x}}_s = \mathbf{f}_s(\mathbf{x}_s) + \mathbf{g}_s(\mathcal{Q}, \mathbf{x}_s)(\mathbf{x}_f + \mathbf{d}_{s0}) + \mathbf{B}_{sd}\mathbf{d}_{s1} \\ \dot{\mathbf{x}}_f = \mathbf{f}_f(\mathbf{x}_s, \mathbf{x}_f) + \mathbf{g}_f(\mathbf{u} + \mathbf{d}_{f0}) + \mathbf{B}_{fd}\mathbf{d}_{f1} \end{cases} \quad (2)$$

where, $\mathbf{x}_s \triangleq [\gamma, \beta, \alpha]^T$ and $\mathbf{x}_f \triangleq [\omega_x, \omega_y, \omega_z]^T$ denote the state vectors of the slow loop and fast loop, respectively; $\mathbf{d}_{s0} \triangleq [d_{sy}, d_{s\beta}, d_{s\alpha}]^T$ and $\mathbf{d}_{f0} \triangleq [d_{\delta_x}, d_{\delta_y}, d_{\delta_z}, d_{\delta_{sy}}, d_{\delta_{sz}}]^T$ denote the modeled disturbances of the slow loop and fast loop, respectively; $\mathbf{d}_{s1}$ and $\mathbf{d}_{f1}$ denote the external disturbances in the H_2 -norm, and $\mathbf{u} \triangleq [\delta_x, \delta_y, \delta_z, \delta_{sy}, \delta_{sz}]^T$ denotes the actual control input. $\mathbf{f}_s(\mathbf{x}_s)$, $\mathbf{g}_s(\mathcal{Q}, \mathbf{x}_s)$, $\mathbf{f}_f(\mathbf{x}_s, \mathbf{x}_f)$ and $\mathbf{g}_f$ stand for the smooth nonlinear matrix functions, where:

$$\mathbf{f}_s(\mathbf{x}_s) \triangleq [0, f_{s\beta}, f_{s\alpha}]^T, \quad \mathbf{f}_f(\mathbf{x}_s, \mathbf{x}_f) \triangleq [f_{f\omega_x}, f_{f\omega_y}, f_{f\omega_z}]^T,$$

$$\mathbf{g}_s(\mathcal{Q}, \mathbf{x}_s) \triangleq \begin{bmatrix} 1 & -\cos \gamma \tan \mathcal{Q} & \sin \gamma \tan \mathcal{Q} \\ \sin \alpha & \cos \alpha & 0 \\ -\tan \beta \cos \alpha & \tan \beta \sin \alpha & 1 \end{bmatrix},$$

$$\mathbf{g}_f \triangleq \begin{bmatrix} g_{f11} & 0 & 0 & 0 & 0 \\ 0 & g_{f22} & 0 & 0 & g_{f25} \\ 0 & 0 & g_{f33} & g_{f34} & 0 \end{bmatrix},$$

with,

$$\begin{aligned} f_{s\beta} &\triangleq \frac{qSC_y^\alpha \sin \alpha \sin \beta}{mV_m} + \frac{qSC_z^\beta \cos \beta}{mV_m}, \quad f_{s\alpha} \triangleq \frac{qSC_y^\alpha \cos \alpha}{mV_m \cos \beta}, \quad f_{f\omega_x} \triangleq \frac{J_y - J_z}{J_x} \omega_y \omega_z, \\ f_{f\omega_y} &\triangleq \frac{J_z - J_x}{J_y} \omega_x \omega_z + \frac{qSLm_y^\beta}{J_y} \omega_x, \quad f_{f\omega_z} \triangleq \frac{J_x - J_y}{J_z} \omega_x \omega_y + \frac{qSLm_z^\alpha}{J_z} \omega_x + \frac{qSLm_z^\beta}{J_z} \omega_y, \\ g_{f11} &\triangleq \frac{qSLm_x^\alpha}{J_x}, \quad g_{f22} \triangleq \frac{qSLm_y^\beta}{J_y}, \quad g_{f25} \triangleq \frac{(1 + K_M)F_{s\max} L_m}{J_y}, \end{aligned}$$

Fig. 1. Structure diagram for the NSI

$$g_{f33} \triangleq \frac{qSLm_z^{\theta_z}}{J_z}, \quad g_{f34} \triangleq \frac{(1+K_M)F_{s\max}L_m}{J_z}.$$

By the Jacobian linearization method, the slow loop LPV model of the NSI can be represented as:

$$\begin{cases} \dot{\mathbf{x}}_s(t) = \hat{\mathbf{A}}_s(V_m, q)\mathbf{x}_s(t) + \hat{\mathbf{B}}_{sv}(V_m, q)(\mathbf{v}_s(t) + \mathbf{d}_{s0}(t)) + \hat{\mathbf{B}}_{sd}(V_m, q)\mathbf{d}_{s1}(t) \\ \mathbf{y}_s(t) = \hat{\mathbf{C}}_s(V_m, q)\mathbf{x}_s(t) \end{cases} \quad (3)$$

where,

$$\hat{\mathbf{A}}_s(V_m, q) \triangleq \begin{bmatrix} 0 & 0 & 0 \\ 0 & \hat{A}_{s22} & \hat{A}_{s23} \\ 0 & \hat{A}_{s32} & \hat{A}_{s33} \end{bmatrix}, \quad \hat{\mathbf{B}}_{sv}(V_m, q) \triangleq \begin{bmatrix} 1 & -\cos\gamma_e \tan\theta_e & \sin\gamma_e \tan\theta_e \\ \sin\alpha_e & \cos\alpha_e & 0 \\ -\tan\beta_e \cos\alpha_e & \tan\beta_e \sin\alpha_e & 1 \end{bmatrix},$$

$$\mathbf{v}_s(t) \triangleq [\omega_x, \omega_y, \omega_z]^T, \quad \mathbf{y}_s(t) \triangleq [\gamma, \beta, \alpha]^T, \quad \hat{\mathbf{B}}_{sd}(V_m, q) \triangleq \mathbf{B}_{sd}, \quad \hat{\mathbf{C}}_s(V_m, q) \triangleq \mathbf{I},$$

with,

$$\begin{aligned} \hat{A}_{s22} &\triangleq \frac{qSC_y^{\alpha} \alpha_e \sin\alpha_e \cos\beta_e}{mV_m} + \frac{qSC_z^{\beta} (\cos\beta_e - \beta_e \sin\beta_e)}{mV_m}, \quad \hat{A}_{s32} \triangleq -\frac{qSC_y^{\alpha} \alpha_e \cos\alpha_e \sin\beta_e}{mV_m \cos^2\beta_e}, \\ \hat{A}_{s23} &\triangleq \frac{qSC_y^{\alpha} (\sin\alpha_e + \alpha_e \cos\alpha_e) \sin\beta_e}{mV_m}, \quad \hat{A}_{s33} \triangleq -\frac{qSC_y^{\alpha} (\cos\alpha_e - \alpha_e \sin\alpha_e)}{mV_m \cos\beta_e}, \end{aligned}$$

and $(\cdot)_e$ denotes the value of the slow loop equilibrium point.

By the T-P model transformation approach [8], the LPV model of the slow loop can be written as:

$$[\dot{\mathbf{x}}_s^T(t), \mathbf{y}_s^T(t)]^T = \hat{\mathbf{S}}_s \otimes_{n=1}^2 \mathbf{w}_{sn}(\mathbf{p}_n(t)) [\mathbf{x}_s^T(t), \mathbf{v}_s^T(t) + \mathbf{d}_{s0}^T(t), \mathbf{d}_{s1}^T(t)]^T \quad (4)$$

where, $\mathbf{p}(t) \triangleq [V_m, q] \in [V_{m\min}, V_{m\max}] \times [q_{\min}, q_{\max}]$, $\mathbf{w}_{s1}(\mathbf{p}_1(t)) = [w_{s1,1}(V_m), w_{s1,2}(V_m)]$, $\mathbf{w}_{s2}(\mathbf{p}_2(t))$ and $\mathbf{w}_{s2}(\mathbf{p}_2(t)) = [w_{s2,1}(q), w_{s2,2}(q)]$. The 4-dimensional coefficient tensor $\hat{\mathbf{S}}_s \in \mathbb{R}^{2 \times 2 \times 6 \times 9}$ is constructed from the corresponding LTI vertex system $\hat{\mathbf{S}}_{s_{i_1 i_2}} \in \mathbb{R}^{6 \times 9}$, for $i_1, i_2 \in \{1, 2\}$. $w_{s1,1}(V_m)$, $w_{s1,2}(V_m)$, $w_{s2,1}(q)$, $w_{s2,2}(q)$ are the corresponding weighting functions, with:

$$w_{s1,1}(V_m) = \frac{V_{m\max} - V_m}{V_{m\max} - V_{m\min}}, \quad w_{s1,2}(V_m) = \frac{V_m - V_{m\min}}{V_{m\max} - V_{m\min}},$$

$$w_{s2,1}(q) = \frac{q_{\max} - q}{q_{\max} - q_{\min}}, \quad w_{s2,2}(q) = \frac{q - q_{\min}}{q_{\max} - q_{\min}}.$$

Then, the slow loop LPV system (4) can be transformed into the following polytopic system:

$$[\dot{\mathbf{x}}_s^T(t), \mathbf{y}_s^T(t)]^T = \sum_{i=1}^4 w_{si}(V_m, q) \hat{\mathbf{S}}_{si} \quad (5)$$

$$\text{i.e., } \begin{cases} \dot{\mathbf{x}}_s(t) = \sum_{i=1}^4 w_{si}(V_m, q) [\mathbf{A}_{si} \mathbf{x}_s(t) + \mathbf{B}_{svi}(\mathbf{v}_s(t) + \mathbf{d}_{s0}(t))] + \mathbf{B}_{sdi} \mathbf{d}_{s1}(t) \\ \mathbf{y}_s(t) = \mathbf{C}_s \mathbf{x}_s(t) \end{cases} \quad (6)$$

where, $\hat{\mathbf{S}}_{si}$ denotes the corresponding slow loop LTI vertex system, $i \in \mathbb{N} \triangleq \{1, 2, 3, 4\}$, and

$$w_{s1}(V_m, q) = w_{s1,1}(V_m)w_{s2,1}(q), \quad w_{s2}(V_m, q) = w_{s1,1}(V_m)w_{s2,2}(q),$$

$$w_{s3}(V_m, q) = w_{s1,2}(V_m)w_{s2,1}(q), \quad w_{s4}(V_m, q) = w_{s1,2}(V_m)w_{s2,2}(q).$$

Similarly, the fast loop LPV model of the NSI can be written as:

$$\begin{cases} \dot{\mathbf{x}}_f(t) = \hat{\mathbf{A}}_f(V_m, q)\mathbf{x}_f(t) + \hat{\mathbf{B}}_{fu}(V_m, q)(\mathbf{u}(t) + \mathbf{d}_{f0}(t)) + \hat{\mathbf{B}}_{fd}(V_m, q)\mathbf{d}_{f1}(t) \\ \mathbf{y}_f(t) = \hat{\mathbf{C}}_f(V_m, q)\mathbf{x}_f(t) \end{cases} \quad (7)$$

where, $\mathbf{y}_f(t) \triangleq [\omega_x, \omega_y, \omega_z]^T$, $\hat{\mathbf{B}}_{fd}(V_m, q) \triangleq \mathbf{B}_{fd}$, $\hat{\mathbf{C}}_f(V_m, q) \triangleq \mathbf{I}$

$$\hat{\mathbf{B}}_{fu}(V_m, q) = \mathbf{g}_{f'} \quad \hat{\mathbf{A}}_f(V_m, q) \triangleq \begin{bmatrix} 0 & \hat{A}_{f12} & \hat{A}_{f13} \\ \hat{A}_{f21} & \hat{A}_{f22} & \hat{A}_{f23} \\ \hat{A}_{f31} & \hat{A}_{f32} & \hat{A}_{f33} \end{bmatrix}$$

with,

$$\begin{aligned} \hat{A}_{f12} &\triangleq \frac{J_y - J_z}{J_x} \omega_{z,e}, \quad \hat{A}_{f13} \triangleq \frac{J_y - J_z}{J_x} \omega_{y,e}, \quad \hat{A}_{f21} \triangleq \frac{J_z - J_x}{J_y} \omega_{z,e}, \quad \hat{A}_{f22} \triangleq \frac{qSLm_y^{\theta_y}}{J_y}, \\ \hat{A}_{f23} &\triangleq \frac{J_z - J_x}{J_y} \omega_{x,e}, \quad \hat{A}_{f31} \triangleq \frac{J_x - J_y}{J_z} \omega_{y,e}, \quad \hat{A}_{f32} \triangleq \frac{J_x - J_y}{J_z} \omega_{x,e}, \quad \hat{A}_{f33} \triangleq \frac{qSLm_z^{\theta_z}}{J_z}, \end{aligned}$$

and $(\cdot)_e$ denotes the value of the fast loop equilibrium point.

Also, by the T-P model transformation approach, the LPV system (7) can be written as the following polytopic systems:

$$[\dot{\mathbf{x}}_f^T(t), \mathbf{y}_f^T(t)]^T = \sum_{i=1}^4 w_{fi}(V_m, q) \hat{\mathbf{S}}_{fi} \quad (8)$$

$$\text{i.e., } \begin{cases} \dot{\mathbf{x}}_f(t) = \sum_{i=1}^4 w_{fi}(V_m, q) [\mathbf{A}_{fi} \mathbf{x}_f(t) + \mathbf{B}_{fui}(\mathbf{u}(t) + \mathbf{d}_{f0}(t))] + \mathbf{B}_{fdi} \mathbf{d}_{f1}(t) \\ \mathbf{y}_f(t) = \mathbf{C}_f \mathbf{x}_f(t) \end{cases} \quad (9)$$

where, $\hat{\mathbf{S}}_{fi}$ denotes the LTI vertex system of the fast loop, and

$$w_{fi}(V_m, q) = w_{si}(V_m, q), \text{ for } i \in \mathbb{N}.$$

3. Switching controller design

3.1 Slow-loop switching controller design

The flight envelop is divided into four smaller subregions, and the subregion R_k is denoted by:

$$R_1: V_m \in [1502, 1702] \text{ m/s and } q \in [44519, 57164] \text{ Pa},$$

$$R_2: V_m \in [1502, 1702] \text{ m/s and } q \in [57164, 71388] \text{ Pa},$$

$$R_3: V_m \in [1702, 1902] \text{ m/s and } q \in [44519, 57164] \text{ Pa},$$

$$R_4: V_m \in [1702, 1902] \text{ m/s and } q \in [57164, 71388] \text{ Pa}.$$

For the subregion R_k , the slow loop polytopic LPV system can be written as:

$$\begin{cases} \dot{\mathbf{x}}_s(t) = \sum_{i=1}^4 w_{ski}(V_m, q) [\mathbf{A}_{ski} \mathbf{x}_s(t) + \mathbf{B}_{svki}(\mathbf{v}_s(t) + \mathbf{d}_{s0}(t))] + \mathbf{B}_{sd} \mathbf{d}_{s1}(t) \\ \mathbf{y}_s(t) = \mathbf{C}_s \mathbf{x}_s(t) \end{cases} \quad (10)$$

Define the following switching characteristic function v_k for the slow loop:

$$v_k = \begin{cases} 1 & (V_m, q) \in R_k \\ 0 & (V_m, q) \notin R_k \end{cases}, \text{ for } k \in \mathbb{N} \quad (11)$$

Under the switching function v_k , the polytopic LPV system (10) can be written as:

$$\begin{cases} \dot{\mathbf{x}}_s(t) = \sum_{k=1}^4 \sum_{i=1}^4 v_k w_{ski}(V_m, q) [\mathbf{A}_{ski} \mathbf{x}_s(t) + \mathbf{B}_{svki}(\mathbf{v}_s(t) + \mathbf{d}_{s0}(t))] + \mathbf{B}_{sd} \mathbf{d}_{s1}(t) \\ \mathbf{y}_s(t) = \mathbf{C}_s \mathbf{x}_s(t) \end{cases} \quad (12)$$

Define the following reference model for the slow loop polytopic LPV system (12):

$$\dot{\mathbf{x}}_{sr}(t) = \mathbf{A}_{sr} \mathbf{x}_{sr}(t) + \mathbf{B}_{sr} \mathbf{r}_s(t) \quad (13)$$

where, $\mathbf{x}_{sr}(t)$ is the state vector of the reference model, and $\mathbf{r}_s(t)$ is the reference input signal.

Assumption 1. It is assumed that the disturbance $\mathbf{d}_{s0}(t)$ is generated by the following exogenous system:

$$\begin{cases} \dot{\bar{\mathbf{w}}}_s(t) = \mathbf{W}_s \bar{\mathbf{w}}_s(t) + \mathbf{B}_{sw} \delta_{sw}(t) \\ \mathbf{d}_{s0}(t) = \mathbf{V}_s \bar{\mathbf{w}}_s(t) \end{cases} \quad (14)$$

where, $\mathbf{W}_s \in \mathbb{R}^{3 \times 3}$, $\mathbf{B}_{sw} \in \mathbb{R}^{3 \times 1}$, and $\mathbf{V}_s \in \mathbb{R}^{3 \times 3}$ are the proper known constant matrices. $\bar{\mathbf{w}}_s(t) \in \mathbb{R}^3$ is the state vector of the exogenous systems. $\delta_{sw}(t)$ is the additional disturbance that results from the perturbations and uncertainties in the exogenous system, and it is also supposed that $\delta_{sw}(t)$ is bounded in the H_2 -norm. $(\mathbf{A}_{ski}, \mathbf{B}_{svki})$ is controllable, and $(\mathbf{W}_s, \mathbf{B}_{svki} \mathbf{V}_s)$ is observable, for $k, i \in \mathbb{N}$.

Remark 1. Many kinds of disturbances in engineering can be described by this model, for example, unknown constants and harmonics with unknown phase and magnitude. In general engineering application, the system (14) can be assumed to be neutral stable, which implies that a persistent disturbance is imposed on the system. The controllability and observability assumption is a necessary condition, such that the latter switching control gain matrices and disturbance observer gain matrix can be obtained, based on linear system theory.

Let us consider the following disturbance observer, for estimating the unknown disturbance $\mathbf{d}_s(t)$ generated by the system (14):

$$\begin{cases} \hat{\mathbf{d}}_{s0}(t) = \mathbf{V}_s \hat{\bar{\mathbf{w}}}_s(t), \hat{\bar{\mathbf{w}}}_s(t) = \bar{\mathbf{v}}_s(t) - \mathbf{L}_s \mathbf{x}_s(t) \\ \dot{\bar{\mathbf{v}}}_s(t) = \sum_{k=1}^4 \sum_{i=1}^4 v_k w_{ski}(V_m, q) [\mathbf{L}_s (\mathbf{A}_{ski} \mathbf{x}_s(t) + \mathbf{B}_{svki} \mathbf{v}_s(t)) \\ + (\mathbf{W}_s + \mathbf{L}_s \mathbf{B}_{svki} \mathbf{V}_s)(\bar{\mathbf{v}}_s(t) - \mathbf{L}_s \mathbf{x}_s(t))] \end{cases} \quad (15)$$

where, $\hat{\bar{\mathbf{w}}}_s(t)$ and $\hat{\mathbf{d}}_{s0}(t)$ are the estimations of $\bar{\mathbf{w}}_s(t)$ and $\mathbf{d}_{s0}(t)$, respectively; $\bar{\mathbf{v}}_s(t)$ is the auxiliary state vector of the disturbance observer, and $\mathbf{L}_s$ is the disturbance observer gain matrix to be determined.

Based on the parallel distributed compensation (PDC)

scheme, we consider the following switching controller for the slow loop polytopic LPV system (12):

$$\mathbf{v}_s(t) = \sum_{k=1}^4 \sum_{i=1}^4 v_k w_{ski}(V_m, q) \mathbf{K}_{svki}(\mathbf{x}_s(t) - \mathbf{x}_{sr}(t)) - \hat{\mathbf{d}}_{s0}(t) \quad (16)$$

where, $\mathbf{K}_{svki}$, and $k, i \in \mathbb{N}$, are the feedback control gain matrices to be determined.

To cast the system into a descriptor form [15], the switching controller (16) can be converted into:

$$0 \cdot [\dot{\mathbf{v}}_s(t) + \hat{\mathbf{d}}_{s0}(t)] = \sum_{k=1}^4 \sum_{i=1}^4 v_k w_{ski}(V_m, q) [\mathbf{v}_s(t) + \hat{\mathbf{d}}_{s0}(t) - \mathbf{K}_{svki}(\mathbf{x}_s(t) - \mathbf{x}_{sr}(t))] \quad (17)$$

From (12)-(17), we can obtain the following descriptor system:

$$\begin{cases} \tilde{\mathbf{E}}_s \dot{\tilde{\mathbf{x}}}_s(t) = \sum_{k=1}^4 \sum_{i=1}^4 v_k w_{ski}(V_m, q) (\tilde{\mathbf{A}}_{ski} \tilde{\mathbf{x}}_s(t) + \tilde{\mathbf{B}}_s \tilde{\mathbf{d}}_s(t)) \\ \mathbf{y}_s(t) = \tilde{\mathbf{C}}_s \tilde{\mathbf{x}}_s(t) \end{cases} \quad (18)$$

where,

$$\tilde{\mathbf{x}}_s(t) \triangleq [\mathbf{x}_s^T(t), \mathbf{e}_s^T(t), \mathbf{v}_s^T(t) + \hat{\mathbf{d}}_{s0}^T(t), \mathbf{e}_{sv}^T(t)]^T, \quad \mathbf{e}_s(t) \triangleq \mathbf{x}_s(t) - \mathbf{x}_{sr}(t), \quad \mathbf{e}_{sv}(t) \triangleq \bar{\mathbf{w}}_s(t) - \hat{\bar{\mathbf{w}}}_s(t),$$

$$\tilde{\mathbf{d}}_s(t) \triangleq [\mathbf{d}_{s1}^T(t), \mathbf{r}_s^T(t), \delta_{sw}^T(t)]^T, \quad \tilde{\mathbf{E}}_s \triangleq \text{diag}\{\mathbf{I}, \mathbf{I}, 0, \mathbf{I}\}, \quad \tilde{\mathbf{C}}_s \triangleq [\mathbf{C}_s, 0, 0, 0],$$

$$\tilde{\mathbf{A}}_{ski} \triangleq \begin{bmatrix} \mathbf{A}_{ski} & 0 & \mathbf{B}_{svki} & \mathbf{B}_{svki} \mathbf{V}_s \\ \mathbf{A}_{ski} - \mathbf{A}_{sr} & \mathbf{A}_{sr} & \mathbf{B}_{svki} & \mathbf{B}_{svki} \mathbf{V}_s \\ 0 & -\mathbf{K}_{svki} & \mathbf{I} & 0 \\ 0 & 0 & 0 & \mathbf{W}_s + \mathbf{L}_s \mathbf{B}_{svki} \mathbf{V}_s \end{bmatrix}, \quad \tilde{\mathbf{B}}_s \triangleq \begin{bmatrix} \mathbf{B}_{sd} & 0 & 0 \\ \mathbf{B}_{sd} & -\mathbf{B}_{sr} & 0 \\ 0 & 0 & 0 \\ \mathbf{L}_s \mathbf{B}_{sd} & 0 & \mathbf{B}_{sw} \end{bmatrix}.$$

Then, consider the following H_∞ control performance for the closed-loop augmented system (18):

$$\begin{aligned} & \int_0^{t_f} [(\mathbf{x}_s(t) - \mathbf{x}_{sr}(t))^T \mathbf{Q}_s (\mathbf{x}_s(t) - \mathbf{x}_{sr}(t)) + (\mathbf{v}_s(t) + \hat{\mathbf{d}}_{s0}(t))^T \mathbf{R}_s (\mathbf{v}_s(t) + \hat{\mathbf{d}}_{s0}(t))] dt \\ & \leq \tilde{\mathbf{x}}_s^T(0) \tilde{\mathbf{E}}_s^T \tilde{\mathbf{P}}_s \tilde{\mathbf{x}}_s(0) + \tilde{\rho}_s^2 \int_0^{t_f} \tilde{\mathbf{d}}_s^T(t) \tilde{\mathbf{d}}_s(t) dt \end{aligned} \quad (19)$$

where, $\tilde{\rho}_s > 0$ is a prescribed disturbance attenuation level, t_f is the terminal time, $\mathbf{Q}_s$ and $\mathbf{R}_s$ are the prescribed positive defined weighting matrices with approximate dimensions, and $\tilde{\mathbf{P}}_s$ is a real matrix to be determined. Then, from (17) and (18), the H_∞ control performance (19) can be rewritten as:

$$\int_0^{t_f} \tilde{\mathbf{x}}_s^T(t) \tilde{\mathbf{Q}}_s \tilde{\mathbf{x}}_s(t) dt \leq \tilde{\mathbf{x}}_s^T(0) \tilde{\mathbf{E}}_s^T \tilde{\mathbf{P}}_s \tilde{\mathbf{x}}_s(0) + \tilde{\rho}_s^2 \int_0^{t_f} \tilde{\mathbf{d}}_s^T(t) \tilde{\mathbf{d}}_s(t) dt \quad (20)$$

where, $\tilde{\mathbf{Q}}_s \triangleq \text{diag}\{0, \mathbf{Q}_s, \mathbf{R}_s, 0\}$.

Therefore, the objective of this section is to design a robust H_∞ switching controller (16), such that the closed-loop augmented system (18) is stable, with a prescribed H_∞ performance (20).

Then, the following theorem provides an LMI-based method for designing the robust H_∞ switching controller (16).

Theorem 1. Consider the polytopic LPV system (12) with the performance H_∞ (20), and let Assumption 1 hold. For some given scalars $\tilde{\rho}_s > 0$, h_{s1} and h_{s2} , if there exist matrices $\tilde{\Pi}_{s11} > 0$, $\tilde{\mathbf{W}}_{s22} > 0$, $\tilde{\mathbf{W}}_{s31}$, $\tilde{\mathbf{W}}_{s32}$, $\tilde{\mathbf{W}}_{s33}$, $\tilde{\mathbf{P}}_{s4} > 0$, $\mathbf{Y}_{svki}$ and $\mathbf{N}_s$, $k, i \in \mathbb{N}$, satisfying the following LMIs:

$$\Xi_{ski} < 0, \quad k, i \in \mathbb{N} \quad (21)$$

$$2h_{s1}\tilde{\mathbf{P}}_{s4} - [\tilde{\mathbf{P}}_{s4}\mathbf{W}_s + \mathbf{N}_s\mathbf{B}_{svki}\mathbf{V}_s + *] < 0, \quad k, i \in \mathbb{N} \quad (22)$$

$$-2h_{s2}\tilde{\mathbf{P}}_{s4} + [\tilde{\mathbf{P}}_{s4}\mathbf{W}_s + \mathbf{N}_s\mathbf{B}_{svki}\mathbf{V}_s + *] < 0, \quad k, i \in \mathbb{N} \quad (23)$$

where,

$$\Xi_{ski} \triangleq \begin{bmatrix} \Xi_s^{(11)} & \Xi_s^{(12)} & \Xi_s^{(13)} & 0 & \mathbf{B}_{sr} & 0 & 0 & 0 & \Xi_s^{(13)} \\ * & \Xi_s^{(22)} & \Xi_s^{(23)} & \mathbf{B}_{sd} & -\mathbf{B}_{sr} & 0 & \mathbf{B}_{svki}\mathbf{V}_s & \tilde{\mathbf{W}}_{s22} & \tilde{\mathbf{W}}_{s22}^T \\ * & * & \Xi_s^{(33)} & 0 & 0 & 0 & 0 & 0 & \tilde{\mathbf{W}}_{s33}^T \\ * & * & * & -\tilde{\rho}_s^2\mathbf{I} & 0 & 0 & \mathbf{B}_{sd}^T\mathbf{N}_s^T & 0 & 0 \\ * & * & * & * & -\tilde{\rho}_s^2\mathbf{I} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\tilde{\rho}_s^2\mathbf{I} & \mathbf{B}_{svki}^T\tilde{\mathbf{P}}_{s4} & 0 & 0 \\ * & * & * & * & * & * & \Xi_s^{(77)} & 0 & 0 \\ * & * & * & * & * & * & * & -\tilde{\mathbf{Q}}_s^{-1} & 0 \\ * & * & * & * & * & * & * & * & -\mathbf{R}_s^{-1} \end{bmatrix},$$

with,

$$\Xi_s^{(11)} \triangleq [\mathbf{A}_{sr}\tilde{\Pi}_{s11} + *], \quad \Xi_s^{(12)} \triangleq \tilde{\Pi}_{s11}(\mathbf{A}_{skl}^T - \mathbf{A}_{sr}^T) + (\tilde{\mathbf{W}}_{s31}^T - \tilde{\mathbf{W}}_{s32}^T)\mathbf{B}_{svki}^T, \quad \Xi_s^{(13)} \triangleq \tilde{\mathbf{W}}_{s31}^T - \tilde{\mathbf{W}}_{s32}^T,$$

$$\Xi_s^{(22)} \triangleq [\mathbf{A}_{skl}\tilde{\mathbf{W}}_{s22} + \mathbf{B}_{svki}\tilde{\mathbf{W}}_{s32} + *], \quad \Xi_s^{(23)} \triangleq \mathbf{B}_{svki}\tilde{\mathbf{W}}_{s33} - \mathbf{Y}_s^T\mathbf{W}_{svki} + \tilde{\mathbf{W}}_{s32}^T, \quad \Xi_s^{(33)} \triangleq [\tilde{\mathbf{W}}_{s33} + *],$$

$$\Xi_s^{(77)} \triangleq [\tilde{\mathbf{P}}_{s4}\mathbf{W}_s + \mathbf{N}_s\mathbf{B}_{svki}\mathbf{V}_s + *],$$

then, there exists a switching controller (16), such that the closed-loop system (18) is stable with a prescribed performance H_∞ (20), and the poles of the disturbance estimation error system lie in the vertical strip region $D_{vs} = \{s \in \mathbb{C} : h_{s1} < \text{Re}(s) < h_{s2}\}$. In this case, the switching controller gain matrices $\mathbf{K}_{svki}$, $k, i \in \mathbb{N}$, and the disturbance observer gain matrix $\mathbf{L}_s$ can be constructed as:

$$\mathbf{K}_{svki} = \mathbf{Y}_{svki}\tilde{\mathbf{W}}_{s22}^{-1}, \quad \mathbf{L}_s = \tilde{\mathbf{P}}_{s4}^{-1}\mathbf{N}_s \quad (24)$$

Proof. Consider the following Lyapunov function:

$$V_s(t) = \tilde{\mathbf{x}}_s^T(t)\tilde{\mathbf{E}}_s^T\tilde{\mathbf{P}}_s\tilde{\mathbf{x}}_s(t)$$

where, $\tilde{\mathbf{E}}_s^T\tilde{\mathbf{P}}_s = \tilde{\mathbf{P}}_s^T\tilde{\mathbf{E}}_s \geq 0$. By taking the time derivative of $V_s(t)$ along the solutions of the closed-loop descriptor system (18), we have:

$$\begin{aligned} \dot{V}_s(t) &= \tilde{\mathbf{x}}_s^T(t)\tilde{\mathbf{Q}}_s\tilde{\mathbf{x}}_s(t) - \tilde{\rho}_s^2\tilde{\mathbf{d}}_s^T(t)\tilde{\mathbf{d}}_s(t) \\ &= \tilde{\mathbf{x}}_s^T(t)\tilde{\mathbf{E}}_s^T\tilde{\mathbf{P}}_s\tilde{\mathbf{x}}_s(t) + \tilde{\mathbf{x}}_s^T(t)\tilde{\mathbf{E}}_s^T\tilde{\mathbf{P}}_s\tilde{\mathbf{x}}_s(t) + \tilde{\mathbf{x}}_s^T(t)\tilde{\mathbf{Q}}_s\tilde{\mathbf{x}}_s(t) - \tilde{\rho}_s^2\tilde{\mathbf{d}}_s^T(t)\tilde{\mathbf{d}}_s(t) \\ &= \tilde{\mathbf{x}}_s^T(t)\tilde{\mathbf{E}}_s^T\tilde{\mathbf{P}}_s\tilde{\mathbf{x}}_s(t) + \tilde{\mathbf{x}}_s^T(t)\tilde{\mathbf{P}}_s^T\tilde{\mathbf{E}}_s\tilde{\mathbf{x}}_s(t) + \tilde{\mathbf{x}}_s^T(t)\tilde{\mathbf{Q}}_s\tilde{\mathbf{x}}_s(t) - \tilde{\rho}_s^2\tilde{\mathbf{d}}_s^T(t)\tilde{\mathbf{d}}_s(t) \\ &= 2\sum_{k=1}^4\sum_{i=1}^4\mathbf{V}_k\mathbf{W}_{skl}(\mathbf{V}_m, \mathbf{q})[\tilde{\mathbf{A}}_{skl}\tilde{\mathbf{x}}_s(t) + \tilde{\mathbf{B}}_{skl}\tilde{\mathbf{d}}_s(t)]^T\tilde{\mathbf{P}}_s\tilde{\mathbf{x}}_s(t) + \tilde{\mathbf{x}}_s^T(t)\tilde{\mathbf{Q}}_s\tilde{\mathbf{x}}_s(t) - \tilde{\rho}_s^2\tilde{\mathbf{d}}_s^T(t)\tilde{\mathbf{d}}_s(t) \\ &= \sum_{k=1}^4\sum_{i=1}^4\mathbf{V}_k\mathbf{W}_{skl}(\mathbf{V}_m, \mathbf{q})[\tilde{\mathbf{A}}_{skl}^T\tilde{\mathbf{x}}_s(t) + \tilde{\mathbf{B}}_{skl}^T\tilde{\mathbf{d}}_s(t)]^T\tilde{\Psi}_{ski}[\tilde{\mathbf{x}}_s^T(t), \tilde{\mathbf{d}}_s^T(t)]^T \end{aligned} \quad (25)$$

$$\text{where, } \tilde{\Psi}_{ski} \triangleq \begin{bmatrix} \tilde{\mathbf{P}}_s^T\tilde{\mathbf{A}}_{ski} + \tilde{\mathbf{A}}_{ski}^T\tilde{\mathbf{P}}_s + \tilde{\mathbf{Q}}_s & \tilde{\mathbf{P}}_s^T\tilde{\mathbf{B}}_s \\ * & -\tilde{\rho}_s^2\mathbf{I} \end{bmatrix}.$$

Define:

$$\tilde{\mathbf{A}}_{ski} \triangleq \begin{bmatrix} \tilde{\mathbf{A}}_{ski11} & \tilde{\mathbf{A}}_{ski12} \\ 0 & \mathbf{W}_s + \mathbf{L}_s\mathbf{B}_{svki}\mathbf{V}_s \end{bmatrix}, \quad \tilde{\mathbf{B}}_s \triangleq \begin{bmatrix} \tilde{\mathbf{B}}_{s11} \\ \tilde{\mathbf{B}}_{s21} \end{bmatrix}, \quad \tilde{\mathbf{Q}}_s \triangleq \begin{bmatrix} \tilde{\mathbf{Q}}_{s1} & 0 \\ 0 & 0 \end{bmatrix} \quad (26)$$

where,

$$\tilde{\mathbf{A}}_{ski11} \triangleq \begin{bmatrix} \mathbf{A}_{ski} & 0 & \mathbf{B}_{svki} \\ \mathbf{A}_{skl} - \mathbf{A}_{sr} & \mathbf{A}_{sr} & \mathbf{B}_{svki} \\ 0 & -\mathbf{K}_{svki} & \mathbf{I} \end{bmatrix}, \quad \tilde{\mathbf{A}}_{ski12} \triangleq \begin{bmatrix} \mathbf{B}_{svki}\mathbf{V}_s \\ \mathbf{B}_{svki}\mathbf{V}_s \\ 0 \end{bmatrix}, \quad \tilde{\mathbf{B}}_{s11} \triangleq \begin{bmatrix} \mathbf{B}_{sd} & 0 & 0 \\ \mathbf{B}_{sd} & -\mathbf{B}_{sr} & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\tilde{\mathbf{B}}_{s21} \triangleq [\mathbf{L}_s\mathbf{B}_{sd} \quad 0 \quad \mathbf{B}_{sw}], \quad \tilde{\mathbf{Q}}_{s1} \triangleq \text{diag}\{0, \mathbf{Q}_s, \mathbf{R}_s\}.$$

$$\text{Let } \tilde{\mathbf{W}}_{s1} = \begin{bmatrix} \tilde{\mathbf{W}}_{s1} & 0 \\ \tilde{\mathbf{W}}_{s2} & \tilde{\mathbf{W}}_{s33} \end{bmatrix}, \quad \tilde{\mathbf{W}}_{s1} \triangleq \begin{bmatrix} \tilde{\mathbf{W}}_{s11} & \tilde{\mathbf{W}}_{s12} \\ \tilde{\mathbf{W}}_{s12}^T & \tilde{\mathbf{W}}_{s22} \end{bmatrix},$$

$\tilde{\mathbf{W}}_{s2} \triangleq [\tilde{\mathbf{W}}_{s31} \quad \tilde{\mathbf{W}}_{s32}]$, $\tilde{\mathbf{W}}_{s11} > 0$, and $\tilde{\mathbf{W}}_{s22} > 0$. It is clear from $\tilde{\mathbf{W}}_{s1} > 0$ that $\tilde{\Pi}_{s11} = \tilde{\mathbf{W}}_{s11} - \tilde{\mathbf{W}}_{s12}\tilde{\mathbf{W}}_{s22}^{-1}\tilde{\mathbf{W}}_{s12}^T > 0$. For brevity, set $\tilde{\mathbf{W}}_{s12} = \tilde{\mathbf{W}}_{s12}^T = \tilde{\mathbf{W}}_{s22}^T$, and we have $\tilde{\Pi}_{s11} = \tilde{\mathbf{W}}_{s11} - \tilde{\mathbf{W}}_{s22}^T$.

$$\text{Define a nonsingular matrix } \Lambda_s \triangleq \begin{bmatrix} \Lambda_{s1} & 0 \\ 0 & \mathbf{I} \end{bmatrix},$$

$\Lambda_{s1} \triangleq \begin{bmatrix} \mathbf{I} & -\mathbf{I} \\ 0 & \mathbf{I} \end{bmatrix}$. Then, by the Schur complement, (24) and (21), we obtain:

$$\begin{bmatrix} \tilde{\Psi}_{ski}^{(11)} & \Lambda_s\tilde{\mathbf{B}}_{s11} & \Lambda_s\tilde{\mathbf{A}}_{ski12} \\ * & -\tilde{\rho}_s^2\mathbf{I} & \tilde{\mathbf{B}}_{s21}^T\tilde{\mathbf{P}}_{s4} \\ * & * & \tilde{\Psi}_{ski}^{(33)} \end{bmatrix} < 0 \quad (27)$$

where, $\tilde{\Psi}_{ski}^{(11)} \triangleq [\Lambda_s\tilde{\mathbf{A}}_{ski11}\tilde{\mathbf{W}}_{s1}\Lambda_s^T + *] + \Lambda_s\tilde{\mathbf{W}}_{s1}^T\tilde{\mathbf{Q}}_{s1}\tilde{\mathbf{W}}_{s1}\Lambda_s^T$,

$$\tilde{\Psi}_{ski}^{(33)} \triangleq [\tilde{\mathbf{P}}_{s4}\mathbf{W}_s + \tilde{\mathbf{P}}_{s4}\mathbf{L}_s\mathbf{B}_{svki}\mathbf{V}_s + *].$$

Pre- and post- multiplying (27) with the matrix $\text{diag}\{\tilde{\mathbf{P}}_{s1}\Lambda_s^{-1}, \mathbf{I}, \mathbf{I}\}$, $\tilde{\mathbf{P}}_{s1} = \tilde{\mathbf{W}}_{s1}^{-1}$, and its transpose, respectively; we then have:

$$\begin{bmatrix} \tilde{\Psi}_{ski}^{(11)} & \tilde{\mathbf{P}}_{s1}^T\tilde{\mathbf{B}}_{s11} & \tilde{\mathbf{P}}_{s1}^T\tilde{\mathbf{A}}_{ski12} \\ * & -\tilde{\rho}_s^2\mathbf{I} & \tilde{\mathbf{B}}_{s21}^T\tilde{\mathbf{P}}_{s4} \\ * & * & \tilde{\Psi}_{ski}^{(33)} \end{bmatrix} < 0 \quad (28)$$

where, $\tilde{\Psi}_{ski}^{(11)} \triangleq [\tilde{\mathbf{P}}_{s1}^T\tilde{\mathbf{A}}_{ski11} + *] + \tilde{\mathbf{Q}}_{s1}$.

$$\text{Define } \tilde{\mathbf{P}}_s \triangleq \begin{bmatrix} \tilde{\mathbf{P}}_{s1} & 0 \\ 0 & \tilde{\mathbf{P}}_{s4} \end{bmatrix}, \text{ where } \tilde{\mathbf{P}}_{s1} \text{ is invertible, and } \tilde{\mathbf{P}}_{s4} > 0.$$

Then, from (26) and (28), we have:

$$\Psi_{ski} < 0, \quad k, i \in \mathbb{N} \quad (29)$$

Therefore, substituting (29) to (25), gives:

$$\dot{V}_s(t) + \tilde{\mathbf{x}}_s^T(t)\tilde{\mathbf{Q}}_s\tilde{\mathbf{x}}_s(t) - \tilde{\rho}_s^2\tilde{\mathbf{d}}_s^T(t)\tilde{\mathbf{d}}_s(t) < 0 \quad (30)$$

Integrating (30) from $t=0$ to $t=t_f$ and considering $\tilde{\mathbf{x}}_s^T(t_f)\tilde{\mathbf{E}}_s^T\tilde{\mathbf{P}}_s\tilde{\mathbf{x}}_s(t_f) \geq 0$, it is easy to obtain that the H_∞ performance (20) is ensured.

For $\tilde{\mathbf{Q}}_s \geq 0$, $\rho_s > 0$, we have:

$$\tilde{\mathbf{P}}_s^T \tilde{\mathbf{A}}_{ski} + \tilde{\mathbf{A}}_{ski}^T \tilde{\mathbf{P}}_s < 0 \quad (31)$$

According to Lyapunov stability theory, the closed-loop descriptor system (18) is stable. Therefore, if (21) is satisfied, we can obtain the switching controller (16).

Based on the robust control theory, if $\tilde{\mathbf{P}}_{s4} > 0$ satisfies (22) and (23), then the poles of the disturbance estimation error system lie in the vertical strip region D_{vs} . This completes the proof.

Remark 2. For every performance index, we only need to solve 16 LMIs, based on the descriptor system method. However, we should solve 40 LMIs, by the traditional switching controller design method. With the increase of the divided subregions, the number of the solved LMIs will swell. The redundancy of the descriptor transformation can drastically reduce the number of the LMIs, especially for large divided subregions. Therefore, the descriptor system method is especially suitable for complex system design.

In order to obtain better tracking performance, we consider the following minimization problem, such that the disturbance attenuation level $\tilde{\rho}_s$ is reduced to be as small as possible:

$$\min_{\{\tilde{\mathbf{P}}_{11}, \tilde{\mathbf{P}}_{22}, \tilde{\mathbf{P}}_{31}, \tilde{\mathbf{P}}_{32}, \tilde{\mathbf{P}}_{33}, \tilde{\mathbf{P}}_{34}, \mathbf{Y}_{ski}, \mathbf{N}_s, k, i \in \mathbb{N}\}} \chi_s \quad \text{subject to LMIs (21)-(23)} \quad (32)$$

where, $\chi_s \triangleq \tilde{\rho}_s^2$.

3.2 Fast loop switching controller design

Similarly, the fast loop polytopic LPV system (9) can be written as:

$$\begin{cases} \dot{\mathbf{x}}_f(t) = \sum_{k=1}^4 \sum_{i=1}^4 v_k w_{fki}(V_m, q) [\mathbf{A}_{fki} \mathbf{x}_f(t) + \mathbf{B}_{fki} (\mathbf{u}(t) + \mathbf{d}_{f0}(t))] + \mathbf{B}_{fd} \mathbf{d}_{f1}(t) \\ \mathbf{y}_f(t) = \mathbf{C}_f \mathbf{x}_f(t) \end{cases} \quad (33)$$

For the fast loop polytopic LPV system, we consider the following reference model:

$$\dot{\mathbf{x}}_{fr}(t) = \mathbf{A}_{fr} \mathbf{x}_{fr}(t) + \mathbf{B}_{fr} \mathbf{r}_f(t) \quad (34)$$

where, $\mathbf{x}_{fr}(t)$ is the state vector of the reference model, and $\mathbf{r}_f(t)$ is the reference input signal.

Remark 3. For the fast loop switching controller, our purpose is that the fast loop state $\mathbf{x}_f(t)$ of the NSI can quickly track the output of the slow loop controller. Therefore, for the fast loop reference model, the reference input signal satisfies $\mathbf{r}_f(t) = \mathbf{v}_s(t)$.

Assumption 2. The disturbances $\mathbf{d}_{f0}(t)$ can be generated by the following exogenous system:

$$\begin{cases} \dot{\tilde{\mathbf{w}}}_f(t) = \mathbf{W}_f \tilde{\mathbf{w}}_f(t) + \mathbf{B}_{fw} \delta_{fw}(t) \\ \mathbf{d}_{f0}(t) = \mathbf{V}_f \tilde{\mathbf{w}}_f(t) \end{cases} \quad (35)$$

where, $\mathbf{W}_f \in \mathbb{R}^{3 \times 3}$, $\mathbf{B}_{fw} \in \mathbb{R}^{3 \times 1}$, and $\mathbf{V}_f \in \mathbb{R}^{5 \times 3}$ are the proper known constant matrices. $\tilde{\mathbf{w}}_f(t) \in \mathbb{R}^3$ is the state vector of the exogenous systems. It is also supposed that $\delta_{fw}(t)$ is bounded in the H_2 -norm. $(\mathbf{A}_{fki}, \mathbf{B}_{fki})$ is controllable, and $(\mathbf{W}_f, \mathbf{B}_{fki} \mathbf{V}_f)$ is observable, for $k, i \in \mathbb{N}$.

To estimate the unknown disturbance $\mathbf{d}_{f0}(t)$, the disturbance observer is constructed, as follows:

$$\begin{cases} \dot{\hat{\mathbf{d}}}_{f0}(t) = \mathbf{V}_f \hat{\tilde{\mathbf{w}}}_f(t), \hat{\tilde{\mathbf{w}}}_f(t) = \bar{\mathbf{v}}_f(t) - \mathbf{L}_f \mathbf{x}_f(t) \\ \dot{\bar{\mathbf{v}}}_f(t) = \sum_{k=1}^4 \sum_{i=1}^4 v_k w_{fki}(V_m, q) [\mathbf{L}_f (\mathbf{A}_{fki} \mathbf{x}_f(t) + \mathbf{B}_{fki} \mathbf{u}(t)) \\ + (\mathbf{W}_f + \mathbf{L}_f \mathbf{B}_{fki} \mathbf{V}_f) (\bar{\mathbf{v}}_f(t) - \mathbf{L}_f \mathbf{x}_f(t))] \end{cases} \quad (36)$$

where, $\hat{\tilde{\mathbf{w}}}_f(t)$ and $\hat{\mathbf{d}}_{f0}(t)$ are the estimations of $\tilde{\mathbf{w}}_f(t)$ and $\mathbf{d}_{f0}(t)$, respectively; $\mathbf{L}_f$ is the disturbance observer gain matrix to be determined, and $\bar{\mathbf{v}}_f(t)$ is the auxiliary state vector of the disturbance observer.

For the fast loop polytopic LPV system (33), we consider the following switching controller:

$$\mathbf{u}(t) = \sum_{k=1}^4 \sum_{i=1}^4 v_k w_{fki}(V_m, q) \mathbf{K}_{fki} (\mathbf{x}_f(t) - \mathbf{x}_{fr}(t)) - \hat{\mathbf{d}}_{f0}(t) \quad (37)$$

where, $\mathbf{K}_{fki}$, and $k, i \in \mathbb{N}$, are the feedback control gain matrices to be determined.

Then, we have the following descriptor system for the fast loop:

$$\begin{cases} \tilde{\mathbf{E}}_f \dot{\tilde{\mathbf{x}}}_f(t) = \sum_{k=1}^4 \sum_{i=1}^4 v_k w_{fki}(V_m, q) (\tilde{\mathbf{A}}_{fki} \tilde{\mathbf{x}}_f(t) + \tilde{\mathbf{B}}_f \tilde{\mathbf{d}}_f(t)) \\ \mathbf{y}_f(t) = \tilde{\mathbf{C}}_f \tilde{\mathbf{x}}_f(t) \end{cases} \quad (38)$$

where,

$$\tilde{\mathbf{x}}_f(t) \triangleq [\mathbf{x}_f^T(t), \mathbf{e}_p^T(t), \mathbf{v}_f^T(t) + \hat{\mathbf{d}}_{f0}^T(t), \mathbf{e}_{fw}^T(t)]^T, \quad \mathbf{e}_p(t) \triangleq \mathbf{x}_f(t) - \mathbf{x}_{fr}(t), \quad \mathbf{e}_{fw}(t) \triangleq \tilde{\mathbf{w}}_f(t) - \hat{\tilde{\mathbf{w}}}_f(t),$$

$$\tilde{\mathbf{d}}_f(t) \triangleq [\mathbf{d}_{f1}^T(t), \mathbf{r}_f^T(t), \delta_{fw}^T(t)]^T, \quad \tilde{\mathbf{E}}_f \triangleq \text{diag}\{\mathbf{I}, \mathbf{I}, 0, \mathbf{I}\}, \quad \tilde{\mathbf{C}}_f \triangleq [\mathbf{C}_f, 0, 0, 0],$$

$$\tilde{\mathbf{A}}_{fki} \triangleq \begin{bmatrix} \mathbf{A}_{fki} & 0 & \mathbf{B}_{fki} & \mathbf{B}_{fki} \mathbf{V}_f \\ \mathbf{A}_{fki} - \mathbf{A}_{fr} & \mathbf{A}_{fr} & \mathbf{B}_{fki} & \mathbf{B}_{fki} \mathbf{V}_f \\ 0 & -\mathbf{K}_{fki} & \mathbf{I} & 0 \\ 0 & 0 & 0 & \mathbf{W}_f + \mathbf{L}_f \mathbf{B}_{fki} \mathbf{V}_f \end{bmatrix}, \quad \tilde{\mathbf{B}}_f \triangleq \begin{bmatrix} \mathbf{B}_{fd} & 0 & 0 \\ \mathbf{B}_{fd} & -\mathbf{B}_{fr} & 0 \\ 0 & 0 & 0 \\ \mathbf{L}_f \mathbf{B}_{fd} & 0 & \mathbf{B}_{fw} \end{bmatrix}.$$

Let us consider the following control performance H_∞ for the augmented system (38):

$$\int_0^{t_f} \tilde{\mathbf{x}}_f^T(t) \tilde{\mathbf{Q}}_f \tilde{\mathbf{x}}_f(t) dt \leq \tilde{\mathbf{x}}_f^T(0) \tilde{\mathbf{E}}_f^T \tilde{\mathbf{P}}_f \tilde{\mathbf{x}}_f(0) + \tilde{\rho}_f^2 \int_0^{t_f} \tilde{\mathbf{d}}_f^T(t) \tilde{\mathbf{d}}_f(t) dt$$

where, $\tilde{\rho}_f > 0$ is a prescribed disturbance attenuation level, $\tilde{\mathbf{Q}}_f \triangleq \text{diag}\{0, \mathbf{Q}_f, \mathbf{R}_f, 0\}$, $\mathbf{Q}_f$ and $\mathbf{R}_f$ are the prescribed positive defined weighting matrix with approximate dimensions.

Similarly, we introduce the following minimization problem, to solve the fast loop robust H_∞ switching tracking controller:

$$\min_{\{\tilde{\mathbf{U}}_{f11}, \tilde{\mathbf{W}}_{f22}, \tilde{\mathbf{W}}_{f31}, \tilde{\mathbf{W}}_{f32}, \tilde{\mathbf{W}}_{f33}, \tilde{\mathbf{P}}_{f4}, \mathbf{Y}_{fuki}, \mathbf{N}_f, k, i \in \mathbb{N}\}} \chi_f \quad (39)$$

subject to:

$$\Xi_{fki} < 0, \quad k, i \in \mathbb{N} \quad (40)$$

$$2h_{f1}\tilde{\mathbf{P}}_{f4} - [\tilde{\mathbf{P}}_{f4}\mathbf{W}_f + \mathbf{N}_f\mathbf{B}_{fuki}\mathbf{V}_f + *] < 0, \quad k, i \in \mathbb{N} \quad (41)$$

$$-2h_{f2}\tilde{\mathbf{P}}_{f4} + [\tilde{\mathbf{P}}_{f4}\mathbf{W}_f + \mathbf{N}_f\mathbf{B}_{fuki}\mathbf{V}_f + *] < 0, \quad k, i \in \mathbb{N} \quad (42)$$

where, $\chi_f = \tilde{\rho}_f^2$,

$$\Xi_{fki} \triangleq \begin{bmatrix} \Xi_f^{(11)} & \Xi_f^{(12)} & \Xi_f^{(13)} & 0 & \mathbf{B}_{fr} & 0 & 0 & 0 & \Xi_f^{(13)} \\ * & \Xi_f^{(22)} & \Xi_f^{(23)} & \mathbf{B}_{fd} & -\mathbf{B}_{fr} & 0 & \mathbf{B}_{fuki}\mathbf{V}_f & \tilde{\mathbf{W}}_{f22} & \tilde{\mathbf{W}}_{f32}^{(13)} \\ * & * & \Xi_f^{(33)} & 0 & 0 & 0 & 0 & 0 & \tilde{\mathbf{W}}_{f33}^{(13)} \\ * & * & * & -\tilde{\rho}_f^2\mathbf{I} & 0 & 0 & \mathbf{B}_{fd}^T\mathbf{N}_f^T & 0 & 0 \\ * & * & * & * & -\tilde{\rho}_f^2\mathbf{I} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\tilde{\rho}_f^2\mathbf{I} & \mathbf{B}_{fr}^T\tilde{\mathbf{P}}_{f4} & 0 & 0 \\ * & * & * & * & * & * & \Xi_f^{(77)} & 0 & 0 \\ * & * & * & * & * & * & * & -\mathbf{Q}_f^{-1} & 0 \\ * & * & * & * & * & * & * & * & -\mathbf{R}_f^{-1} \end{bmatrix},$$

with,

$$\begin{aligned} \Xi_f^{(11)} &\triangleq [\mathbf{A}_{fr}\tilde{\mathbf{U}}_{f11} + *], \quad \Xi_f^{(12)} \triangleq \tilde{\mathbf{U}}_{f11}(\mathbf{A}_{fr}^T - \mathbf{A}_{fr}^T) + (\tilde{\mathbf{W}}_{f31}^T - \tilde{\mathbf{W}}_{f32}^T)\mathbf{B}_{fuki}^T, \quad \Xi_f^{(13)} \triangleq \tilde{\mathbf{W}}_{f31}^T - \tilde{\mathbf{W}}_{f32}^T, \\ \Xi_f^{(22)} &\triangleq [\mathbf{A}_{fr}\tilde{\mathbf{W}}_{f22} + \mathbf{B}_{fuki}\tilde{\mathbf{W}}_{f32} + *], \quad \Xi_f^{(23)} \triangleq \mathbf{B}_{fuki}\tilde{\mathbf{W}}_{f33} - \mathbf{Y}_{fuki}^T + \tilde{\mathbf{W}}_{f32}^T, \quad \Xi_f^{(33)} \triangleq [\tilde{\mathbf{W}}_{f33} + *], \\ \Xi_f^{(77)} &\triangleq [\tilde{\mathbf{P}}_{f4}\mathbf{W}_f + \mathbf{N}_f\mathbf{B}_{fuki}\mathbf{V}_f + *], \end{aligned}$$

In this case, the switching controller gain matrices $\mathbf{K}_{fuki}$, $k, i \in \mathbb{N}$, and the disturbance observer gain matrix $\mathbf{L}_f$ can be constructed as:

$$\mathbf{K}_{fuki} = \mathbf{Y}_{fuki}\tilde{\mathbf{W}}_{f22}^{-1}, \quad \mathbf{L}_f = \tilde{\mathbf{P}}_{f4}^{-1}\mathbf{N}_f \quad (43)$$

Based on the above analysis, the robust H_∞ switching controller of the NSI is summarized as follows:

Design procedure:

- Step 1. Construct the slow-fast loop polytopic LPV model for the NSI.
- Step 2. Design the slow loop switching tracking controller (16).
- Step 3. Design the fast loop switching tracking controller (37).

4. Simulation results

In this section, some numerical simulations will be given, to demonstrate the effectiveness of the proposed composite control scheme. Some model parameters of the NSI are given in Table 1.

The system matrices of the reference model for the slow-fast loop model are given by:

$$\mathbf{A}_{sr} = -12\mathbf{I}, \quad \mathbf{B}_{sr} = [-12, -12, -12]^T, \quad \mathbf{A}_{fr} = -150\mathbf{I}, \quad \mathbf{B}_{fr} = [-150, -150, -150]^T.$$

The system matrices for $\mathbf{d}_{s0}(t)$ and $\mathbf{d}_{f0}(t)$ described by (14) and (35) are given by:

$$\mathbf{W}_s = \mathbf{W}_f = \begin{bmatrix} 0 & 5 & 0 \\ -5 & 0 & 0 \\ -5 & 0 & 5 \end{bmatrix}, \quad \mathbf{B}_{sw} = \mathbf{B}_{fw} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}, \quad \mathbf{V}_s = 3\mathbf{I}, \quad \mathbf{V}_f = \begin{bmatrix} 0.2 & 0 & 0 & 0.3 & 0 \\ 0 & 0.2 & 0 & 0 & 0.3 \\ 0 & 0 & 0.2 & 0 & 0 \end{bmatrix}.$$

Selecting $\mathbf{Q}_s = 100\mathbf{I}$, $\mathbf{R}_s = \mathbf{I}$, $h_{s1} = -15$, $h_{s2} = -10$, and solving the minimization problem (32), we can obtain the following switching controller gain matrices and disturbance observer gain matrix of the slow loop:

$$\mathbf{K}_{sv11} = \begin{bmatrix} -52.181 & -29.123 & 2.1716 \\ 43.0445 & -71.648 & 3.3733 \\ -37.9774 & 4.0077 & -76.295 \end{bmatrix}, \quad \dots, \quad \mathbf{L}_s = \begin{bmatrix} -3.0225 & -1.928 & 0.2620 \\ 1.7711 & -3.508 & 0.3079 \\ -1.5688 & 0.465 & -5.7827 \end{bmatrix}.$$

Select $\mathbf{Q}_f = 200\mathbf{I}$, $\mathbf{R}_f = \text{diag}\{100, 2, 2, 8, 8\}$, $h_{f1} = -30$, $h_{f2} = -10$, and solving the minimization problem (39), we can obtain the following switching controller gain matrices and disturbance observer gain matrix of the fast loop

$$\mathbf{K}_{fu11} = \begin{bmatrix} 3.0995 & -6.6 \times 10^{-4} & 0.0032 & -0.0066 & 0.0014 \\ -0.0011 & 9.8651 & 1.1 \times 10^{-4} & -2.27 \times 10^{-4} & -20.7209 \\ 0.0052 & 1.1 \times 10^{-4} & 9.8655 & -20.7218 & -2.27 \times 10^{-4} \end{bmatrix}, \quad \dots,$$

$$\mathbf{L}_f = \begin{bmatrix} -0.0334 & -0.0489 & -0.7076 \\ -4.76 \times 10^{-4} & -0.4132 & 0.0168 \\ 0.6067 & 0.0247 & 4.7701 \end{bmatrix}.$$

The reference command of the NSI system is given as follows:

$$\mathbf{r}_s(t) = \begin{cases} [10^\circ, -15^\circ, 15^\circ]^T & 0 \leq t \leq 1\text{s} \\ [25^\circ, -25^\circ, 25^\circ]^T & 1 \leq t \leq 2\text{s} \\ [10^\circ, -15^\circ, 15^\circ]^T & 2 \leq t \leq 3\text{s} \end{cases} \quad (44)$$

In the simulation process, we select $\delta_{sw}(t)$ and $\delta_{fw}(t)$ as the random signal, with upper 2-norm bound 1, and the external disturbance $\mathbf{d}_{s1} = \mathbf{d}_{f1} = 0.1e^{-10t}[\sin(10t), \cos(10t), \sin(10t)]^T$.

Table 1. Model parameters of the NSI

Model parameter	Value
J_x	1.615 kg.m ²
J_y	247.436726 kg.m ²
J_z	247.436726 kg.m ²
m	204.022722 kg
$\bar{L}$	0.2286 m
L_m	2 m
S	0.04087734 m ²
K_{M_y}	0.3
K_{M_z}	0.3

The robust H_∞ switching tracking controller incorporated with the disturbance observer (denoted by $RH_\infty\text{STC} + \text{DO}$, Case 1) is firstly applied to the NSI angle command tracking control. For comparison, the switching tracking controller combined with the no disturbance observer (denoted by $RH_\infty\text{STC}$, Case 2) is also studied.

Figs. 2-3 show the response curves of the angle and angle velocity for the NSI, where the solid line is for Case 1, and the dotted lines indicate the corresponding response curves for Case 2. From the comparison, we can show that the proposed composite control scheme of Case 1 can obtain a fast tracking effect, and present a better anti-disturbance performance, under the same simulation condition. Fig.

4 shows the actual and estimated value response curves of the slow loop disturbance. It can be seen that the proposed disturbance observer can provide a satisfactory estimated value for the slow loop disturbance d_{s0} . The conditions of the fast loop disturbance, which are omitted here, are similar. Fig. 5 shows the response curves of the aerodynamic fins, and lateral thrust equivalent control surface deflections, based on the two control schemes.

From the simulation results, we observe that the proposed composite control scheme ($RH_\infty\text{STC} + \text{DO}$) can improve the control performance compared with the method ($RH_\infty\text{STC}$), in tracking accuracy, disturbance rejection ability, and so on.

(a) Response curves of the bank angle

(b) Response curves of the sideslip angle

(c) Response curves of the angle of attack

Fig. 2. Response curves of the angle

(a) Response curves of the roll rate

(b) Response curves of the yaw rate

(c) Response curves of the pitch rate

Fig. 3. Response curves of the angle velocity

(a) Response curves of the disturbance d_{sr}

(b) Response curves of the disturbance d_{sb}

(c) Response curves of the disturbance d_{so}

Fig. 4. Actual and estimated value of the slow loop disturbance d_s

5. Conclusion

In this paper, we have dealt with the robust H_∞ switching tracking control design problem combined with disturbance observer for the NSI. In the proposed composite control scheme, a slow-fast loop polytopic LPV model is firstly employed to approximate the nonlinear dynamics of the NSI, and a disturbance observer is constructed, to estimate the disturbance generated by an exogenous system. Then, based on the descriptor system method, a robust switching tracking controller is developed, to provide an H_∞ disturbance attenuation constraint, as well as ensure the stability of the closed-loop NSI system. Finally, simulation results on the NSI indicate that the proposed design method is effective.

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(a) Response curves of the aileron

(b) Response curves of the rudder

(c) Response curves of the elevator

(d) Response curves of equivalent control surface deflections δ_{sy}

(e) Response curves of equivalent control surface deflections δ_{sz}

Fig. 5. Response curves of the control inputs

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Nomenclature

α	angle of attack
β	sideslip angle
γ	bank angle
ϑ	pitch angle
ω_x	roll rate
ω_y	yaw rate
ω_z	pitch rate

m	missile mass
g	acceleration due to gravity
V_m	missile velocity
q	dynamic pressure
S	reference area
$\bar{L}$	reference length
ρ	air density
δ_x	aileron deflection
δ_y	rudder deflection
δ_z	elevator deflection
K_{My}	jet interaction factor
K_{Mz}	jet interaction factor
J_x	moment of inertia around the roll axis
J_y	moment of inertia around the yaw axis
J_z	moment of inertia around the pitch axis
C_y^α	contribution to lift due to angle of attack α
$C_z^{\delta_z}$	contribution to lift due to elevator deflection δ_z
C_z^β	contribution to yaw force due to sideslip angle β
$C_z^{\delta_y}$	contribution to yaw force due to rudder deflection δ_y
m_y^β	contribution to yaw moment due to sideslip angle β
$m_y^{\omega_y}$	contribution to yaw moment due to yaw rate ω_y
$m_y^{\delta_y}$	contribution to yaw moment due to rudder deflection δ_y
m_z^α	contribution to pitch moment due to angle of attack α
$m_z^{\omega_z}$	contribution to pitch moment due to pitch rate ω_z
$m_z^{\delta_z}$	contribution to pitch moment due to elevator deflection δ_z