MCCARD: MONTE CARLO CODE FOR ADVANCED REACTOR DESIGN AND ANALYSIS

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McCARD is a Monte Carlo (MC) neutron-photon transport simulation code. It has been developed exclusively for the neutronics design of nuclear reactors and fuel systems. It is capable of performing the whole-core neutronics calculations, the reactor fuel burnup analysis, the few group diffusion theory constant generation, sensitivity and uncertainty (S/U) analysis, and uncertainty propagation analysis. It has some special features such as the anterior convergence diagnostics, real variance estimation, neutronics analysis with temperature feedback, B₁ theory-augmented few group constants generation, kinetics parameter generation and MC S/U analysis based on the use of adjoint flux. This paper describes the theoretical basis of these features and validation calculations for both neutronics benchmark problems and commercial PWR reactors in operation.

KEYWORDS : McCARD, Monte Carlo Particle Transport, Few Group Constant, Sensitivity and Uncertainty, Gen-IV Reactors

1. INTRODUCTION

McCARD is a Monte Carlo (MC) neutron-photon transport simulation code designed exclusively for neutronics analyses of various nuclear reactor and fuel systems. Since its predecessor MCNAP [1] was first introduced in 1999 as a MC burnup analysis tool with an ORIGEN2-type [2] fuel depletion analysis solver, it has evolved to a versatile MC tool that is capable of conducting whole-core neutronics calculations, few group diffusion theory constants (few group constants) generation, and uncertainty propagation calculations. In the course of its evolution, a wide range of nuclear systems such as SMART [3], PMR-200 [4], KALIMER-600 [5], fusion blankets [6], subcritical systems like HYPER [7], YALINA [8], and IPEN/MB-01 [9] have been subjected to the neutronics analyses. The R&D efforts to meet both functional and non-functional requirements for these analyses have played crucial roles in developing McCARD into its current status.

McCARD estimates neutronics design parameters of a nuclear reactor or a fuel system such as effective multiplication factor (k_{eff}), neutron flux and current,

fission power, etc. by using continuous-energy cross section libraries and detailed geometrical data of the system. The neutron and photon cross section libraries are processed by NJOY [10]. A combination of hierarchical geometry options and the Python scripts [11] are adopted to input complex geometrical data efficiently and at the same time with as much simplicity as possible. The McCARD burnup analysis is performed with a built-in depletion equation solver based on the matrix exponential method [2, 12]. The predictor-corrector [13] calculation option is available for more accurate burnup analyses of nuclear reactor cores having strong absorbers like burnable poison rods (BPRs). McCARD can handle the double heterogeneity of the High Temperature Gas-cooled Reactor (HTGR) cores by direct sampling of the spatial distribution of fuel particles or pebbles from the random distribution function [14].

McCARD is written in an object-oriented programming language, C++, and it is relatively easy to maintain, modify, and expand. It has incorporated many recent advances in the MC particle transport analysis methodology as special computational modules. Listed below are some

of the special features implemented in McCARD.

- McCARD is equipped with anterior stopping criteria [15] which detect the onset of the fundamental mode fission source distribution (FSD), terminate the inactive cycle calculations, and automatically initiate active cycle eigenvalue calculations. The McCARD is also equipped with the posterior fission source convergence criterion based on the Shannon entropy [16] which makes it possible to diagnose whether the active cycle FSDs from the use of the anterior criteria are truly stationary, if users opt to do so.
- The sample variances of the tallied neutronics parameters in the MC eigenvalue calculations are biased because of intercycle correlations of the FSD. There are several studies on how to estimate the biases of the sample variances to determine the real variances of the tallied parameters. McCARD offers four different methods currently available to estimate the biases of the sample variances or the real variances; the Gelbard's batch method [17], the Ueki's method [18], the FSD's intercycle correlation method [19], and the history-based batch method [20].
- In addition to the static neutronics design parameter calculation module, McCARD has the kinetics parameter calculation module designed to calculate the adjoint-weighted kinetics parameters such as the effective delayed neutron fraction, β_{eff} , and the prompt neutron generation time, Λ . The adjoint flux needed for this module is the self-consistent adjoint flux that can be determined during the MC forward calculations [21].
- McCARD provides a few group constants generation module which aims at producing critical spectrum weighted few group constants for a nuclear fuel system such as a fuel pin cell or a fuel assembly as a function of fuel depletion using the B_1 theory-augmented MC method [22]. One of the salient features of this module is the ability to model the random distribution of the fuel particles in the HTGR fuel compact or pebbles [14].
- McCARD has an uncertainty quantification module aimed at quantifying both uncertainties of the tallied parameters from a single MC step neutronics calculations for a given nuclear system and the propagated uncertainties of the parameters from a combination of the MC and the deterministic method calculations like the MC depletion analysis and few group constants generation. The module makes the best of the adjoint-based MC perturbation technique [23], and is applicable to the S/U analysis aimed at assessing effects of the uncertainties of evaluated nuclear cross section data on the criticalities of criticality benchmark problems as well as neutronics design parameters of research or commercial power reactors, transmutation reactors or fuel systems [24]. It can also be applicable to estimating the uncertainties of the nuclide number densities of the burnt fuel assembly (FA) [25] and those of burnup-

dependent few group constants as functions of uncertainties of uncertain nuclear cross section and nuclide number density input data to McCARD [26].

What follows is a brief account of the layout of this paper. Followed by the introduction, Section 2 gives a brief description of the general features of and some numerical results from McCARD. Section 3 describes the theoretical basis of special features of McCARD such as the FSD convergence diagnostics, the real variance estimations, and the temperature feedback calculations, which are essential for the whole-core MC calculations. Section 4 describes the B_1 theory-augmented MC method used to develop the few group constants generation module of McCARD, and presents the burnup-dependent few group constants of pressurized water reactor (PWR) fuel assemblies. Section 5 presents both the mathematical basis of the uncertainty quantification module and some of the numerical results. The latter includes the S/U analysis for effects of the evaluated nuclear data uncertainties on the criticality estimates of benchmark problems, the uncertainty propagation analysis of MC depletion analysis of a PWR FA, and the effects of nuclear cross section and nuclide number density uncertainties on the burnup-dependent few group constants of PWR.

2. GENERAL FEATURES

2.1 Overview

McCARD can be used for both source-mode and eigenvalue calculations through the MC particle simulation algorithms [27-29] for neutrons and neutron-induced photons. It utilizes continuous-energy neutron and photon cross section libraries and thermal scattering data of molecules or crystalline solids for thermal neutrons. In addition to the continuous energy MC calculations, the multi-group MC calculations can also be conducted by using multi-group neutron cross-section libraries. The energy-dependent fission product yield, and the decay chain data can be easily modified by editing external library files.

McCARD inputs are written in text mode with free-form syntax in which keywords and parameters need not be placed in specific columns and whitespace is just used as a delimiter. System geometry is inputted as a sum of three-dimensional unit cells, each of which is represented by the surface or boundary representation (B-rep) modeling. As a way to input geometrical data of a complex system efficiently, McCARD adopts a hierarchical modeling option that makes the best of hierarchical tiers called composition cells. For example, users can input geometrical data of a nuclear reactor core first by providing geometrical definitions of fuel pins and the layout of assemblies using the pre-defined fuel pins and then by describing the loading pattern of the pre-defined assemblies. A repeated geometry

can be easily modeled by using the Python scripts [11]. The Python interpreter imbedded in McCARD enhances greatly the applicability of a generalized input file.

McCARD is capable of the burnup analysis by using the built-in depletion equation solver module. Unlike many existing MC burnup analysis codes, therefore, it is not necessary to couple the MC neutronics analysis modules with an external depletion code. As a result, a user can easily activate the burnup analysis module by declaring cells subject to depletion and specifying a thermal power and burnup steps. The code then automatically generates burnup tallies for the depletion cells. It provides three options for the predictor-corrector algorithm: no corrector, semi-predictor-corrector, and full predictor-corrector. In the semi-predictor-corrector option [13], the tally calculations for the corrected number densities are not conducted to save the computation time. In the full predictor-corrector option, however, they are conducted for an accurate analysis. A simplified flow diagram of the McCARD burnup calculations for the semi-predictor-corrector algorithm is shown in Fig. 1.

Tallies for various nuclear parameters can be calculated by track-length and collision estimators. The parallel computation capability of McCARD is implemented using the MPI [30] libraries. A load balancing algorithm by using a supervisor processor [31] highly augments parallel computational efficiency.

Fig. 1. Flowchart of the McCARD Burnup Calculations

2.2 Validation

The static and kinetics neutronics analysis capabilities of McCARD are tested against well-publicized neutronics benchmark problems such as International Criticality Safety Benchmark Experimental Problem (ICSBEP) [32], the Venus PWR engineering experiments [33-35], and the burnup benchmark problem suite for the reactor physics studies of next-generation fuels of light water reactors [36]. The measurement data available as well as computations of internationally recognized MC and deterministic codes are used to present the degree of computational accuracy of McCARD analyses.

Table 1 shows a comparison of experiments and McCARD estimates on the criticality of 28 criticality benchmark problems selected from ICSBEP which include 11 Plutonium (PU), 7 highly enriched Uranium (HEU), 1 intermediate enriched Uranium (IEU), 1 low enriched uranium (LEU), 7 Uranium-233 (U233), and 1 mixed Plutonium-Uranium (MIX) systems. The McCARD eigenvalue calculations are obtained on 1,050 cycles including 50 inactive cycles with 100,000 neutron histories per cycle. Table 1 lists the results only for the two cases of using ENDF/B-VII.0 and ENDF/B-VI.8 nuclear cross section libraries as the continuous energy cross section input data. The word, 'SD,' stands for the standard deviation. Considering the experimental uncertainties, Δk_{eff} , the McCARD results agree well with the measurements.

The VENUS experiments [33-35] are analyzed to examine the accuracy of the McCARD on pin power estimations. Figure 2 displays McCARD estimates on typical 48 pin powers of VENUS-1 core [33] in comparison with the measurements. The root mean square (RMS) and maximum relative errors of the McCARD analysis using ENDF/B-VII.0 cross section libraries for all the 324 pin powers are shown to be 1.71% and 7.83%, respectively. The McCARD analyses for VIP-PWR [34] and the VENUS-2 [35] have shown 3.1% and 2.8% RMS errors, respectively, and they are available in Ref. 37.

The UO₂ fuel pin cell and the PWR UO₂ FA problems in the burnup benchmark problem suite [36] for studying the physics of next-generation fuels of light water reactors are analyzed with ENDF/B-VI.8 and ENDF/B-VII.0 libraries as continuous cross section input data to McCARD. Figures 3 and 4 compare the McCARD estimates on burnup dependent k_{inf} with those from other computations by HELIOS [13], CASMO [38], and MVP-BURN [39] codes for the UO₂ fuel pin cell and FA problems, respectively. McCARD calculations for the two problems show very good agreements with results from the three other codes mentioned above, except for k_{inf} of the FA problem from MVP-BURN. The maximum deviation of the SD's of two McCARD estimates on k_{inf} of the pin cell problem with ENDF/B-VI.8 and ENDF/B-VII.0 libraries is about 21 pcm.

McCARD has the capability to estimate the point kinetics parameter such as the effective delayed neutron

Table 1. McCARD Estimates on k_{eff} 's for ICSBEP Benchmark Problems

Handbook ID	Benchmark k_{eff} (Unc.)	McCARD k_{eff} (SD)	
		ENDF/B-VII	ENDF/B-VI
PU-MET-FAST-001	1.0000 (0.0020)	0.99999 (0.00006)	0.99751 (0.00006)
PU-MET-FAST-002	1.0000 (0.0020)	1.00003 (0.00006)	0.99829 (0.00006)
PU-MET-FAST-019	0.9992 (0.0015)	0.99832 (0.00006)	1.00137 (0.00006)
PU-MET-FAST-020	0.9993 (0.0017)	0.99879 (0.00006)	0.99890 (0.00006)
PU-MET-FAST-021, case a	1.0000 (0.0026)	1.00610 (0.00007)	1.00711 (0.00007)
PU-MET-FAST-021, case b	1.0000 (0.0026)	0.99613 (0.00006)	0.99510 (0.00006)
PU-MET-FAST-022	1.0000 (0.0021)	0.99859 (0.00006)	0.99619 (0.00006)
PU-SOL-THERM-011, case 18-1	1.0000 (0.0052)	0.99411 (0.00008)	0.99527 (0.00008)
PU-SOL-THERM-011, case 18-6	1.0000 (0.0052)	1.00004 (0.00008)	1.00053 (0.00008)
PU-SOL-THERM-011, case 16-5	1.0000 (0.0052)	1.00630 (0.00009)	1.00626 (0.00009)
PU-SOL-THERM-011, case 16-1	1.0000 (0.0052)	1.01202 (0.00009)	1.01237 (0.00009)
HEU-MET-FAST-001	1.0000 (0.0010)	0.99992 (0.00006)	0.99649 (0.00006)
HEU-MET-FAST-028	1.0000 (0.0030)	1.00299 (0.00007)	1.00156 (0.00007)
HEU-SOL-THERM-013, case 1	1.0012 (0.0026)	0.99830 (0.00006)	0.99872 (0.00005)
HEU-SOL-THERM-013, case 2	1.0007 (0.0036)	0.99754 (0.00006)	0.99788 (0.00006)
HEU-SOL-THERM-013, case 3	1.0009 (0.0036)	0.99410 (0.00006)	0.99425 (0.00006)
HEU-SOL-THERM-013, case 4	1.0003 (0.0036)	0.99555 (0.00007)	0.99569 (0.00006)
HEU-SOL-THERM-032	1.0015 (0.0026)	0.99909 (0.00004)	0.99857 (0.00004)
IEU-MET-FAST-007	1.0045 (0.0007)	0.99929 (0.00005)	1.00908 (0.00005)
LEU-COMP-THERM-077	1.0003 (0.0010)	1.00363 (0.00007)	0.99899 (0.00008)
U233-MET-FAST-001	1.0000 (0.0010)	0.99953 (0.00006)	0.99284 (0.00006)
U233-MET-FAST-002	1.0000 (0.0010)	0.99988 (0.00006)	0.99614 (0.00006)
U233-MET-FAST-003	1.0000 (0.0010)	0.99988 (0.00006)	0.99633 (0.00006)
U233-MET-FAST-004	1.0000 (0.0007)	1.00513 (0.00006)	1.00273 (0.00006)
U233-MET-FAST-005	1.0000 (0.0030)	0.99427 (0.00006)	1.00466 (0.00006)
U233-MET-FAST-006	1.0000 (0.0010)	0.99959 (0.00007)	1.00006 (0.00007)
U233-SOL-THERM-008	1.0006 (0.0029)	1.00145 (0.00004)	0.99718 (0.00004)
MIX-MET-FAST-008, case 1	0.9920 (0.0063)	0.99230 (0.00005)	0.99374 (0.00005)

fraction, β_{eff} , and the prompt neutron generation time, Λ , using the self-consistent adjoint flux which can be computed during the MC forward simulations, the validity of which is demonstrated [21] via the true values for infinite homogenous two-group problems and with measurements

for several critical facility systems. Table 2 shows a comparison of McCARD estimates on β_{eff} and $\beta_{\text{eff}}/\Lambda$ and their measurements for Godiva and TCA [40, 41]. McCARD results show that it can predict β_{eff} and $\beta_{\text{eff}}/\Lambda$ within a 3% error.

Exp.	Exp. = Experimental power distribution
E6	E6 = Rel. error of McCARD using ENDF/B-VI.8
E7	E7 = Rel. error of McCARD using ENDF/B-VII.0

1.018	1.188	1.292	1.221	Pyrex
3.53%	3.42%	-1.65%	-1.41%	
3.50%	2.99%	-2.33%	-1.81%	
0.970	1.162	1.258	1.219	1.202
1.76%	2.78%	0.01%	1.55%	-1.54%
1.42%	2.16%	-0.53%	1.38%	-1.65%
0.963	1.162	1.274	1.226	1.206
0.61%	0.69%	-2.35%	0.22%	-2.28%
0.09%	0.26%	-2.79%	-0.07%	-2.40%
0.958	1.149	1.245	1.168	Pyrex
1.30%	1.72%	-1.57%	0.33%	
0.66%	0.77%	-2.18%	0.13%	
0.955	1.149	1.241	1.200	1.191
2.71%	1.17%	-0.69%	1.75%	-1.16%
1.39%	0.52%	-1.87%	1.28%	-1.43%

3.3 w/o enrichment region

0.417	0.366	0.320	0.277	0.228
1.02%	3.85%	4.49%	4.91%	4.92%
1.32%	3.48%	4.46%	4.49%	5.36%
0.496	0.457	0.392	0.341	0.274
2.88%	0.48%	3.85%	3.02%	2.83%
2.72%	0.73%	3.64%	3.39%	3.17%
0.572	0.509	0.461	0.373	0.312
0.00%	1.63%	-0.25%	6.51%	1.84%
0.19%	1.36%	-0.30%	6.47%	2.25%
0.623	0.574	0.498	0.426	0.342
0.75%	-1.18%	1.09%	1.89%	2.05%
0.61%	-1.31%	1.27%	2.74%	1.57%
0.680	0.613	0.550	0.463	0.365
-0.14%	0.59%	-0.69%	1.49%	2.91%
-0.02%	0.06%	-1.12%	1.56%	2.91%

4.0 w/o enrichment region

Fig. 2. Comparison of Pin Power Distributions for VENUS-1 Critical Facility

Fig. 3. Comparison of k_{inf} 's for the UO₂ Fuel Pin Problem in the Burnup Benchmark Suite Fig. 4. Comparison of k_{inf} 's for the PWR UO₂ FA Problem in the Burnup Benchmark SuiteTable 2. Comparison of McCARD Estimates on β_{eff} 's and β_{eff}/Λ 's with Measurements for Godiva and TCA

Facility	Core Name	$(\beta_{eff})_{exp}^a$	$(\beta_{eff})_{MC}^a$ by McCARD Ratio to Exp. RSD (%)		$(\beta_{eff}/\Lambda)_{exp}^b$	$(\beta_{eff}/\Lambda)_{MC}^b$ by McCARD Ratio to Exp. RSD (%)	
Godiva	-	0.00640	1.01	0.42	1.11×10^6	1.03	0.42
TCA	1.50U	0.00771	1.00	1.32	219	1.01	1.34
	1.83U	0.00760	1.00	1.38	201	0.98	1.39
	2.48U	0.00765	0.98	1.31	175	0.97	1.32
	3.00U	0.00749	1.02	1.29	161	0.98	1.29

^a $(\beta_{eff})_{exp}$ is the experimental β_{eff} while $(\beta_{eff})_{MC}$ from McCARD calculations.^b $(\beta_{eff}/\Lambda)_{exp}$ is the experimental β_{eff}/Λ while $(\beta_{eff}/\Lambda)_{MC}$ from McCARD calculations.

3. WHOLE -CORE TRANSPORT CALCULATIONS

The commercial nuclear reactor cores generally have higher dominance ratios (DRs) than critical assemblies and research reactor cores. A higher DR of a nuclear system implies a longer inter-generational correlation of the FSD in the MC eigenvalue calculation, which requires more inactive cycle runs for the FSD to converge to the fundamental mode and becomes the source of big bias of the sample variance. In order to cope with these two problems, McCARD is equipped with the on-the-fly stopping criteria of terminating inactive cycle runs automatically and various real variance estimation methods.

3.1 Inactive Cycle Stopping Criteria

Anterior or on-the-fly convergence criteria [15] implemented in McCARD is designed to initiate automatic change of inactive cycle MC runs to the active ones by detecting the onset of the stationary or the fundamental-mode FSD. There are two types of anterior stopping criteria distinguished by the capital letters A and B in McCARD.

The type-A criterion is characterized by the following two equations;

$$|S_m^i - S_m^{i-L}| < \kappa \sqrt{2(\sigma^2[S_m^i] - \text{cov}[S_m^i, S_m^{i-L}])}; \quad m=1, 2, \dots, N_m, \quad (1a)$$

and

$$\frac{\text{the number of cells not satisfying Eq. (1a)}}{\text{the total number of cells}} < C(\kappa). \quad (1b)$$

S_m^t ($t = i$ or $i - L$) is the FSD of cell m at cycle t defined by $S_m^t = \int_{V_m} S^t(\mathbf{r}) d\mathbf{r}$ where $S^t(\mathbf{r})$ is the normalized FSD of neutrons born at location $\mathbf{r}$ with any energy and direction at cycle t and V_m ($m=1, 2, \dots, N_m$) the spatial volume of cell m with N_m representing the total number of non-overlapping cells comprising the whole system.

Equation (1a) of the criterion A is based on a rationale that, when the FSD is converged, the difference between two fission sources L cycles apart, S_m^i and S_m^{i-L} , must be statistically indistinguishable. Mathematically, this rationale is expressed as that the following conditions must be met for all cells.

$$\left| \frac{S_m^i - S_m^{i-L}}{S_m^i} \right| < \kappa \cdot \sigma \left[\frac{S_m^i - S_m^{i-L}}{S_m^i} \right]; \quad m=1, 2, \dots, N_m. \quad (2)$$

σ is the SD of the bracketed quantity and κ is an integer, say, 1 or 2 or 3. Equation (1a) is derived after a little algebraic manipulation of Eq. (2) [15]. Because of the statistical behavior of $(S_m^i - S_m^{i-L})/S_m^i$, there may be cells whose fission sources do not satisfy Eq. (1a). Equation (1b) is augmented to take into account the presence of such cells. Actually, $C(\kappa)$ in Eq. (1b) depends on κ as well as the distribution function of the relative differences, $(S_m^i - S_m^{i-L})/S_m^i$, for all m . Under an assumption that $(S_m^i - S_m^{i-L})/S_m^i$ follows the normal distribution,

$C(\kappa=1) \approx 32\%$, $C(\kappa=2) \approx 4.6\%$, and $C(\kappa=3) \approx 0.3\%$.

The type B criterion is specified by

$$\sum_{m=1}^{N_m} \frac{(S_m^i - S_m^{i-L})^2}{2(\sigma^2[S_m^i] - \text{cov}[S_m^i, S_m^{i-L}])} < \chi^2(N_m, \alpha), \quad (3)$$

where $\chi^2(N_m, \alpha)$ denotes the value ν that satisfies probability $\{Z > \nu\} = \int_{\nu}^{\infty} p_z(z) dz = \alpha$ in which $p_z(z)$ is the chi-square distribution of N_m degrees of freedom. This criterion is based on an assumption that $(S_m^i - S_m^{i-L})/S_m^i$ ($m=1, 2, \dots, N_m$) are a set of N_m independent normal random variables. On this assumption, the square sum of the normalized relative differences, Z , given by

$$Z = \sum_{m=1}^{N_m} \left\{ \frac{\left(\frac{S_m^i - S_m^{i-L}}{S_m^i} \right) - \frac{S_m^i - S_m^{i-L}}{S_m^i}}{\sigma \left[\frac{S_m^i - S_m^{i-L}}{S_m^i} \right]} \right\}^2, \quad (4)$$

can be regarded as the random variable that follows a chi-square distribution with N_m degrees of freedom. This is then used to derive the stopping criterion B as

$$Z < \chi^2(N_m, \alpha), \quad (5)$$

which can be converted to Eq. (3) after some algebraic manipulation [15].

The application of the two stopping criteria requires estimating the values of $\sigma^2[S_m^i]$, $\text{cov}[S_m^i, S_m^{i-L}]$, and L . Reference 15 presents the details on how to estimate them using the cycle-by-cycle error propagation model [42].

The two stopping criteria implemented in McCARD are applied [15, 43] for various nuclear systems. As an illustration to demonstrate the effectiveness of the two criteria, their application results for the MC Wielandt calculations [29] of the fuel storage facility problem [44] are shown here. Table 3 shows the comparisons of the convergence cycles detected by the two-stopping criteria and the Ueki's posterior method [16] as a function of k_e , a key parameter of the Wielandt eigenvalue calculation method. $k_e = \infty$ means the normal MC eigenvalue calculations. t_s is the cycle number at which the inactive cycle MC runs are halted by the stopping criteria and the active cycle MC runs are immediately activated. t_{UPM} denotes the cycle number signaling the onset of a stationary cycle estimated by the Ueki's posterior source convergence diagnosis. It is noteworthy to observe $t_s > t_{UPM}$ for all the test cases. This indicates that the two criteria adequately terminate the inactive cycle runs for the MC Wielandt calculations.

The activation of the on-the-fly convergence diagnostics is an option in the McCARD input. The output file prints out the cycle number signaling the onset of the active cycle runs by on-the-fly convergence diagnostics in comparison with that of the Ueki's posterior method after the completion of the MC runs.

3.2 Real Variance Estimation

It is well known that the sample variance of a tally estimate tends to be smaller than its true or real variance in the MC eigenvalue calculations. McCARD provides four methods available currently to estimate the real variance of a tallied parameter: the Gelbard's batch method [17], the Ueki's method [18], the FSD's inter-cycle correlation method [19], and the history-based batch method [20].

The Gelbard's batch method estimates the real variance by splitting the whole active cycles into non-overlapping batches, averaging cycle-wise tally estimates over each batch, and estimating the sample variance from the batch-averages. The method is based on the idea that the correlations between the batch-averaged estimations are

expected to become much weaker than those between the individual cycle values and thus the bias of the sample variance can be reduced. In order to activate the Gelbard's batch method in McCARD, a user has only to specify the batch size. The Ueki's method estimates the variance bias utilizing the lag covariances, which are directly measured from the cycle-wise tally estimations. To use this method, a user should specify the maximum lag covariance length. The FSD's inter-cycle correlation method utilizes the inter-cycle covariance of the FSDs which can be computed by the cycle-by-cycle stochastic error's propagation model [42]. It is more accurate than the aforementioned two methods especially for problems with high DRs but requires an appropriate regional discretization to build a fission matrix required for analysis of the error propagation model. The history-based batch method makes the most of the fact that the variance bias can disappear if the samples become independent of each other. It treats a MC eigenvalue run on N active cycles with M histories per cycle as the sum of N_B runs on N active cycles with M/N_B histories per cycle.

The accuracy of the real variance estimations by the Gelbard's batch, Ueki's, and FSD's inter-cycle correlation methods for commercial reactor core problems has been reported in Ref. [19]. For the purpose of a comparison of the four methods, the weakly coupled fissile array problem [45] composed of two uranyl nitrate aqueous solution slabs of 35 cm in width, 69 cm in depth and 50 cm in height separated by a 30 cm-thick concrete slab is analyzed by McCARD. The MC eigenvalue calculations are performed to determine the fission powers and their uncertainties for seven sub-regions dividing a uranyl solution slab with the same thickness of 5cm based on 500 active cycles with 1,000,000 histories per cycle. Table 4 compares the relative standard deviations (RSDs) of the regional fission powers estimated from four different real variance estimation methods with the reference RSD. σ_{REF} , σ_{batch} , σ_{Ueki} , σ_{FSD} , and σ_{HB} in Table 4 denote the RSD estimated by a reference calculation, Gelbard's batch, Ueki's, FSD's inter-cycle correlation, and history-based

Table 3. Performance Characteristics of Anterior Stopping Criteria for the Fuel Storage Facility Problem

k_e	L	t_{UPM}	t_s	
			Type-A ^{a)}	Type-B ^{b)}
∞	636	1298	1482	1612
10.0	576	1385	1438	1632
2.0	503	717	1048	1071
1.5	266	571	687	719
1.4	282	507	676	707
1.3	254	454	625	635
1.2	195	352	475	491
1.1	174	342	427	430
1.0	84	153	228	235
0.95	54	131	152	168

^{a)} $C(1)=31.7\%$. ^{b)} $\alpha=0.9$.

Table 4. Comparison of the Real Variance Estimates of Region-wise Fission Power Tallies

Region	σ_{REF} (95% conf. intv.) [%]	σ_{batch} [%]	σ_{Ueki} [%]	σ_{FSD} [%]	σ_{HB} (95% conf. intv.) [%]
1, Outer-most	0.546 (0.490 , 0.619)	0.106	0.147	0.437	0.494 (0.443 , 0.560)
2	0.539 (0.483 , 0.611)	0.095	0.133	0.434	0.494 (0.443 , 0.560)
3	0.533 (0.477 , 0.604)	0.092	0.131	0.431	0.490 (0.439 , 0.555)
4	0.522 (0.468 , 0.591)	0.098	0.138	0.424	0.484 (0.434 , 0.548)
5	0.506 (0.453 , 0.573)	0.105	0.148	0.413	0.470 (0.421 , 0.532)
6	0.484 (0.434 , 0.549)	0.100	0.141	0.395	0.451 (0.404 , 0.511)
7, Inner-most	0.429 (0.385 , 0.487)	0.090	0.124	0.352	0.400 (0.358 , 0.453)

Fig. 5. Schematic Diagram for the B₁ Theory-augmented MC Method

batch methods, respectively. The batch size is taken as 10 for the Gelbard's batch method while the maximum lag covariance length for the Ueki's method is assumed to be 10. For σ_{HIB} , N_B is set at 100. It is noted that real variances are better predicted by the FSD's inter-cycle method and the history-based batch method than the two other methods implemented in McCARD.

3.3 Temperature Feedback Calculations

Like its deterministic counterpart, the MC whole-core neutronics analysis must take into account thermal-hydraulic (T/H) feedback in order to determine the neutron distribution consistent with temperature distribution of the system. This is done by a repetition of the MC neutronics and T/H calculations. Cell-averaged temperatures in the McCARD calculations are updated by a simple heat transfer model [46]. The convergence of T/H feedback coupled with neutronics calculations is checked by monitoring the change of the cell-wise temperature,

$$\max_m |T_m^{t+1} - T_m^t| \leq \varepsilon_T \text{ or } |T_m^{t+1} - T_m^t| \leq C \sigma_{\text{R,est}} [T_m^{t+1} - T_m^t]. \quad (6)$$

m and t are the cell index and T/H coupled iteration index, respectively. T_m^t is the average temperature of cell m at iteration t . $\sigma_{\text{R,est}}$ is the estimated real SD by the methods introduced in Section 3.2. ε_T and C are user-provided convergence parameters. For example, they may be set at 5K and 3, respectively.

Coolant density corresponding to the temperature of the coolant cell is updated from a user-defined property table. Currently, the neutron cross section libraries for the updated temperature are chosen as the ones corresponding to the closest temperature from pre-prepared temperature-dependent cross section library sets. The McCARD capability of the whole-core neutronics analysis with the T/H feedback is demonstrated in Ref. 47. Some of the details on the whole-core neutronics analysis with the T/H feedback is available in Ref. 48.

4. FEW GROUP DIFFUSION THEORY CONSTANT GENERATION

McCARD can generate the few group constants by the B₁ theory-augmented MC method [22, 49, 50]. As shown in Fig. 5, this method consists of a sequence of steps; step 1 for the MC computation for the infinite medium spectrum (ϕ_g) of and microscopic reaction rates ($r_{\alpha,g,i,m}$) in the nuclear fuel pin or the FA, step 2 for processing them into fine group reaction cross sections ($\Sigma_{\alpha,g}$) to prepare inputs to B₁ equation, step 3 for solving B₁ equation for the critical spectrum (ϕ_g^B) and critical buckling (B_c^2), and step 4 for generation of the critical spectrum weighted few group cross sections ($\Sigma_{\alpha,G}$) and diffusion constants (D_G). Steps 1 to 4 calculations are performed at a given depletion time t_n ($n=0,1,2,\dots$) and they are enough to

Table 5. Comparison of Two Group Constants at Fresh Fuel State

Code	McCARD		Relative Difference from McCARD [%]	
	XS	RSD [%]	CASMO3	HELIOS 1.8
B_0^2	5.09876×10^{-3}	-	-1.90	1.01
D_1	1.46317×10^0	0.12	3.10	-0.69
$\Sigma_{a,1}$	9.24314×10^{-3}	0.10	0.91	0.16
$\Sigma_{s,12}$	1.48781×10^{-2}	0.12	1.67	-0.25
$\nu\Sigma_{f,1}$	7.06284×10^{-3}	0.08	2.94	0.43
D_2	5.00963×10^{-1}	0.13	-5.61	1.22
$\Sigma_{a,2}$	8.58098×10^{-2}	0.14	-0.06	-0.41
$\nu\Sigma_{f,2}$	1.45452×10^{-1}	0.14	-0.95	-0.10
k_{inf}	1.33971	0.04	-0.12	0.28

Fig. 6. The Assembly-wise Normalized Power (NP) Distribution for 3D PWR Core Problem

generate the few group constants at t_n . In order to generate those at t_{n+1} , however, the depletion calculation step to be discussed later in Section 5.3 needs to be conducted to prepare critical spectrum corrected nuclide number densities at t_{n+1} , $N_{i,m,n+1}$, namely the nuclide number density inputs for step 1 MC calculation at the next depletion time t_{n+1} . Note that McCARD method differs from some of MC methods suggested earlier [51-53] for the few group constants generation in that it makes use of B_1 spectrum [54]. Note also that B_1 spectrum is used to take into account the effect of critical spectrum on the fuel depletion as well as to generate critical spectrum-weighted few group nuclear reaction cross sections and the few group diffusion constants.

Since the B_1 theory-augmented MC method was implemented into McCARD for the first time in 2008

[22], the qualification of the few group constants from McCARD has been examined from the two aspects of their applications to neutronics design computations. One is how well they are compared with those from the commercial FA spectrum codes such as HELIOS [13] and CASMO [38]. The other is how accurately they can produce core neutronics computations for core neutronics test problems in comparison with the reference MC calculations or the neutronics design reports (NDR) or the measured plant data [49, 50]. As an illustration, Table 5 shows a comparison of the two-group constants determined by HELIOS and CASMO, and McCARD for a 17x17 PWR FA [22]. Remarkably good agreements are observed between the two group constants from McCARD and those from HELIOS and CASMO. As the other illustrations, Figures 6 and 7

Fig. 7. (a) Critical Boron Letdown Curve and (b) Assembly-wise Normalized Power Distribution in YGN 4 Reactor Core at 13 EFPD

show the assemblywise normalized power distribution and k_{eff} by two-step code system McCARD/MASTER calculations for a 3-dimensional (3D) PWR core [50] and the cycle 1 PWR core of Yonggwang unit 4 (YGN4) [55], respectively. The McCARD/MASTER calculations for the FA power distribution and k_{eff} of the two PWR cores display excellent agreements with the reference McCARD calculations in case of the 3D PWR and the measurements in case of YGN4 cycle 1. Needless to mention, these are seen as warranting the quality of the few group constants from McCARD. For the sake of reference, MASTER [56] is a nodal diffusion theory core analysis code of Korea Atomic Energy Research Institute.

5. UNCERTAINTY QUANTIFICATION

The MC reactor calculations are bound to carry statistical uncertainties with them. In addition, uncertainties of the input

data may amplify those of the resulting MC calculations when the nuclear cross section and nuclide number density inputs to the MC calculation have uncertainties. Therefore, it is essential for MC users to be able to quantify both the statistical and the input data uncertainties of MC reactor calculations –preferably separately, in order to report the confidence interval of the MC results, to identify the most significant contributing input data and find ways to improve them, or to provide a quantitative basis for assessing the feasibility of the next-generation fuel or reactor system designs. McCARD has an uncertainty quantification module designed to quantify uncertainties associated with such problems as (i) criticality benchmark analysis, (ii) actinide fuel incineration analysis, and (iii) generation of fuel assembly homogenized few group cross section by the MC method. The problems (ii) and (iii) require composite analyses in which MC results are combined with deterministic calculations. These are so-called uncertainty propagation problems in which the uncertainties of MC calculations

affect those of the subsequent deterministic calculations. The special feature of the uncertainty quantification module of McCARD is its ability to quantify separately the statistical uncertainty and contributions of input data uncertainties to the overall uncertainty of any reactor parameter estimated from it. The following includes a brief description on the mathematical basis of this special feature and how to handle the uncertainty propagation within the framework of the mathematical basics of uncertainty quantification.

5.1 Mathematical Basis of McCARD Uncertainty Quantification

Suppose that one is interested in computing a reactor parameter such as k_{eff} , power densities etc. by McCARD and that the nuclear cross section and the nuclide number density input data to McCARD have uncertainties. Representing the reactor parameter of interest by Q and the uncertain input data by a set of variables, $u_1, \dots, u_\ell, \dots, u_L$, one may have a functional relation between the uncertain input data and the output from them by

$$Q_\kappa = Q(u_1^\kappa, \dots, u_\ell^\kappa, \dots, u_L^\kappa). \quad (7)$$

κ is the index for the different input data sets that one may sample from the probability distribution function of each member of the input data and the output from it. From Eq. (7), one can formally obtain the mean value of Q , $\bar{Q}$, and the variance, $\sigma^2(Q)$, by

$$\bar{Q} = \lim_{K \rightarrow \infty} \frac{1}{K} \sum_{\kappa=1}^K Q_\kappa \quad \text{and} \quad \sigma^2(Q) = \lim_{K \rightarrow \infty} \frac{1}{K-1} \sum_{\kappa=1}^K (Q_\kappa - \bar{Q})^2 \quad (8)$$

One may compute $\bar{Q}$ and $\sigma^2(Q)$, a measure of uncertainty of the MC estimate Q , using Eq. (8) directly by obtaining Q_κ for a finite sampling of input data sets, say, $K=50, 100$, and the like and approximating the infinite sum by a finite sum. Though observed in Ref. 57 and 58, this approach has a couple of drawbacks. It is not only time-consuming but also lacks in information on contribution of statistical uncertainties and contributions from individual input data uncertainties to $\sigma^2(Q)$.

McCARD avoids these drawbacks of the direct sampling method by computing separately the two contributions to $\sigma^2(Q)$ by

$$\sigma_s^2(Q) = \lim_{K \rightarrow \infty} \frac{1}{K-1} \sum_{\kappa=1}^K (Q_\kappa - \langle Q_\kappa \rangle)^2 \quad (9)$$

$$\begin{aligned} \sigma_i^2(Q) &= \lim_{K \rightarrow \infty} \frac{1}{K-1} \sum_{\kappa=1}^K (\langle Q_\kappa \rangle - \bar{Q})^2 \\ &= \sum_{\ell=1}^L \sum_{p=1}^L \text{cov}[u_\ell, u_p] \left(\frac{\partial \langle Q \rangle}{\partial u_\ell} \right) \left(\frac{\partial \langle Q \rangle}{\partial u_p} \right) \Big|_0. \end{aligned} \quad (10)$$

Note that $Q_\kappa - \bar{Q}$ in Eq. (8) is rewritten as $Q_\kappa - \langle Q_\kappa \rangle + \langle Q_\kappa \rangle - \bar{Q}$ which makes it split $\sigma^2(Q)$ into $\sigma_s^2(Q)$ and $\sigma_i^2(Q)$ denoting the contributions from the statistical and input

data uncertainties, respectively. Note also that Eq. (10) is obtained from the first-order Taylor expansion for $\langle Q_\kappa \rangle - \bar{Q}$,

$$\langle Q_\kappa \rangle - \bar{Q} \approx \sum_{\ell=1}^L \frac{\partial \langle Q \rangle}{\partial u_\ell} \Big|_0 \cdot (u_\ell^\kappa - \bar{u}_\ell). \quad (11)$$

The angular bracket implies the expected value of a quantity in it, while the bar sign the mean value of a quantity under it. Thus $Q_\kappa - \langle Q_\kappa \rangle$ is the statistical fluctuation of Q_κ about its expected value while $u_\ell^\kappa - \bar{u}_\ell$ the uncertainty of the ℓ -th member of input data, u_ℓ^κ , about its mean, $\bar{u}_\ell$. The symbol " $\frac{\partial \langle Q \rangle}{\partial u_\ell} \Big|_0$ " implies

$$\frac{\partial \langle Q \rangle}{\partial u_\ell} \Big|_0 = \frac{\partial \langle Q(u_1^\kappa, \dots, u_\ell^\kappa, \dots, u_L^\kappa) \rangle}{\partial u_\ell^\kappa} \Big|_{u_1^\kappa = \bar{u}_1, \dots, u_\ell^\kappa = \bar{u}_\ell, \dots, u_L^\kappa = \bar{u}_L}. \quad (12)$$

McCARD determines $\bar{Q}$, $\sigma_s^2(Q)$, and $\frac{\partial \langle Q \rangle}{\partial u_\ell} \Big|_0$ in the above equations by approximating them by

$$\bar{Q} \approx \langle Q(\bar{u}_1, \dots, \bar{u}_\ell, \dots, \bar{u}_L) \rangle, \quad (13)$$

$$\sigma_s^2(Q) \approx \langle Q^2(\bar{u}_1, \dots, \bar{u}_\ell, \dots, \bar{u}_L) \rangle - \langle Q(\bar{u}_1, \dots, \bar{u}_\ell, \dots, \bar{u}_L) \rangle^2 \quad (14)$$

$$\begin{aligned} \frac{\partial \langle Q \rangle}{\partial u_\ell} \Big|_0 &\approx \frac{\langle Q(\bar{u}_1, \dots, \bar{u}_\ell + \sigma_\ell, \dots, \bar{u}_L) \rangle - \langle Q(\bar{u}_1, \dots, \bar{u}_\ell, \dots, \bar{u}_L) \rangle}{\sigma_\ell} \\ &= \frac{\delta \langle Q \rangle}{\sigma_\ell}. \end{aligned} \quad (15)$$

These approximations require one McCARD run with the mean value input data set, $\bar{u}_1, \dots, \bar{u}_\ell, \dots, \bar{u}_L$. $\delta \langle Q \rangle$ may be computed by a direct subtraction or MC perturbation computation. σ_ℓ is the SD of $\bar{u}_\ell$.

5.2 S/U Analysis of Criticality Benchmark Problems

The capability of McCARD to isolate the contribution of the input data uncertainties to the uncertainty of a MC estimate on any neutronics parameter has useful applications. For example, McCARD can estimate the effects of the nuclear cross section uncertainties on the uncertainty of k_{eff} of a nuclear reactor system by the type of the cross section and by the type of the nuclide. On the assumption that input data uncertainties come from the nuclear cross sections only and that there are no correlation between cross sections of the different isotopes, $\sigma_i^2(k_{\text{eff}})$ is derived from Eq. (10) as

$$\sigma_i^2(k_{\text{eff}}) = \sum_i \sum_{\alpha, g} \sum_{\alpha', g'} \text{cov}[\sigma_{\alpha, g, i}, \sigma_{\alpha', g', i}] \frac{\partial k_{\text{eff}}}{\partial \sigma_{\alpha, g, i}} \Big|_0 \frac{\partial k_{\text{eff}}}{\partial \sigma_{\alpha', g', i}} \Big|_0 \quad (16)$$

where $\sigma_{\alpha, g, i}$ is the α -type cross section of nuclide i for

Table 6. Contribution of ^{235}U Cross Section Uncertainties to $\sigma(k_{\text{eff}})$ of the Godiva Critical Assembly by the Type of the Nuclear Data

Code	Neutronics S/U	ANISN SUSD3D	McCARD McCARD
Energy Group		44 groups	Continuous
Covariance Data		44 groups	44 groups
$\sigma(k_{\text{eff}})(\%)$	ν, ν	0.15	0.15
	$(n,\gamma), (n,\gamma)$	0.17	0.16
	$(n,\gamma), (n,n)$	0.05	0.05
	$(n,2n), (n,2n)$	0.01	0.01
	$(n,\text{fis}), (n,\text{fis})$	0.17	0.17
	$(n,n), (n,n)$	0.31	0.33
	$(n,n), (n,n')$	-0.44*	-0.46*
	$(n,n'), (n,n')$	0.69	0.69
total		0.43	0.47

* The minus sign implies negative contribution to $\sigma^2(k_{\text{eff}})$

group g neutrons. Table 6 compares McCARD results [59] for contributions of the cross section data uncertainties of ^{235}U on $\sigma_i^2(k_{\text{eff}})$ of Godiva critical facility with those of SUSD3D [60]. The derivatives are computed using the perturbation calculation routine of McCARD in which the perturbation effect on the free-flight and collision kernels are estimated by the higher-order differential operator sampling method [61] while that on the FSD is taken into account either by the fission source perturbation (FSD) method [62] or the adjoint-weighted perturbation (AWP) method [23]. Figure 8 shows the k uncertainties generated from the JENDL covariance data for the ICSBEP benchmarks.

5.3 Uncertainty Propagation in McCARD Burnup Analysis

McCARD conducts the MC burnup analysis using the built-in subroutine program providing the solution to the Bateman equations;

$$\frac{dN_{i,m}(t)}{dt} = \sum_j l_{ij} \lambda_j N_{j,m}(t) + \sum_j r_{ij,m} N_{j,m}(t) - (\lambda_i + r_{i,m}) N_{i,m}(t), \quad t \in [t_n, t_{n+1}], \quad (17)$$

$$N_{i,m}(t_n) = N_{i,m,n},$$

where

$$r_{ij,m} = \sum_{\alpha} \sum_g f_{ij,\alpha} \omega_g r_{\alpha,g,i,m}, \quad r_{i,m} = \sum_j r_{ji,m}; \quad \omega_g = \frac{\phi_g^B}{\phi_g}, \quad (18)$$

Fig. 8. McCARD Estimates on k_{eff} and $\sigma(k_{\text{eff}})$ for ICSBEP Benchmark Problems

$N_{\ell,m}(t)$ ($\ell = i, j$) is the number density of nuclide ℓ in cell m at time t . $r_{ij,m}$ is the critical spectrum corrected microscopic reaction rate of nuclide j in region m leading to creation of nuclide i . $f_{ij,\alpha}$ is the expected number of nuclide i formed when an α -type reaction of nuclide j takes place. ω_g is the critical spectrum correction factor defined by the ratio of the critical spectrum to the infinite spectrum. Other notations are standard. The solution to Eq. (17) is obtained first by determining MC estimates on $r_{\alpha,g,i,m}$ at every beginning of the depletion time step (DTS) n , $[t_n, t_{n+1}]$ ($n = 0, 1, 2, \dots, n_F$) by the MC run like step 1 MC calculation in Fig. 5 and then by calling the depletion subroutine to determine $N_{i,m,n+1}$, the number density of nuclide i in cell m at $t=t_{n+1}$, for all i using the initial condition $N_{i,m}(t=t_n) = N_{i,m,n}$. The above-described two-step MC burnup calculations which utilize the infinite medium spectrum weighted $r_{ij,m}$ with $\omega_g=1$ are the general practice of the MC burnup analyses today. In order to take into account the critical spectrum effect on the fuel depletion in reactors operating at the critical state, however, McCARD offers another MC depletion analysis option based on the critical spectrum corrected $r_{ij,m}$ in which ω_g is determined by the B_1 critical spectrum (ϕ_g^B) from step 2 and 3 calculations in Fig. 5.

Because more than two-step calculations every DTS are performed repeatedly from the first to the last DTS to determine the composition or the nuclide number densities of the burnt fuel or FA, the cross section data uncertainties plus statistical uncertainties of parameters $r_{ij,m}$ of the first step MC calculation propagate to uncertainties of the nuclide number densities of the ensuing DTS and eventually those of the isotopic contents of the irradiated fuel. The mathematical basis for McCARD to quantify propagating uncertainties like this in MC depletion calculations without critical spectrum correction, namely the case of $\omega_g=1$ is well documented in reference 25. Its application results for the incineration behavior of minor actinides like ^{241}Am , ^{243}Am , and ^{244}Cm in an Actinide FA designed for the HYPER subcritical incinerator and the SDs of the number

Table 7. Fuel Assembly Homogenized Two Group Cross Sections and Their Percentile Relative Errors

EFPD	$\Sigma_{f1} \pm \%RE$	$\Sigma_{f2} \pm \%RE$	$\Sigma_{\gamma,1} \pm \%RE$	$\Sigma_{\gamma,2} \pm \%RE$	$D_1 \pm \%RE$	$D_2 \pm \%RE$
0	$2.72 \times 10^{-3} \pm 0.27$	$6.29 \times 10^{-2} \pm 0.48$	$6.49 \times 10^{-3} \pm 1.61$	$4.27 \times 10^{-2} \pm 0.68$	$1.48 \times 10^0 \pm 2.28$	$4.66 \times 10^{-1} \pm 0.51$
7.3	$2.71 \times 10^{-3} \pm 0.28$	$6.24 \times 10^{-2} \pm 0.42$	$6.50 \times 10^{-3} \pm 1.65$	$4.41 \times 10^{-2} \pm 0.61$	$1.48 \times 10^0 \pm 2.33$	$4.66 \times 10^{-1} \pm 0.48$
24	$2.71 \times 10^{-3} \pm 0.29$	$6.18 \times 10^{-2} \pm 0.50$	$6.53 \times 10^{-3} \pm 1.59$	$4.56 \times 10^{-2} \pm 0.71$	$1.48 \times 10^0 \pm 2.29$	$4.67 \times 10^{-1} \pm 0.50$
48	$2.70 \times 10^{-3} \pm 0.26$	$6.17 \times 10^{-2} \pm 0.49$	$6.55 \times 10^{-3} \pm 1.63$	$4.63 \times 10^{-2} \pm 0.71$	$1.48 \times 10^0 \pm 2.28$	$4.68 \times 10^{-1} \pm 0.54$
96	$2.67 \times 10^{-3} \pm 0.30$	$6.15 \times 10^{-2} \pm 0.50$	$6.58 \times 10^{-3} \pm 1.61$	$4.74 \times 10^{-2} \pm 0.71$	$1.48 \times 10^0 \pm 2.36$	$4.68 \times 10^{-1} \pm 0.53$
145	$2.65 \times 10^{-3} \pm 0.30$	$6.14 \times 10^{-2} \pm 0.60$	$6.60 \times 10^{-3} \pm 1.67$	$4.82 \times 10^{-2} \pm 0.78$	$1.48 \times 10^0 \pm 2.30$	$4.67 \times 10^{-1} \pm 0.55$
193	$2.63 \times 10^{-3} \pm 0.39$	$6.13 \times 10^{-2} \pm 0.61$	$6.65 \times 10^{-3} \pm 1.49$	$4.88 \times 10^{-2} \pm 0.73$	$1.48 \times 10^0 \pm 2.31$	$4.66 \times 10^{-1} \pm 0.53$
242	$2.60 \times 10^{-3} \pm 0.40$	$6.11 \times 10^{-2} \pm 0.72$	$6.67 \times 10^{-3} \pm 1.63$	$4.95 \times 10^{-2} \pm 0.78$	$1.49 \times 10^0 \pm 2.33$	$4.65 \times 10^{-1} \pm 0.53$
290	$2.58 \times 10^{-3} \pm 0.43$	$6.09 \times 10^{-2} \pm 0.78$	$6.68 \times 10^{-3} \pm 1.65$	$5.01 \times 10^{-2} \pm 0.79$	$1.49 \times 10^0 \pm 2.30$	$4.64 \times 10^{-1} \pm 0.51$
339	$2.56 \times 10^{-3} \pm 0.50$	$6.07 \times 10^{-2} \pm 0.92$	$6.71 \times 10^{-3} \pm 1.63$	$5.06 \times 10^{-2} \pm 0.89$	$1.49 \times 10^0 \pm 2.34$	$4.63 \times 10^{-1} \pm 0.55$

density estimates of the three nuclides with burnup are also presented there along with discussions on the implication of the uncertainty quantification as a means to assess the design feasibility of incinerators of minor Actinides like HYPER. The McCARD option to quantify uncertainties of the critical spectrum corrected fuel depletion calculations is a relatively recent addition. The formulation for this option can be made in exactly same way as the infinite spectrum weighted counterpart by using the basic equations in Section 5.1. Figure 9 shows critical spectrum corrected McCARD depletion calculations for ^{235}U burnup in a 17x17 PWR FA in comparison with the infinite medium weighted counterpart. The two calculations did not show the noticeable differences in the number density estimates of ^{235}U but they do in the uncertainty estimates of number densities as displayed in the inset of Fig. 9.

Fig. 9. Critical Spectrum Corrected McCARD Depletion Calculations for ^{235}U Burnup in a 17x17 PWR FA

5.4 Uncertainty Quantification of Few Group Constants

McCARD generates the few group constants through multi-step calculations in sequence as discussed in Section 4. Because a sequence of steps are involved and outputs of any one step become inputs of the ensuing step, the uncertainties of the nuclear cross section and nuclide number density input data to step 1 MC calculation propagates to uncertainties of step 1 MC outputs and to those of the ensuing step outputs and finally to those of few group constants. The McCARD module to quantify the uncertainties of few group constants is as recent as its few group constants generation module. The mathematical formulation for this module can be derived in the similar way as that for uncertainty propagation analysis in McCARD depletion analysis and detailed description on

it will be presented elsewhere. Table 7 shows homogenized two group constants and their percentile relative errors (% RE) of the 17x17 PWR FA with burnup to demonstrate the capability of McCARD to quantify the uncertainties of the few group constants. The necessary covariance data files for this demonstration are from JENDL 3.3. Only covariance data files of the two major uranium isotopes ^{235}U and ^{238}U are used.

6 CONCLUSIONS

McCARD is a specialized and highly efficient MC code for neutronics analyses of nuclear reactors and fuel

systems, because it is developed exclusively for those purposes. Its burnup analysis capability based on precise geometric modeling and efficient load balancing makes it a very useful MC code for nuclear design and analysis of nuclear reactors – particularly research and Gen-IV reactors. Its capabilities of the few group constants generation and uncertainty quantification offer wide-range applications to reactor calculations.

McCARD is equipped with a great many state-of-the-art technologies aimed at the MC whole core calculations. Yet it is still in evolutionary process. To make it a more versatile and user-friendly MC code, more works and research efforts should be made in order to improve and expand its computational modules, to extend the area of its applications beyond neutronics analysis of the current-generation fission reactors, to incorporate an advanced scheme to handle temperature-dependent cross section data, and to strengthen the visual representation of I/O through advanced graphic user interface and CAD interfacing module. The results of these efforts will be reported.

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