Dynamic Stability Analysis of Thick Plates with Varying Thickness and Concentrated Mass on Inhomogeneous Pasternak Foundation

비균질 Pasternak 지반에 놓인 집중질량을 갖는 변단면 후판의 동적안정해석

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Key Words : Finite Element Method(유한요소법), Dynamic Stability(동적안정), Tapered Thick Plate(변단면 후판), Pasternak Foundation(Pasternak 지반)

ABSTRACT

This paper is to analyze the stability of the thick plate on inhomogeneous Pasternak foundation, with linearly varying thickness and concentrated mass by finite element method. To verify this finite element method, the results of natural frequencies and buckling stresses by the proposed method are compared with the existing solutions. The dynamic instability regions are decided by the dynamic stability analysis of the thick plate on inhomogeneous Pasternak foundation, with linearly varying thickness and concentrated mass. The non-dimensional Winkler foundation parameter is applied as 100, 1000 and non-dimensional shear foundation parameter is applied as 5. The tapered ratios are applied as 0.25 and 1.0, the ratios of concentrated mass to plate mass as 0.25 and 1.0, and the ratio of in-plane force to critical load as 0.4. As the result of numerical analysis of the thick plate on inhomogeneous Pasternak foundation for $u \times v = 300 \text{ cm} \times 300 \text{ cm}$ and $a \times b = 600 \text{ cm} \times 600 \text{ cm}$, instability areas of the thick plate which has the larger rigidity of inner area are farther from β -axis and narrower than those which has the larger rigidity of outer area.

요 약

이 연구는 유한요소법을 이용하여 집중질량을 갖고 비균질한 지반에 놓은 변단면 후판 구조물에 대한 동적 안정해석이다. 해석을 위하여 유한요소법을 이용하여 해당구조물의 고유진동수와 좌굴하중을 구하여 기존해와 비교하여 프로그램을 검증하였다. 비균질 Pasternak 지반에 놓인 변단면으로 집중질량을 갖는 후판의 동적안정 해석을 통하여 판의 불안정 영역이 결정된다. 무차원 Winkler 지반강성은 100, 1000을 적용하였고, 무차원 전단지반강성은 5로 하였다. 그리고 변단면율은 0.25, 1.0, 집중질량비는 0.25, 1.0 그리고 면내응력은 $0.4\sigma_{cr}$ 을 적용하여 해석하였다.

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Nomenclatures

- h_0 : Thickness of plate at $x = 0$
 h_1 : Thickness of plate at $x = \bar{a}$
 T : Total kinetic energy of plate elements
 U : Total strain energy of plate elements
 W : Total work done by one-way in-plane force
 k_{fs} : The stiffness matrices of shear
 $m_c(a_1, b_1)$: Magnitude of mass at $x = a_1, y = b_1$
 k_{fw} : The stiffness matrices of Winkler foundation
 Δ : Nodal displacement vector
 M_e : Mass matrix of the thick plate
 M_p : Concentrated mass matrix of the thick plate
 K : Bending and shear stiffness matrix of the thick plate
 K_f : Stiffness matrix of the foundation
 K_G : Geometrical stiffness matrix of the thick plate

1. Introduction

Vibration of foundation which happens by explosions, driving piles, compaction of foundation, excavations, earthquake and so forth affects dynamic behaviors of buildings. Dynamic effect transmitted through foundation causes soil-structure interaction and this is an important topic of vibration control field. We can consider mat foundation as thick plate on elastic foundation. Recently, as high-rise buildings increase steeply, substructures of those are often supported inhomogeneous foundation and mats of substructures have often varying thickness.

In 1985, Horenberg et al.⁽¹⁾ researched vibration analysis of plate by modal constraint method. The plate on two-parametric foundation was analyzed by Celik et al.⁽²⁾ by use of iteration method. Buckling instability of thick elastic plates subjected to in-plane stresses, and vibration and stability of thick plates in elastic foundations were studied by Matsunaga^(3,7).

Kukreti et al.⁽⁴⁾ reported about fundamental fre-

quencies of simply supported rectangular plates with linearly varying thickness. Vibration analysis of thick plate subjected in-plane stress on inhomogeneous Pasternak foundation and dynamic stability of square thick plate on inhomogeneous Pasternak foundation were studied by Lee et al.^(5,6).

In this study, we studied dynamic stability of thick plate with concentrated mass and linearly varying thickness on inhomogeneous Pasternak foundation. In this analysis, we used 8-nodes Serendipity finite element and after obtaining element stiffness matrices and mass matrix, assembled all matrices for each element as global coordinates. As solving dynamic stability equation composed by global matrices, instability region of this plate is decided.

2. Finite Element Method Implementation

2.1 Variable Section

Figure 1 represents dimensions of the plate with linearly varying thickness. Minimum and maximum thickness of this plate are h_0 and h_1 respectively. If the thickness of plate departing x from origin of the cartesian coordinates (x, y) is h , which can be represented by non-dimensional parameter $\bar{x}$ as follows.

$$h = h_0 f(\bar{\xi}) \quad (1)$$

where,

$$\bar{\xi} = \frac{X}{a}, \quad f(\bar{\xi}) = 1 + \alpha_{TR} \bar{\xi}, \quad \alpha_{TR} = \frac{h_1 - h_0}{h_0}$$

$\bar{a}$: Length of plate,

h_0 : Thickness of plate at $x = 0$

h_1 : Thickness of plate at $x = \bar{a}$

The flexural rigidity of thick plate D is represented by using function $f(\bar{\xi})$ as follows.

$$D = D_0 f(\bar{\xi}) \quad (2)$$

Fig. 1 Tapered thick plate

where, $D_0 = \frac{Eh_0^3}{12(1-\nu^2)}$,

E : Young modulus

ν : Poisson's ratio

2.2 Vibration Analysis of the Thick Plate Element

(1) The Principal of Virtual Work

U_e , the strain energy, T_e , the kinetic energy of the thick plate element, and W_e , the work done by N_x , which is one-way in-plane axial force applied to the thick plate are assembled. Applying the principal of virtual work, we obtain the eigen-value equation as follows.

$$\delta(T - U) - \delta W = 0 \quad (3)$$

where,

T : Total kinetic energy of plate elements

U : Total strain energy of plate elements

W : Total work done by one-way in-plane force

(2) Stiffness and Mass Matrix of the Thick Plate Element

When length and width of the thick plate element are $2a$, $2b$ respectively, vertical deflection is w , and deflection angles for x , y axis are θ_x, θ_y respectively, kinetic energy of it T_e is written as follows.

$$T_e = \frac{1}{2} \int_{-a}^a \int_{-b}^b \rho \left(h \dot{w}^2 + \frac{h^3}{12} \dot{\theta}_x^2 + \frac{h^3}{12} \dot{\theta}_y^2 \right) dx dy + \frac{1}{2} m_c(a_1, b_1) \dot{w}^2(a_1, b_1) \quad (4)$$

where,

$$\dot{w}(a_1, b_1) = \left[\frac{\partial w}{\partial t} \right]_{y=b_1}^{x=a_1}$$

$$\dot{w} = \frac{\partial w}{\partial t}, \quad \dot{\theta}_x = \frac{\partial \theta_x}{\partial t}, \quad \dot{\theta}_y = \frac{\partial \theta_y}{\partial t}$$

m_c : magnitude of mass at $x=a_1, y=b_1$,

$h = h(x)$: thickness of the thick plate element

Strain energy of the thick plate element, U_e : considering stiffness of Winkler foundation and shear foundation is as follows.

$$U_e = \frac{1}{2} \int_{-a}^a \int_{-b}^b M^T \chi dx dy + \frac{1}{2} \int_{-a}^a \int_{-b}^b Q^T \psi dx dy + \frac{1}{2} \int_{-a}^a \int_{-b}^b k_w w^2 dx dy + \frac{1}{2} \int_{-a}^a \int_{-b}^b k_g [w_{,x}^2 + w_{,y}^2] dx dy \quad (5)$$

k_w and k_g of Eq. (5) are the rigidity of Winkler foundation and the rigidity of shear foundation respectively. M, Q, χ and ψ are moment vector, shear vector, curvature vector, and shear deformation vector of this plate element respectively and these can be presented as follows.

$$M = [M_x, M_y, M_{xy}]^T, \quad Q = [Q_x, Q_y]^T \quad (6a, b)$$

$$\chi = [-\theta_{x,x}, -\theta_{y,y}, -(\theta_{x,y} + \theta_{y,x})]^T \quad (6c)$$

$$\psi = [-\psi_x, -\psi_y]^T = [w_{,x} - \theta_x, w_{,y} - \theta_y]^T \quad (6d)$$

where

$$w_{,x} = \frac{\partial w}{\partial x}, \quad w_{,y} = \frac{\partial w}{\partial y}, \quad \theta_{x,x} = \frac{\partial \theta_x}{\partial x}, \quad \theta_{y,y} = \frac{\partial \theta_y}{\partial y}$$

The work done in-plane force N_x is as follows.

$$W_e = \frac{1}{2} \int_{-a}^a \int_{-b}^b N_x w_x^2 dx dy \quad (7)$$

here, N_x is in-plane force per unit width of plate element. Shape function N_i of serendipity element same as Fig. 2 is as follows⁽⁸⁾.

Fig. 2 8 node serendipity element

$$\bar{N} = [N_1, N_2, \dots, N_8]$$

$$\begin{aligned} N_1(\xi, \eta) &= -\frac{1}{4}(1-\xi)(1-\eta)(1+\xi+\eta) \\ N_2(\xi, \eta) &= \frac{1}{2}(1-\xi^2)(1-\eta) \\ N_3(\xi, \eta) &= \frac{1}{4}(1+\xi)(1-\eta)(\xi-\eta-1) \\ N_4(\xi, \eta) &= \frac{1}{2}(1+\xi)(1-\eta^2) \\ N_5(\xi, \eta) &= \frac{1}{4}(1+\xi)(1+\eta)(-1+\xi+\eta) \\ N_6(\xi, \eta) &= \frac{1}{2}(1-\xi^2)(1+\eta) \\ N_7(\xi, \eta) &= \frac{1}{4}(1-\xi)(1+\eta)(-\xi+\eta-1) \\ N_8(\xi, \eta) &= \frac{1}{2}(1-\xi)(1-\eta^2) \end{aligned} \quad (8)$$

$$\text{where, } \xi = \frac{x}{a}, \quad \eta = \frac{y}{b}$$

Substituting Eq. (8) in Eq. (4), we obtain the following kinetic energy equation.

$$T_e = \frac{1}{2} \delta_e^T m_e \dot{\delta}_e + \frac{1}{2} \dot{w}_e^T m_p \dot{w}_e \quad (9)$$

where, $\delta_e = [w_1, \theta_{x1}, \theta_{y1}, \dots, w_8, \theta_{x8}, \theta_{y8}]$: nodal displacement vector, $\dot{\delta} = \partial \delta_e / \partial t$

$$m_e = \int_{-a}^a \int_{-b}^b \bar{\rho} \bar{N}^T \bar{m}_e \bar{N} dx dy : \text{Mass matrix,}$$

$$\bar{m}_e = \begin{bmatrix} h & 0 & 0 \\ 0 & h^3/12 & 0 \\ 0 & 0 & h^3/12 \end{bmatrix}$$

$$m_p = m_c(a_1, b_1) [N^T N] : \text{Concentrated mass matrix}$$

$$N = [N_1, N_2, \dots, N_8], \quad N_i = N_i I_3 (i = 1, 2, \dots, 8)$$

$$I_3 = 3 \times 3 \text{ unit matrix}$$

$$W_e = [w_1, w_2, \dots, w_8]^T$$

$$\dot{w} = \partial w_e / \partial t$$

Substituting Eq. (8) in Eq. (5), we obtain the following strain energy equation.

$$U_e = \frac{1}{2} \delta_e^T k_{ef} \delta_e + \frac{1}{2} \delta_e^T k_{es} \delta_e + \frac{1}{2} w_e^T k_{fs} w_e \quad (10)$$

k_{ef} and k_{es} are represented the bending stiffness matrix and the shear stiffness matrix of the thick plate element, and k_{fw} and k_{fs} are the stiffness matrices of Winkler foundation and shear foundation are represented as follows.

Substitution Eq. (8) in Eq. (6), we obtain the following equation of work done by in-plane axial force N_x .

$$\begin{aligned} W_e &= \frac{1}{2} \int_{-a}^a \int_{-b}^b N_x w^T dx dy \\ &= \frac{1}{2} w_e^T \int_{-a}^a \int_{-b}^b \rho \bar{N}_{,x}^T \bar{N}_{,x} dx dy w_e \\ &= \frac{1}{2} w_e^T K_g w_e \end{aligned} \quad (11)$$

$$\text{where, } K_g = \int_{-a}^a \int_{-b}^b \rho \bar{N}_{,x}^T \bar{N}_{,x} dx dy, \quad \rho = N_x / h$$

(3) Eigen-value Equation of the Thick Plate

Assembling element stiffness and mass matrices of total plate elements by using global coordinates, and applying the principal of virtual work, we can obtain the following eigen-value equation.

$$(M_e + M_p) \ddot{\Delta} + (K + K_f) \Delta - K_G \Delta = 0 \quad (12)$$

where,

Δ : Nodal displacement vector of the thick plate

M_e : Mass matrix of the thick plate

M_p : Concentrated mass matrix of the thick plate

K : Bending and shear stiffness matrix of the thick plate

K_f : Stiffness matrix of the foundation

K_G : Geometrical stiffness matrix of the thick plate

Substituting in-plane stress $\sigma=1$ in K_G , we can obtain $\overline{K_G}$ and $K_{G=\sigma}\overline{K_G}$.

Substituting $\sigma\overline{K_G}$ instead of K_G in Eq. (12), we obtain the following another eigen-value equation.

$$(M_e + M_p)\ddot{\Delta} + (K + K_f)\Delta - \sigma\overline{K_G}\Delta = 0 \quad (13)$$

2.3 Composition of Dynamic Stability Equation

In dynamic stability problem, in-plane stress is composed of the sum of static stress and dynamic stress and as follows.

$$\sigma = \alpha\sigma_{cr} + \beta\sigma_{cr}\cos\bar{\omega}t = \sigma_0 + \sigma_t\cos\bar{\omega}t \quad (14)$$

where,

- α : The ratio of static stress to buckling stress
- β : The ratio of dynamic stress to buckling stress
- $\bar{\omega}$: Frequency of dynamic stress
- σ_{cr} : Buckling stress
- σ_0 : Average stress of σ
- σ_t : Amplitude of σ

Substituting Eq. (14) in Eq. (12), we obtain Mathieu-Hill equation of the following form.

$$(M_e + M_p)\ddot{\Delta} + (K + K_f - \alpha\sigma_{cr}\overline{K_{Gs}} - \beta\sigma_{cr}\overline{K_{Gt}}\cos\bar{\omega}t)\Delta = 0 \quad (15)$$

where,

- $\overline{K_{Gs}}$: Geometrical stiffness matrix for σ_0 ,
- $\overline{K_{Gt}}$: Geometrical stiffness matrix for σ_t

The solution of Eq. (15) can be assumed by the following form.

$$\Delta(t) = \sum_{k=1,3,5,\dots}^{\infty} \left(a_k \sin \frac{k\bar{\omega}}{2}t + b_k \cos \frac{k\bar{\omega}}{2}t \right) \quad (16)$$

Substituting Eq. (16) in Eq. (15), arranging it as a_k and b_k , and taking only first term to approximate, we can obtain the following $k \times k$ determinant for a_k and b_k that equals zero.

$$K + K_f - \alpha\sigma_{cr}\overline{K_{Gs}} \pm \frac{1}{2}\beta\sigma_{cr}\overline{K_{Gt}} - \bar{\omega}^2(M_e + M_p) = 0 \quad (17)$$

Assuming $\overline{K_{Gs}} = \overline{K_{Gt}} = \overline{K_G}$, Eq. (17) is written as follows.

$$K + K_f - \left(\alpha \pm \frac{1}{2}\beta \right) \sigma_{cr}\overline{K_G} - \frac{\bar{\omega}^2}{4}(M_e + M_p) = 0 \quad (18)$$

In Eq. (18), calculating the ratio of the frequency of in-plane stress $\bar{\omega}$ to the natural frequency of plate ω_e about α and β , we can decide dynamic instability region.

3. Application

3.1 Verification

In order to check dynamic stability computer program developed in chapter 2, we compared buckling stresses, natural frequencies of thick plates with or without in-plane force, with the existing solutions.

Table 1 is shown the results that are comparing non-dimensional natural frequencies

$\lambda^2 = \omega_e \bar{a}^2 \sqrt{\rho/D_0}$ (ω_e : natural frequency of the plate, ρ : mass per unit volume of the plate, D_0 : flexural rigidity of the plate, $\bar{a}$: length of the plate) of thick plate on Pasternak foundation with the existing solutions. Length, width, and thickness of this plate are 100 cm, 100 cm, and 20 cm respectively, and this plate is divided by finite elements of 8×8 . In order to present results of numerical analysis as non-dimensional values, non-dimensional Winkler foundation parameter K_W and shear foundation parameter S are defined as follow.

$$K_W = k_w \bar{a}^4 / D_0, \quad S = k_g \bar{a}^2 / D_0 \quad (19)$$

where,

k_w : Rigidity of Winkler foundation

k_g : Rigidity of shear foundation

In Table 1, comparing solutions of this study with the existing solutions, the maximum error is represented as 0.25 % for $S=10$ and $K_W=0 \sim 10^4$.

Table 2 is shown non-dimensional frequencies of tapered plates according to tapered ratio $(h_1 - h_0) / h_0$.

We can see that when the tapered ratio is larger,

Table 1 Non-dimensional natural frequency $\lambda^2 = \omega_e \bar{a}^2 \sqrt{\rho / D_0}$ of plates on Pasternak foundation

S	K_W	Reference (7)	This study	Error(%)
10	0	22.2117	22.25175	0.18
	10	22.4104	22.46566	0.25
	10^2	24.2698	24.30675	0.15
	10^3	38.0638	38.09225	0.075
	10^4	99.4473	99.6619	0.2

Table 2 Natural frequency $\lambda^2 = \omega_e \bar{a}^2 \sqrt{\rho / D_0}$ of tapered thick plates

α_{TR}	Reference (4)	This study	Error(%)
0.2	21.59051	21.58787	0.01
0.3	23.54861	23.45184	0.19
0.4	25.37844	25.29344	0.33
0.5	27.23463	27.08043	0.56

Table 3 Buckling coefficients $\sigma_{cr} \bar{a}^2 / \pi^2 D_0$ of square plates

$h/\bar{b}$	This study	Hinton ⁽¹⁰⁾	Rao ⁽¹¹⁾
0.01	4.014	-	-
0.02	3.990	-	-
0.05	3.939	3.924	3.944
0.1	3.771	3.741	3.787
0.2	3.259	3.150	3.264

the error is larger. The maximum error is 0.56 % for tapered ratio = 0.8.

Table 3 is shown buckling coefficients of square plates versus the ratios of width to thickness. In this table, Hinton's solutions(1978) are based on 3-dimensional analysis and Rao's solutions(1975) are based on the triangular finite element with high density. By means of this table, we can see that the maximum error of this study's solutions is 3.46 % for Hinton's and 0.35 % for Rao's.

Table 4 is shown the ratio $(\bar{\omega} / \omega_e)$ of frequency of in-plane stress to the natural frequency of the plate without in-plane stress versus the ratio (σ_0 / σ_{cr}) of in-plane static stress to buckling stress. In this case, the maximum error in comparison with the existing solutions is 0.045 %.

Table 5 is shown the non-dimensional natural frequency of plate with the central concentrated mass $m_c = \rho \bar{a}^2 / 4$ (ρ : mass per unit volume) on the plate.

Length, width, and thickness of this plate are 50 cm, 50 cm and 0.455 cm respectively. The maximum error of this study's solution in comparison with Leissa's solution(1983) by four terms Dirac

Table 4 $\bar{\omega} / \omega_e$ of square plate subjected in-plane stress

	σ_0 / σ_{cr}				
	0	0.2	0.4	0.6	0.8
This study	1	0.8944	0.7746	0.6324	0.4470
Reference (9) (Mei)	1	0.8944	0.7746	0.6325	0.4472
Error (%)	0	0	0	0.016	0.045

Table 5 Non-dimensional natural frequency $\lambda^2 = \omega_e \bar{a}^2 \sqrt{\rho / D_0}$ of plates with central concentrated mass

This study	Leissa ⁽¹²⁾	
λ^2	Number of terms	λ^2
13.76	1	13.96
	2	13.89
	3	13.81
	4	13.79

delta function is 0.22 %.

3.2 Application

Dynamic stability regions of tapered thick plates with the concentrated mass on the inhomogeneous Pasternak foundation are decided by using the computer program develop chapter 3.1.

Elastic modulus(E) : 2.1×10^5 kgf/cm²
 Poisson's ratio(ν) : 0.167
 Mass per unit volume of the plate(ρ) : 2.4×10^{-3} kg/cm³
 Length and width of the plate : 600 cm $\times$ 600 cm
 Thickness of the plate(h_0) : 30 cm
 Tapered ratio(α_{TR}) : 0.25, 1.0
 In-plane average stress $\sigma_0 : 0.4\sigma_{cr}$ (σ_{cr} means buckling stress)
 Magnitude of the concentrated mass(m_c) : $0.25M$, $1.0M$
 where, M means mass of the plate,
 Non-dimensional Winkler foundation parameter (K_{w1}, K_{w2}) : 100, 1000 where, K_{w1} , K_{w2} mean for central area and margin of the plate
 Non-dimensional shear foundation parameter(S) : 5
 Dimensions of inner area of Pasternak foundation($u \times v$) : 300 cm $\times$ 300 cm
 Boundary condition: four simply supported edges.
 Shaded areas of Figs. 4 to 12 : dynamic instability areas

The dynamic stability problems are to decide stability or instability area on $\bar{\omega}/\omega_e - \beta$ rectangular coordinate system with constant α . The relationship of $\bar{\omega}/\omega_e$ and β with constant α can be obtained as solving eigen-value Eq. (18).

The curves of Figs. 4 to 12 are to represent $\bar{\omega}/\omega_e$ for defined α , β . Here, shaded areas mean dynamic instability region. In these figures, there are five parts of instability region and each area

Fig. 3 Thick plate with concentrated mass on the inhomogeneous Pasternak foundation

from left to right, represents from first to fifth mode of vibration.

Figure 4 is shown dynamic instability region of the thick plate on homogeneous Pasternak foundation in case that $\alpha_{TR} = 0.25$, $\sigma_0 = 0.4\sigma_{cr}$, $m_c = 0.25M$, $K_{w1} = K_{w2} = 100$, and $S = 5$. Figures 5 and 6 represent dynamic instability region of the thick plate on inhomogeneous Pasternak foundations with same conditions as the thick plate of Fig. 4 except rigidities of foundations. In these three figures, we can see that areas of all modes of Figs. 5 and 6 which have larger rigidities of foundations are closer to β -axis and more narrow range of instability region than those of Fig. 4. Comparing Figs. 5 and 6.

We can see that when the rigidity is larger in the inner area of foundation than that of outer area.

Instability areas of all modes are farther from β -axis. All conditions of Fig. 7 except m_c are same as those of Fig. 5.

Comparing these two figures, we can see that when the thick plate has larger concentrated mass, the distances of instability areas from β -axis become farther. Figure 8 has the same conditions as Fig. 6 except m_c . In this case, we can see also see that the result is similar to the comparison of Figs. 5 and 7.

Comparing Figs. 7 and 8, we can see also that it has similar results as the comparison of Figs. 5 and 6. All conditions of Fig. 9 except α_{TR} are same as those of Fig. 5.

Fig. 4 Instability region($\sigma_0 = 0.4\sigma_{cr}$, $\alpha_{TR} = 0.25$, $m_c = 0.25M$, $K_{w1} = K_{w2} = 100$, $S = 5$)

Fig. 5 Instability region($\sigma_0 = 0.4\sigma_{cr}$, $\alpha_{TR} = 0.25$, $m_c = 0.25M$, $k_{W1} = 100$, $k_{W2} = 1000$, $S = 5$)

Fig. 9 Instability region($\sigma_0 = 0.4\sigma_{cr}$, $\alpha_{TR} = 0.25$, $m_c = 0.25M$, $K_{W1} = 100$, $K_{W2} = 1000$, $S = 5$)

Fig. 6 Instability region($\sigma_0 = 0.4\sigma_{cr}$, $\alpha_{TR} = 0.25$, $m_c = 0.25M$, $K_{W1} = 1000$, $K_{W2} = 100$, $S = 5$)

Fig. 10 Instability region($\sigma_0 = 0.4\sigma_{cr}$, $\alpha_{TR} = 1.0$, $m_c = 0.25M$, $K_{W1} = 1000$, $K_{W2} = 100$, $S = 5$)

Fig. 7 Instability region($\sigma_0 = 0.4\sigma_{cr}$, $\alpha_{TR} = 0.25$, $m_c = 1.0M$, $K_{W1} = 100$, $K_{W2} = 1000$, $S = 5$)

Fig. 11 Instability region($\sigma_0 = 0.4\sigma_{cr}$, $\alpha_{TR} = 1.0$, $m_c = 1.0M$, $K_{W1} = 100$, $K_{W2} = 1000$, $S = 5$)

Fig. 8 Instability region($\sigma_0 = 0.4\sigma_{cr}$, $\alpha_{TR} = 1.0$, $m_c = 1.0M$, $k_{W1} = 1000$, $k_{W2} = 100$, $S = 5$)

Fig. 12 Instability region($\sigma_0 = 0.4\sigma_{cr}$, $\alpha_{TR} = 1.0$, $m_c = 1.0M$, $K_{W1} = 1000$, $K_{W2} = 100$, $S = 5$)

Comparing these two figures, we can see that when the tapered ratio is larger, the instability areas of all modes are closer to β -axis.

All conditions of Pasternak foundation of Fig. 10 except rigidities are same as those of Fig. 9.

In these figures we can also get the similar results as those of Figs. 5 and 6 or Figs. 7 and 8. All conditions of Pasternak foundation of Fig. 11 except m_c are same as those of Fig. 9. Comparing Figs. 9 and 11, we gain similar results as those of Figs. 5 and 7. Comparing Figs. 11 and 12, we can also see similar results as those of Figs. 5 and 6 or 7 and 8 or 9 and 10.

4. Conclusions

This study is for dynamic stability analysis of thick plate with concentrated mass and varying thickness on inhomogeneous Pasternak's foundation. The results of this analysis are as follows.

(1) Checking non-dimensional natural frequencies (λ^2), buckling coefficients, and the ratios of frequency of in-plane stress to natural frequency of the thick plates, the maximum error is about 3.46 % for Hinton's solutions about buckling coefficients of square plates.

(2) When the rigidity of the foundation under thick plate is larger, instability areas of all modes become narrower and closer to β -axis.

(3) As the result of numerical analysis of the thick plate on inhomogeneous Pasternak foundation for $u \times v = 300 \text{ cm} \times 300 \text{ cm}$ and $a \times b = 600 \text{ cm} \times 600 \text{ cm}$, instability areas of the thick plate which has the larger rigidity of inner area are farther from β -axis and narrower than those which has the larger rigidity of outer area.

(4) When the thick plate has the larger concentrated mass, the instability areas of all modes become wider and farther from β -axis.

(5) When the thick plate has the larger tapered ratio, the instability areas of all modes become narrower and closer to β -axis.

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References

- (1) Horenberg, J. A. G. and Kerstens, J. G. M., 1985, Transverse Vibrations of Rectangular Plates on Inhomogeneous Foundations, Part II: Modal Constraint Method, Journal of Sound and Vibration, Vol. 101, No. 3, pp. 317~324.
- (2) Celik, M. and Ahmet, S., 1999, A Method for the Analysis of Plates on a Two-parameter Foundation, International Journal of Solids and Structures, Vol. 36, No. 19, pp. 2891~2915.
- (3) Matsunaga, H., 1997, Buckling Instabilities of Thick Elastic Plates Subjected to In-plane Stresses, Computer & Structures, Vol. 62, No. 1, pp. 205~214.
- (4) Kukreti, A. R., Farsa, J. and Bert, W. C., 1996, Differential Quadrature and Rayleigh-Ritz Method to Determine the Fundamental Frequencies of Simply Supported Rectangular Plates with Linearly Varying Thickness, Journal of Sound and Vibration, Vol. 189, No. 1, pp. 103~122.
- (5) Lee, Y. S., Kim, I. J. and Oh, S. K., 2003, Free Vibration Analysis of Thick Plate Subjected to In-plane Force on Inhomogeneous Pasternak Foundation, Journal of Korean Society of Steel Construction, Vol. 15, No. 3, pp. 291~298.
- (6) Lee, Y. S., Kim, I. J. and Oh, S. K., 2003, Dynamic Stability Analysis of Thick Plate on Inhomogeneous Pasternak Foundation, Journal of The Architectural Institute of Korea, Vol. 19, No. 4, pp. 45~54.
- (7) Matsunaga, H., 2004, Vibration and Stability of Thick Plates in Elastic Foundations, Journal of Engineering Mechanics, Vol. 126, No. 1, pp. 27~34.
- (8) Zienkiewicz, O. C. and Taylor, R. L., 1982, The Finite Element Method, McGraw-Hill, Vol. 2, pp. 66~102.

- (9) Mei, C., 1978, Large Amplitude Vibrations of Plates with Initial Stresses, *Journal of Sound and Vibration*, Vol. 60, pp. 461~464.
- (10) Hinton, E., 1978, Buckling of Initially Stressed Mindlin Plates Using a Finite Strip Method, *Computers & Structures*, Vol. 8, pp. 99~105.
- (11) Rao, J. V., Venkataramana, J. and Raju, K. K., 1975, Stability of Moderately Thick Rectangular Plates Using a High Precision Triangular Finite Element, *Computers & Structures*, Vol. 5, No. 5, pp. 257~259.
- (12) Lee, Y. S., Kim, I. J. and Oh, S. K., 2009, Free Vibration Analysis of Stiffened Tapered Thick Plates with Concentrated Masses, *Transactions of the Korean Society for Noise and Vibration Engineering*, Vol. 19, No. 8, pp. 828~837.