ON SOME OUTSTANDING PROBLEMS IN NUCLEAR REACTOR ANALYSIS

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This article discusses selects of some outstanding problems in nuclear reactor analysis, with proposed approaches thereto and numerical test results, as follows: i) multi-group approximation in the transport equation, ii) homogenization based on isolated single-assembly calculation, and iii) critical spectrum in Monte Carlo depletion.

KEYWORDS : Transport Equation, Space-energy-angle Separability, Fine-group Approximation, Few-group Approximation, Angle-dependent Total Cross Section, Inter-assembly Transport Effect, Overlapping Local/Global Iteration, p-CMFD Method, Critical Spectrum Monte Carlo Depletion

1. INTRODUCTION

A nuclear reactor is an engineering device in which controlled nuclear fission is maintained and from which the produced nuclear energy is extracted for useful uses such as electricity generation. The discipline of reactor physics or reactor analysis deals with the design and analysis of such reactors; in particular, reactor analysis and methods development may be characterized as a discipline concerning determination and prediction of the neutronic states of a reactor in various operating configurations. This is achieved by balancing neutron production by fission and loss by capture and leakage. For that, we have to take into account the motion of the neutrons and their interactions with the host nuclei of the reactor medium.

As a high-level model that describes the distribution of neutrons in a reactor, we start with the following Boltzmann transport equation (in steady state) [1, 2]:

$$\begin{aligned} \vec{\Omega} \cdot \nabla \varphi(\vec{r}, E, \vec{\Omega}) + \sigma_t(\vec{r}, E) \varphi(\vec{r}, E, \vec{\Omega}) \\ = \int_{4\pi} d\vec{\Omega}' \int_0^\infty dE' \sigma_s(\vec{r}, E' \rightarrow E, \vec{\Omega}' \cdot \vec{\Omega}) \varphi(\vec{r}, E', \vec{\Omega}') \\ + \frac{1}{k_{\text{eff}}} \frac{\chi(E)}{4\pi} \int_{4\pi} d\vec{\Omega}' \int_0^\infty dE' \nu \sigma_f(\vec{r}, E') \varphi(\vec{r}, E', \vec{\Omega}'), \end{aligned} \quad (1.1)$$

with appropriate boundary conditions provided. In Eq. (1.1), the angular density is defined as

$$\varphi(\vec{r}, E, \vec{\Omega}) = v n(\vec{r}, E, \vec{\Omega}), \quad (1.1a)$$

and other notations are standard, except that the lower case σ stands for macroscopic cross sections. Eq. (1.1) is

rather rigorous, thus with the cross sections given by experimental data (supplemented by theoretical calculations if experimental data are not available) its solution will be very accurate for the problem at hand. However, the problem of finding solution to Eq. (1.1) is nontrivial and defies elementary approaches of analytical methods, but requires sophisticated numerical methods with attending approximations. This is due to i) the complicated energy-dependency of the cross sections (resonance behavior), ii) the space-dependency of the cross sections (heterogeneity in the medium), iii) the angular dependency of the scattering cross section $\sigma_s(\vec{\Omega}' \cdot \vec{\Omega})$, and iv) complexity due to the $\vec{\Omega} \cdot \nabla \varphi$ term. The angular flux $\varphi(\vec{r}, E, \vec{\Omega})$ becomes highly inseparable in its argument variables.

It is customary to first represent the differential scattering cross section in Legendre components:

$$\sigma_s(\vec{r}, E' \rightarrow E, \vec{\Omega}' \cdot \vec{\Omega}) = \sum_{l=0}^{L_s} \frac{2l+1}{4\pi} \sigma_{sl}(\vec{r}, E' \rightarrow E) P_l(\mu_0), \quad (1.2)$$

where L_s is the scattering expansion order.

Eq. (1.1) is then rewritten as

$$\begin{aligned} \vec{\Omega} \cdot \nabla \varphi(\vec{r}, E, \vec{\Omega}) + \sigma_t(\vec{r}, E) \varphi(\vec{r}, E, \vec{\Omega}) \\ = \int_{4\pi} d\vec{\Omega}' \int_0^\infty dE' \sum_{l=0}^{L_s} \frac{2l+1}{4\pi} \sigma_{sl}(\vec{r}, E' \rightarrow E) P_l(\mu_0) \varphi(\vec{r}, E', \vec{\Omega}') \\ + \frac{1}{k_{\text{eff}}} \frac{\chi(E)}{4\pi} \int_{4\pi} d\vec{\Omega}' \int_0^\infty dE' \nu \sigma_f(\vec{r}, E') \varphi(\vec{r}, E', \vec{\Omega}'), \end{aligned} \quad (1.3)$$

Except for extremely simple cases, it is not feasible to find exact solutions of the transport equation (1.3) of

continuous form in independent variables. We need to use various numerical methods in which the governing equation is discretized. The wisdom employed by the reactor analysts is the following three-tier approach:

- i) generate fine-group cross sections applicable to a broad type of reactors (e.g., 100~200 groups for thermal reactors or 1000~2000 groups for fast reactors),
 - ii) find fine-group transport solution over a small unit cell (a pin cell or a fuel assembly) using fine discretizations in angle and space usually with reflective boundary conditions, and homogenize the unit cell and condense to few groups,
 - iii) find few-group diffusion solutions over the whole core to provide k_{eff} and power distributions.
- Recently, feasibility of whole-core transport calculation

without homogenization [3] draws a strong interest in the reactor analysis community. In this two-tier approach, tier (ii) above is removed and tier (iii) becomes transport calculation over the whole core.

Fig. 1 shows a schematic of the three-tier framework in reactor analysis currently in use. Fig. 2 shows a schematic of the two-tier approach desired to be developed.

This review article provides discussion on select outstanding problems in the reactor analysis methodology and proposes some approaches to the problems with numerical test results. The topics chosen are obviously limited, reflecting the author's experience, expertise, and interests.

The article is organized as follows. Section 2 addresses the multi-group approximation to the transport equation. Section 3 discusses the isolated single-assembly calculation

Fig. 1. A Schematic of Three-tier Framework in Reactor Analysis

Fig. 2. A Schematic of Two-tier Framework in Reactor Analysis without Homogenization

in lattice physics calculation that is invariably used for homogenization and few-group condensation. Section 4 provides a discussion on the critical spectrum in Monte Carlo depletion. Finally, Section 5 provides concluding remarks.

2. MULTI-GROUP APPROXIMATION IN TRANSPORT EQUATION

In deterministic neutron transport methods, a process called fine-group to few-group condensation is used to reduce the computational burden. In the group condensation procedure for a transport method, it is well known that angle dependency arises in the few-group condensed total cross section as described in the literature of multi-group approximation; this angle dependency is traditionally removed by consistent P_N approximation or extended transport approximation [1]. The existing numerical algorithms and computer codes are based on this representation. Although this limitation has been recognized for a long time, there has been no serious effort for clear resolution until the work in Ref. [4]. It is noted, though, a *Casian* approach [5] that was reported to provide analytical solutions to the one-velocity transport equation with anisotropic cross section in a homogeneous planar medium [6]. This approach was also cast in a discrete ordinates model and solved using the analytical eigenstate method.

Recently, the authors [4] proposed i) direct application of angle-dependent total cross section to the discrete ordinates method to overcome the limitation of conventional multi-group approximations, ii) a transport sweep procedure adapting the source iteration in the discrete ordinates method, and iii) angle-collapsing concept to further reduce the computational burden.

2.1 Conventional Group Condensation

To get fine-group to few-group condensed cross sections, we first consider the following fine-group discrete ordinates equation in one-dimensional slab geometry:

$$\begin{aligned} \mu_n \frac{\partial \varphi_{n,g}(x)}{\partial x} + \sigma_g(x) \varphi_{n,g}(x) \\ = \sum_{l=0}^{L_s} (2l+1) P_l(\mu_n) \sum_{g'} \sigma_{l,gg'}(x) \phi_{lg'}(x) \\ + \frac{\chi_g}{k_{eff}} \sum_{g'} \nu \sigma_{fg'}(x) \phi_{g'}(x), \end{aligned} \quad (2.1)$$

where n denotes discrete angle index and g denotes fine-group structure, and L_s is the highest order after which the scattering term is truncated. In Eq. (2.1), $\sigma_g(x)$ is angle independent, if ΔE_g is fine enough or energy-angle separability applies, i.e.,

$$\varphi(\vec{r}, E, \vec{\Omega}) = \varphi_E(\vec{r}, E) \varphi_{\vec{\Omega}}(\vec{r}, \vec{\Omega}) \quad \text{for } E \in \Delta E_g, \quad (2.2)$$

which is rarely the case in practical problems.

Eq. (2.1) is summed over the energy range of a few-group structure ($g \in G$), and the few-group discrete ordinates equation is then obtained as follows (without any approximation):

$$\begin{aligned} \mu_n \frac{\partial \varphi_{n,G}(x)}{\partial x} + \hat{\sigma}_{n,G}(x) \varphi_{n,G}(x) \\ = \sum_{l=0}^{L_s} (2l+1) P_l(\mu_n) \sum_{G'} \sigma_{l,GG'}(x) \phi_{lG'}(x) \\ + \frac{\chi_G}{k_{eff}} \sum_{G'} \nu \sigma_{fG'}(x) \phi_{G'}(x), \end{aligned} \quad (2.3)$$

where

$$\varphi_{n,G}(x) \equiv \sum_{g \in G} \varphi_{n,g}(x), \quad \phi_{lG}(x) \equiv \sum_{g \in G} \phi_{lg}(x), \quad (2.3a)$$

and the few-group condensed parameters are defined as

$$\chi_G \equiv \sum_{g \in G} \chi_g, \quad (2.3b)$$

$$\hat{\sigma}_{n,G}(x) \equiv \frac{\sum_{g \in G} \sigma_g(x) \varphi_{n,g}(x)}{\varphi_{n,G}(x)}, \quad (2.3c)$$

$$\sigma_{l,GG'}(x) \equiv \frac{\sum_{g \in G} \sum_{g' \in G'} \sigma_{l,gg'}(x) \phi_{lg'}(x)}{\phi_{lG'}(x)}, \quad (2.3d)$$

$$\nu \sigma_{fG'}(x) \equiv \frac{\sum_{g' \in G'} \nu \sigma_{fg'}(x) \phi_{g'}(x)}{\phi_{G'}(x)}. \quad (2.3e)$$

It is noted that, as shown in Eq. (2.3c), the group-condensed total cross section has angle dependency. The traditional way to remove this difficulty is to move the total reaction rate term (the second term on the left-hand side of Eq. (2.1)) to the right-hand side of Eq. (2.1), after which its angular flux is expanded in Legendre moment fluxes up to the term L_N . The equation is then summed over the energy range of a few-group structure. By adding an artificial reaction rate term (σ_G to be specified later) to both sides, the following few-group equations are obtained:

$$\begin{aligned} \mu_n \frac{\partial \varphi_{n,G}(x)}{\partial x} + \sigma_G(x) \varphi_{n,G}(x) \\ = \sum_{l=0}^{L_N, L_s} (2l+1) P_l(\mu_n) \sum_{G'} \tilde{\sigma}_{l,GG'}(x) \phi_{lG'}(x) \\ + \frac{\chi_G}{k_{eff}} \sum_{G'} \nu \sigma_{fG'}(x) \phi_{G'}(x), \end{aligned} \quad (2.4)$$

where

$$\tilde{\sigma}_{l,GG'}(x) = \sigma_{l,GG'}(x) + [\sigma_G(x) - \sigma_{lG}(x)] \delta_{GG'}, \quad (2.4a)$$

$$\sigma_{lG}(x) \equiv \frac{\sum_{g \in G} \sigma_g(x) \phi_{lg}(x)}{\phi_{lG}(x)}. \quad (2.4b)$$

Note in Eq. (2.4) that

$$\sigma_{l,GG'}(x); \quad l = 0, 1, \dots, L_s, \quad (2.4c)$$

and

$$\sigma_{lG}(x); \quad l = 0, 1, \dots, L_N. \quad (2.4d)$$

The unspecified angle-independent total cross section, $\sigma_G(x)$, can be chosen in several ways [7]; two typical examples are:

i) Consistent PN approximation:

$$\sigma_G(x) = \sigma_{0,G}(x), \quad (2.5)$$

ii) Extended transport approximation:

$$\sigma_G(x) = \sigma_{L_N+1,G}(x) - \sum_{G'} \sigma_{L_N+1,G'G}(x). \quad (2.6)$$

2.2 New Group Condensation and Overlapping Local/Global Iteration

2.2.1 Group Condensation

In this article, we propose to solve the equivalently condensed few-group Eq. (2.3) directly, which is written again below,

$$\begin{aligned} \mu_n \frac{\partial \varphi_{n,G}(x)}{\partial x} + \hat{\sigma}_{n,G}(x) \varphi_{n,G}(x) \\ = \sum_{l=0}^{L_s} (2l+1) P_l(\mu_n) \sum_{G'} \sigma_{lGG'}(x) \phi_{lG'}(x) \\ + \frac{\chi_G}{k_{eff}} \sum_{G'} \nu \sigma_{fG'}(x) \phi_{G'}(x), \end{aligned} \quad (2.3)$$

that has an angle-dependent total cross section $\hat{\sigma}_{n,G}(x)$. The iteration procedure of the discrete ordinates equation is called source iteration based on transport sweep [2, 8]. To apply the transport sweep to the transport equation with angle-dependent total cross section, we simply give a different total cross section for each discrete angle.

If we integrate Eq. (2.3) over i -th fine-mesh interval, the resulting balance equation becomes:

$$\mu_n [\varphi_{i+1/2,n,G} - \varphi_{i-1/2,n,G}] + \hat{\sigma}_{i,n,G} \varphi_{i,n,G} h_i = q_{i,n,G} h_i, \quad (2.7a)$$

$$\begin{aligned} q_{i,n,G} = \sum_{l=0}^{L_s} (2l+1) P_l(\mu_n) \sum_{G'} \sigma_{l,iGG'} \phi_{l,i,G'} \\ + \frac{\chi_G}{k_{eff}} \sum_{G'} \nu \sigma_{f,i,G'} \phi_{i,G'}, \end{aligned} \quad (2.7b)$$

where i indicates the fine-mesh cell index ($i \pm 1/2$ for the fine-mesh cell boundary), h the fine-mesh cell size, and q is the average source of the fine-mesh cell.

To get an explicit expression for fine-mesh cell average flux, $\varphi_{i,n,G}$, we need an auxiliary equation that relates fine-mesh boundary angular fluxes and fine-mesh cell average flux. Typical examples of auxiliary equations are diamond difference scheme, step characteristic scheme, and step difference scheme. In this study, we consider the simplest example of diamond difference scheme:

$$\varphi_{i,n,G} = \frac{\varphi_{i-1/2,n,G} + \varphi_{i+1/2,n,G}}{2}. \quad (2.8)$$

From Eqs. (2.7a) and (2.8), we can get an explicit

equation for the fine-mesh cell average flux:

$$\varphi_{i,n,G} = \left(1 + \frac{\hat{\sigma}_{i,n,G} h_i}{2|\mu_n|} \right)^{-1} \left(\varphi_{i-1/2,n,G} + \frac{q_{i,n,G} h_i}{2|\mu_n|} \right) \text{ for } \mu_n > 0, \quad (2.9a)$$

$$\varphi_{i,n,G} = \left(1 + \frac{\hat{\sigma}_{i,n,G} h_i}{2|\mu_n|} \right)^{-1} \left(\varphi_{i+1/2,n,G} + \frac{q_{i,n,G} h_i}{2|\mu_n|} \right) \text{ for } \mu_n < 0, \quad (2.9b)$$

Eqs. (2.8) and (2.9) suggest a sweeping calculational procedure for Eq. (2.3). The same transport sweep in the usual discrete ordinates method can be used. The only difference from the usual sweeping procedure (with angle independent total cross section) is that the group condensed cross sections are prepared cell-wise and different total cross sections are given for each discrete angle. This difference causes high computational memory requirement for the few-group transport calculation with angle dependent total cross section, because few-group cross section should be prepared for every cell and discrete ordinate. To reduce this memory requirement, homogenization technique for transport equation with angle dependent total cross section or angle collapsing concept can be considered. In the following section, angle collapsing concept to reduce computational memory requirement and time will be introduced.

2.2.2 Angle Collapsing

To simplify the group-condensed few-group transport problem, we consider angle collapsing in positive and negative directions in the application of the discrete ordinates method to a one-dimensional problem. To perform angle collapsing, we first multiply Eq. (2.3) by w_n (the quadrature weight corresponding to μ_n , normalized as $\sum_{n=1}^N w_n = 2$) and then sum the values over a positive and negative angle direction, respectively, to get partially angle integrated transport equations:

$$\begin{aligned} \frac{\partial J_G^+(x)}{\partial x} + \hat{\sigma}_G^+(x) \varphi_G^+(x) \\ = \sum_{l=0}^{L_s} (2l+1) P_l^+ \sum_{G'} \sigma_{lGG'}(x) \phi_{lG'}(x) \\ + \frac{\chi_G}{k_{eff}} \sum_{G'} \nu \sigma_{fG'}(x) \phi_{G'}(x), \end{aligned} \quad (2.10)$$

where

$$\varphi_G^+(x) \equiv \sum_{n=1}^{N/2} w_n \varphi_{n,G}(x), \quad (2.11a)$$

$$\varphi_G^-(x) \equiv \sum_{n=1}^{N/2} w_{N+1-n} \varphi_{N+1-n,G}(x),$$

$$J_G^+(x) \equiv \sum_{n=1}^{N/2} w_n \mu_n \varphi_{n,G}(x), \quad (2.11b)$$

$$J_G^-(x) \equiv \sum_{n=1}^{N/2} w_{N+1-n} \mu_{N+1-n} \varphi_{N+1-n,G}(x),$$

$$\hat{\sigma}_G^+ \equiv \frac{\sum_{n=1}^{N/2} w_n \hat{\sigma}_{n,G}^+(x) \varphi_{n,G}(x)}{\varphi_G^+}, \quad (2.11c)$$

$$\hat{\sigma}_G^- \equiv \frac{\sum_{n=1}^{N/2} w_{N+1-n} \hat{\sigma}_{N+1-n,G}^-(x) \varphi_{N+1-n,G}(x)}{\varphi_G^-},$$

$$P_l^+ \equiv \sum_{n=1}^{N/2} w_n P_l(\mu_n), \quad P_l^- \equiv \sum_{n=1}^{N/2} w_{N+1-n} P_l(\mu_{N+1-n}), \quad (2.11d)$$

where N is the number of discrete ordinates; using the standard notation in S_N with symmetric quadratures and mirror reflective ordinates for a one-dimensional problem.

The first term in Eq. (2.10) is expressed in partial currents $J_G^\pm(x)$. To relate $J_G^\pm(x)$ with partially angle integrated fluxes $\varphi_G^\pm$, we consider the following from Eq. (2.11):

$$\frac{J_G^+(x)}{\varphi_G^+(x)} = \frac{\sum_{n=1}^{N/2} w_n \mu_n \varphi_{n,G}(x)}{\sum_{n=1}^{N/2} w_n \varphi_{n,G}(x)} \equiv \bar{\mu}_G^+(x), \quad (2.12a)$$

and similarly,

$$\frac{J_G^-(x)}{\varphi_G^-(x)} = \frac{\sum_{n=1}^{N/2} w_{N+1-n} \mu_{N+1-n} \varphi_{N+1-n,G}(x)}{\sum_{n=1}^{N/2} w_{N+1-n} \varphi_{N+1-n,G}(x)} \equiv \bar{\mu}_G^-(x). \quad (2.12b)$$

Note that the “representative” collapsed angles $\bar{\mu}_G^\pm(x)$ have not only spatial dependency but also energy group dependency. It is a different property from that of the original discrete ordinates equation. If we substitute Eq. (2.12) into Eq. (2.10), an equivalent form of the partially angle integrated transport equations can be obtained:

$$\begin{aligned} \frac{\partial}{\partial x} [\bar{\mu}_G^\pm(x) \varphi_G^\pm(x)] + \hat{\sigma}_G^\pm(x) \varphi_G^\pm(x) \\ = \sum_{l=0}^{L_g} (2l+1) P_l^\pm \sum_{G'} \sigma_{lGG'}(x) \phi_{lG'}(x) \\ + \frac{\chi_G}{k_{eff}} \sum_{G'} \nu \sigma_{fG'}(x) \phi_{G'}(x). \end{aligned} \quad (2.13)$$

Similarly to the spatial discretization in Section 2.2.1, we integrate Eq. (2.13) over i -th fine-mesh interval and combine with diamond difference scheme, to obtain a sweeping procedure for Eq. (2.13). However, it is noted that the diamond difference equation for partially angle collapsed transport equation is slightly different from the original one (Eq. (2.8)). The diamond difference equation for Eq. (2.13) is written as:

$$J_{i,G}^\pm = \frac{J_{i+1/2,G}^\pm + J_{i-1/2,G}^\pm}{2}. \quad (2.14a)$$

The diamond difference scheme is used to substitute leakage related term in transport equation (first term in LHS of Eq. (2.3) or (2.13)) into mesh-averaged angular flux. In case of angle collapsed transport equation,

“representative” collapsed angle $\bar{\mu}_G^\pm(x)$ has spatial distribution. Therefore, the diamond difference scheme for Eq. (2.13) should involve the information of $\bar{\mu}_G^\pm(x)$. If Eq. (2.14a) is rewritten in terms of angles and angular fluxes, it becomes:

$$\bar{\mu}_{i,G}^\pm \varphi_{i,G}^\pm = \frac{\bar{\mu}_{i+1/2,G}^\pm \varphi_{i+1/2,G}^\pm + \bar{\mu}_{i-1/2,G}^\pm \varphi_{i-1/2,G}^\pm}{2}. \quad (2.14b)$$

In Eq. (2.14b), if angle collapsing is not applied (in other words, cosine angle is independent to the position), cosine angles are canceled out and the form of Eq. (2.8) is obtained.

By using Eq. (2.14b) as a diamond difference scheme, sweeping equation is simply written as:

$$\varphi_{i,G}^\pm = \left(1 + \frac{\hat{\sigma}_{i,G}^\pm h_i}{2 |\bar{\mu}_{i,G}^\pm|} \right)^{-1} \left(\frac{\bar{\mu}_{i+1/2,G}^\pm}{2 \bar{\mu}_{i,G}^\pm} \varphi_{i+1/2,G}^\pm + \frac{q_{i,G}^\pm h_i}{2 |\bar{\mu}_{i,G}^\pm|} \right), \quad (2.15)$$

where $q^\pm$ is the average source of the cell for the positive and negative directions, respectively, as determined by scattering and fission.

In this section, only one-dimensional slab problem is used to describe angle collapsing. But it should be straightforward to extend the angle collapsing concept to multi-dimensional problems. We can use four representative angles (quadrants) in two-dimensional problems, and eight angles (octants) in three-dimensional problems.

2.2.3 Overlapping Local/Global (OLG) Iteration

Recently, a framework of overlapping local/global iteration was proposed and tested [9] so that the whole core could be accurately analyzed by iteratively applying the Monte Carlo method to local problems coupled with the nodal diffusion method to the global problem. This framework takes advantage of the Monte Carlo capability to describe continuous energy and complex geometry accurately. Another nice feature of local/global iteration framework is that local problems are naturally suited for parallel computation.

A similar local/global iteration may be used in deterministic transport method. As for the transport method in this section, fine-group discrete ordinates calculation is used in local problems while few-group angle collapsed transport calculation is used in the global problem. The transport methods are therefore used for both local and global calculations.

Fig. 3 shows a flow chart of the local/global iteration framework. First, at the zeroth iteration, a local fine-group calculation is performed with an all-reflective boundary condition for a subregion; this subregion is normally chosen as an assembly. The global few-group calculation is then performed with group-condensed parameters generated in the local calculation. From the global calculation, an albedo boundary condition for each local subregion is updated and used for the next local calculation. These procedures are performed iteratively.

Fig. 3. Flow Chart of the Local/Global Iteration Framework

A weakness of the local/global iteration framework is that the albedo boundary condition, which is updated from the global calculation, is expressed in terms of a few-group angle-collapsed structure. For the next local calculation, a fine-group albedo boundary condition is consequently assumed to be constant within the few-group angle-collapsed structure. However, with the half or fully overlapping subregion in the local calculation, we can use the solution of the local calculation to “modulate” the few-group angle-collapsed albedo boundary condition with good accuracy [9]. Fig. 4 shows the definition of half-node overlapping local problem. The red dotted line in Fig. 4 corresponds to the boundary of adjacent local problem. Thus, if we store albedo values at the center surface of a local problem during local calculation, the few-group albedo values from global calculation can be modulated or “resolved” to the fine-group albedo values. The equation for albedo modulation is expressed as:

$$\alpha_{g,s}^{modulated} = \frac{\alpha_{g,s}^{local}}{\alpha_{G,s}^{local}} \alpha_{G,s}^{global}, \quad (2.16)$$

where s represents local problem center surface, g fine-group index, and G few-group index (albedo α is a ratio of incoming to outgoing angular fluxes). The modulated albedo values then become the boundary conditions for neighboring local problems in the next iteration. In the case of angle collapsing, a similar modulation is also used in angular values.

In the test problem of application in the next section, we use half-node overlapping assembly as local problems. The reflector region, if exists, is included in the local problem of the adjacent fuel assembly. Alternatively, the

Albedo values edited from global solution will be modulated and used as B.C. of neighboring local problems

Fig. 4. Half-node Overlapping Local Problem and Albedo Modulation

reflector local problem may be defined to include a half of the adjacent fuel assembly.

2.2.4 Test Results

Problem Description

Two one-dimensional multi-slab problems are considered in this article. Although the scattering cross

Fig. 5. Configuration of the Thermal Reactor Test Problem

section exhibits low anisotropy in these problems (isotropic in the first problem, linearly anisotropic in the second problem), the strong geometric heterogeneity induces highly anisotropic angular fluxes. This calls for strong challenges to the group condensation.

The first problem is a modified C5G7 benchmark problem which is thermal reactor test problem. The problem configuration of modified C5G7 is shown in Fig. 5 and Fig. 6.

The inside of the UOX/MOX assembly is heterogeneous. The material properties shown along the red line in Fig. 6 are used to form one-dimensional multi-slabs in the corresponding fuel assembly in Fig. 5.

7-group cross sections are specified in the C5G7 benchmark report [3] as a fine-group structure with isotropic scattering. In group condensation, the first five groups are condensed into a fast group; and the remaining groups are condensed into a thermal group. The diamond difference scheme and the S_{12} discrete ordinates are used for the fine-group calculation; the iteration criterion is 10^{-7} relative error on scalar flux. Note that homogenization is not considered in this study because our focus is on the error of group condensation. As shown in Eq. (2.3), the group-condensed cross sections depend on the angular moment flux that is space-dependent; hence, the group-condensed cross section sets are obtained for each calculational cell. In this study, to minimize the spatial discretization effect, a very fine mesh cell size of 0.03 cm is used and group-condensed cross sections are generated for each fine mesh cell.

The second problem considered in this article is a simplified fast reactor core-reflector interface problem. It is reported that the use of a small number of few groups in existing methods induces large discrepancy with reference solution for this core-reflector interface problem [10, 11]. The problem configuration of this fast reactor problem is shown in Fig. 7 and Table 1.

150-group cross sections are used as a fine-group structure with scattering cross section up to P_1 Legendre expansion in this problem. This 150-group cross section set is obtained by the TRANSX code [12] processing based on JEFF 3.1 library. In group condensation, few-group cross sections are generated in two ways: i) 25-group,

Fig. 6. Detailed Configuration of the UOX/MOX Assembly [3]

Fig. 7. Configuration of the Fast Reactor Core-reflector Test Problem

Table 1. Isotope Composition of the Fast Reactor Test Problem

Region	Isotope	Density [atoms/barn-cm]
Core	Pu239	1.5×10^{-3}
	U238	5.0×10^{-3}
	Fe56	7.0×10^{-3}
	Cr52	1.5×10^{-3}
	Na23	1.0×10^{-2}
	O16	1.5×10^{-2}
Reflector	Fe56	5.0×10^{-2}
	Cr52	1.5×10^{-2}
	Na23	5.0×10^{-3}

Fig. 8. Half-node Overlapping Subregions for the Thermal Reactor Test Problem

and ii) 1-group condensation. Similarly with the previous thermal reactor test problem, diamond difference scheme is used as an auxiliary equation and S_{12} discrete ordinates are used for fine-group calculation. The fine-mesh size used is 0.5cm in this problem.

Group Condensation without Angle Collapsing

To simulate a practical group condensation procedure, we therefore use the assembly-level half-overlapping subregion, as suggested by [9], for the fine-group calculation in this and subsequent sections. The half-overlapping subregions for the first test problem in Fig. 5 are shown in Fig. 8.

In Fig. 8, subregion (a) is used to get the group-condensed cross sections in the UOX assembly and subregion (b) is used for the MOX assembly and reflector, respectively. In this numerical test, the local/global iterative calculation is conducted until an accurate estimation of k_{eff} is achieved.

Likewise, Fig. 9 shows the half-overlapping subregions for the second test problem. In Fig. 9, subregion (a) is used to get the group-condensed cross sections in the core region and subregion (b) for the reflector region, respectively.

Results of modified C5G7 benchmark problem

Table 2 shows the k_{eff} values and its relative errors of the thermal reactor test problem for conventional consistent P_N and extended transport approximations as well as the application of angle-dependent total cross sections up to the second local/global iteration. As shown in Table 2, the angle-dependent total cross section gives a much more

Fig. 9. Half-node Overlapping Subregions for the Fast Reactor Core-reflector Test Problem

accurate k_{eff} value than the conventional approaches for the problem tested. The consistent P_N approximation with $L_N=0$ converges very slowly or appears to converge to a value different from the reference solution.

It is observed that the consistent P_N approximation

Table 2. The Value of k_{eff} and the Relative Error^a (without Angle Collapsing)

Local/global iteration: zeroth			
	L_N	k_{eff}	Relative Error
Consistent P approximation	0	1.1028811	614.6 pcm
	1	No convergence	-
Consistent P approximation	0	No convergence	-
	1	No convergence	-
Angle-dependent total XS	-	1.099130	272.4 pcm
Local/global iteration: first			
	L_N	k_{eff}	Relative Error
Consistent P approximation	0	1.102348	565.9 pcm
	1	-	-
Consistent P approximation	0	-	-
	1	-	-
Angle-dependent total XS	-	1.096185	3.7 pcm
Local/global iteration: second			
	L_N	k_{eff}	Relative Error
Consistent P approximation	0	1.102380	568.9 pcm
	1	-	-
Consistent P approximation	0	-	-
	1	-	-
Angle-dependent total XS	-	1.096145	0.1 pcm

^aReference $k_{eff} = 1.096144$ from the fine-group reference calculation (ONEDANT)

with $L_N=1$ and the extended transport approximation with $L_N=0$ and 1 do not converge numerically. Eq. (2.4b) suggests the possibility that $\sigma_{IG}(x)(l \geq 1)$ has a negative value. In this situation, the value of a group-scattering cross section moment, $\tilde{\sigma}_{IG}(x)(l \geq 1)$, can be greater than the total cross section. This phenomenon occurs during the group condensation procedure in this problem; it yields a negative source during the iteration and causes convergence problem.

■ Results of simplified fast reactor core-reflector problem

Table 3 shows the k_{eff} values and its relative errors of the fast reactor test problem when angle dependent total

Table 3. The Value of k_{eff} and the Relative Error^a (without Angle Collapsing)

Local/global iteration: zeroth		
	k_{eff}	Relative Error
150-group to 25-group	1.392059	151.8pcm
150-group to 1-group	1.365306	-1773pcm
Local/global iteration: first		
	k_{eff}	Relative Error
150-group to 25-group	1.389955	0.4pcm
150-group to 1-group	1.389895	-3.9pcm
Local/global iteration: second		
	k_{eff}	Relative Error
150-group to 25-group	1.389950	0.0pcm
150-group to 1-group	1.389953	0.3pcm

^aReference $k_{eff} = 1.389950$ from the fine-group reference calculation (ONEDANT)

cross section is used for group condensation. The results of the conventional method such as consistent P_N and extended transport approximations give different k_{eff} values or show convergence problem like the previous problem. Therefore, these results are not included in Table 3. As shown in Table 3, the numerical results after 0th local/global iteration show excellent agreement for k_{eff} estimation even in the 150-group to 1-group condensation. Because group condensation is performed equivalently resulting in angle-dependent total cross section, the structure (number and boundaries) of the few groups is, in principle, not important. These results were anticipated in [4] and reported in preliminary form in [13].

Group Condensation with Angle Collapsing

In this section, the angle-collapsing concept is applied in addition to the few-group condensation.

■ Results of modified C5G7 benchmark problem

Table 4 shows the k_{eff} value and its relative error when angle collapsing in two directions and two-group condensation are applied to the thermal reactor test problem. The results of consistent P_N approximation with $L_N = 1$ and extended transport approximation are not

included because there is no convergence, even when, as shown in Table 2, the angle collapsing is not applied.

In the case of angle-dependent total cross section, the discrepancies are 272.4pcm (Table 2, no angle collapsing) and 693.5pcm (Table 4, angle collapsing), respectively, in the zeroth iteration; they are 3.7pcm and 21.5pcm, respectively, in the first iteration; 0.1pcm and 1.2pcm, respectively, in the second iteration. Even in the first local/global iteration, the error is reduced remarkably compared to the zeroth local/global iteration. We believe that the local/global iteration converges quite fast due to the use of overlapping subregion in local problems. Note again that, in the case of consistent P_N approximation with $L_N=0$, the iteration converges very slowly or appears to converge to a different solution.

In the case of angle-dependent total cross section, as shown in Tables 2 and 4, there is slightly larger discrepancy in the k_{eff} value with angle collapsing than without. This is due to the need of additional modulation in angular values when angle collapsing is used. However, the calculational burden is reduced because just two directions are considered in the few-group calculation.

Table 4. The k_{eff} Value and its Relative Error^a (with angle collapsing)

Local/global iteration: zeroth			
	L_N	k_{eff}	Relative Error
Consistent P approximation	0	1.106441	939.4 pcm
Angle-dependent total XS	-	1.103746	693.5 pcm
Local/global iteration: first			
	L_N	k_{eff}	Relative Error
Consistent P approximation	0	1.101650	502.3 pcm
Angle-dependent total XS	-	1.096380	21.5 pcm
Local/global iteration: second			
	L_N	k_{eff}	Relative Error
Consistent P approximation	0	1.101573	495.3 pcm
Angle-dependent total XS	-	1.096157	1.2 pcm

^aReference $k_{eff} = 1.096144$ from the fine-group reference calculation (ONEDANT)

■ Results of simplified fast reactor core-reflector problem

Table 5 shows the results of k_{eff} and its relative errors of the fast reactor test problem when angle-collapsing and angle-dependent total cross section are applied. Similarly with the previous non-angle-collapsing and thermal reactor test problem, it shows good agreement on k_{eff} estimation at 1st or 2nd local/global iteration.

3. INTER-ASSEMBLY TRANSPORT EFFECT

In the modern reactor design analysis, whole-core diffusion nodal method is used in which isolated single-assembly lattice physics (multi-group transport) calculations provide homogenized parameters. The single-assembly calculation is performed using net current zero boundary condition. This boundary condition is applicable in the academic case of infinite lattice of identical assemblies. To remove the net current zero assumption in the isolated single-assembly calculation and to take into account the inter-assembly transport effect, the overlapping local/

Table 5. The Value of k_{eff} and the Relative Error^a (with Angle Collapsing)

Local/global iteration: zeroth		
	k_{eff}	Relative Error
150-group to 25-group	1.411054	1518pcm
150-group to 1-group	1.389955	-125pcm
Local/global iteration: first		
	k_{eff}	Relative Error
150-group to 25-group	1.390143	14pcm
150-group to 1-group	1.389995	3pcm
Local/global iteration: second		
	k_{eff}	Relative Error
150-group to 25-group	1.389955	0.4pcm
150-group to 1-group	1.389942	-0.6pcm

^aReference $k_{eff} = 1.389950$ from the fine-group reference calculation (ONEDANT)

global iteration method described in Section 2 can be used (in which the global calculation to be described in this section is now diffusion-like but gives transport solution). Similar local/global iteration methods (not overlapping) have been proposed based on boundary condition perturbation theory [14, 15, 16]. Similarly, a method has been reported [17] that proposes to use tabulated nodal parameters for various albedo boundary conditions to reflect the effect of neighboring assemblies by interpolation. A rehomogenization method [18] addresses the problem by precomputing the “rehomogenization coefficients” during single assembly lattice calculation. The method is simple to update the homogenized cross sections, but it cannot be used to correct the discontinuity factor nor to account for inter-assembly transport effect. A recent review paper [19] discusses black-box homogenization with current discontinuity factors in the presence of surface leakage.

3.1 Overlapping Local/ Global Iteration

The overall computational scheme is similar to that of Section 2 and Fig. 3, except that i) the global calculation is performed in Partial-Current Coarse-Mesh Finite Difference (p-CMFD) methodology [20, 21, 22], and ii) to update albedo boundary conditions for a local problem in next iteration, two options are used; a) flux distribution is reconstructed in one-node Analytic Function Expansion Nodal (AFEN) method [23, 24] to provide partial currents at mid-surface of the node, b) the results of a local problem are edited in two subnodes in the case of one-dimensional

problems (2x2 subnodes in the case of two-dimensional problems, 2x2x2 subnodes in the case of three-dimensional problems). Fig. 10 shows the computational flow.

3.2 Whole-Core Transport Solution via p-CMFD

The Partial-Current Coarse-Mesh Finite Difference Method (p-CMFD) [20, 21, 22] is a modification of the Coarse-Mesh Finite Difference Method (CMFD) [25] that is popularly used as an acceleration scheme of the multi-group transport equations. In this section, the p-CMFD methodology is used to provide whole-core transport solution in the local/global iteration scheme. To explain how to use p-CMFD in the overlapping local/global iteration, let us first integrate the transport equation (1.3) over angle to get,

$$\begin{aligned} \nabla \cdot \vec{J}(\vec{r}, E) + \sigma_t(\vec{r}, E)\phi(\vec{r}, E) \\ = \int_0^\infty dE' \sigma_{s0}(\vec{r}, E' \rightarrow E)\phi(\vec{r}, E') \\ + \frac{\chi(E)}{k_{eff}} \int_0^\infty dE' \nu \sigma_f(\vec{r}, E')\phi(\vec{r}, E'), \end{aligned} \quad (3.1)$$

where

$$\phi(\vec{r}, E) = \int d\vec{\Omega} \varphi(\vec{r}, E, \vec{\Omega}), \quad (3.1a)$$

and

$$\vec{J}(\vec{r}, E) = \int d\vec{\Omega} \vec{\Omega} \varphi(\vec{r}, E, \vec{\Omega}). \quad (3.1b)$$

Note that there is no approximation in Eq. (3.1). Defining conventional flux-volume weighted cross sections

Fig. 10. Computation Flow of Overlapping Local/Global Iteration in S_N /p-CMFD

over coarse-mesh cells typically of assembly size, Eq. (3.1) is converted (again without approximation) into a finite-difference form of homogenized few-group equations:

$$\frac{1}{V_i} \sum_{is=1}^6 A_{is} (\bar{J}_g^+(is) - \bar{J}_g^-(is)) + \bar{\sigma}_{ag,i} \bar{\phi}_{g,i} = \sum_{g'=1}^G \left(\bar{\sigma}_{gg',i} + \frac{\chi_g}{k_{eff}} \nu \bar{\sigma}_{fg',i} \right) \bar{\phi}_{g',i}, \quad (3.2)$$

where, i =(coarse) node index,
 is =surface index,
 g =few-group index.

Now, solution of the overlapping local problem (fine group, heterogeneous transport) provides the weighting flux for homogenization and few-group condensation as well as the partial currents at assembly surfaces and assembly center-surface. The partial currents at the center-surface are modulated by the p-CMFD method solution on the global problem described below. Therefore, the p-CMFD method induces partial currents in Eq. (3.2) to approach the partial currents from the local problem in local/global iteration. Fig. 11 shows local problem i in a half-node overlapping local/global iteration scheme.

In p-CMFD calculation, partial currents at the right edge ($i+1/2$) of coarse mesh cell i are corrected with the CMR concept; outgoing and incoming partial currents of any mesh cell are related with the corresponding cell average scalar fluxes, respectively, as (dropping group index):

$$\bar{J}_{i+1/2}^+ = \frac{-\tilde{D}_{i+1/2} (\bar{\phi}_{i+1} - \bar{\phi}_i) + 2\hat{D}_{i+1/2}^+ \bar{\phi}_i}{2}, \quad (3.3a)$$

$$\bar{J}_{i+1/2}^- = \frac{\tilde{D}_{i+1/2} (\bar{\phi}_{i+1} - \bar{\phi}_i) + 2\hat{D}_{i+1/2}^- \bar{\phi}_{i+1}}{2}, \quad (3.3b)$$

where $\tilde{D}_{i+1/2}$ is the coupling coefficient determined in ordinary finite difference method, and the two correction

factors are defined to preserve the respective partial currents as

$$\hat{D}_{i+1/2}^+ = \frac{2\bar{J}_{i+1/2}^+ + \tilde{D}_{i+1/2} (\bar{\phi}_{i+1} - \bar{\phi}_i)}{2\bar{\phi}_i}, \quad (3.4a)$$

$$\hat{D}_{i+1/2}^- = \frac{2\bar{J}_{i+1/2}^- - \tilde{D}_{i+1/2} (\bar{\phi}_{i+1} - \bar{\phi}_i)}{2\bar{\phi}_{i+1}}, \quad (3.4b)$$

where the partial currents $\bar{J}_{i+1/2}^+$ and $\bar{J}_{i+1/2}^-$ are obtained from the local problems. Then, the net current is obtained from Eqs. (3.3a) and (3.3b) as

$$\bar{J}_{i+1/2} = \bar{J}_{i+1/2}^+ - \bar{J}_{i+1/2}^- = -\tilde{D}_{i+1/2} (\bar{\phi}_{i+1} - \bar{\phi}_i) - (\hat{D}_{i+1/2}^- \bar{\phi}_{i+1} - \hat{D}_{i+1/2}^+ \bar{\phi}_i). \quad (3.5)$$

The net current relation (Eq. (3.5)) is then substituted into Eq.(3.2) with the flux-volume weighted cross sections, resulting in a finite difference form of equations called p-CMFD equation.

It is interesting to note that either CMFD or p-CMFD equation is in a similar form to a low-order equation of diffusion-type with, in effect, directional “diffusion coefficients” $\hat{D}^{*+}$ and $\hat{D}^{*-}$. It is interesting to note that it does not require to deal with discontinuity factors or surface flux discontinuity directly, although surface flux is allowed to be discontinuous. Because of the directionality of the “diffusion coefficients”, the low-order equation becomes a nonsymmetric matrix system for which a Krylov subspace method is a good solver. Implementation details of this type of local/global iteration (S_N/p-CMFD) are available in [26].

3.3 Test Results

Problem Description

To compare the overlapping local/global iteration with the conventional method (that is based on isolated assembly calculation), a heterogeneous core problem is needed. A modified C5G7 benchmark problem which is used in Section 2 is also considered in this section. The diamond difference scheme and the S₁₂ discrete ordinates are used for local calculation to perform the assembly homogenization and group condensation; iteration stopping criterion is 10⁻⁷ relative error on scalar flux, a cell size used is 0.045cm. The conventional method uses single assembly domain, while the overlapping local/global iteration uses half overlapping assembly. But, in the reflector assembly, to obtain the condensed group homogenized cross sections, the half overlapping assembly is also used in the conventional method. We use two few-group structures; 7-group to 1-group condensation and 7-group to 2-group condensation (two-group division is the same as in Section 2). In the global calculation, AFEN method is used in the conventional method, while p-CMFD is used in the overlapping local/global iteration.

Reference Solution

Reference solution is provided from the transport calculation of whole domain, with the diamond difference

Albedo B.C. for local problem i
 Fig. 11. Half-node Overlapping Local Problem

scheme and S_{12} discrete ordinates. The numerical result of the k_{eff} value is $k_{eff} = 1.09614$ in seven-group whole-core transport calculation. The power density distribution is also calculated.

Multiplication Factor during the Iterations

In the overlapping local/global iteration, the multiplication factor and the scalar flux are calculated in each iteration. The k_{eff} and its relative error for each iteration are shown in Table 6.

Table 6 shows that the overlapping local/global iteration converges well in early iterations (second or third iterations).

Comparison with Conventional Method

In this section, we compare the results of overlapping local/global iteration (2nd iteration) with the conventional results. Table 7 shows the multiplication factor and rRMSE (relative Root Mean Square Error) of power density in the UO2 assembly and MOX assembly.

As shown in Table 7, the overlapping local/global iteration gives more accurate values than the conventional method for any group condensation. The multiplication factor error is drastically reduced when we use the overlapping local/global iteration; 884 pcm to -11 pcm for 1-group condensation and 803 pcm to -10 pcm for 2-group condensation. The accuracy in power density is much more remarkable. In the conventional method, 2-group condensation gives several percents in rRMSE of power density in each assembly. But the overlapping local/global iteration gives about 0.03~0.09% in rRMSE of power density. Even in the one-group condensation, the overlapping local/global iteration gives rRMSE of power density less than 0.10%. On the other hand, the conventional method gives a large error of power density in the case of one-group condensation.

Fig. 12 shows the relative error of the power distribution in the case of the 2-group condensation.

Table 6. The k_{eff} Values of Local/Global Iteration

Iteration number	Group Condensation			
	7-group to 2-group		7-group to 1-group	
	k_{eff}	Relative error* (pcm)	k_{eff}	Relative error* (pcm)
0	1.08249	-1245	1.08339	-1163
1	1.09728	104	1.09739	114
2	1.09603	-10	1.09602	-11
3	1.09615	1	1.09615	1
4	1.09614	0	1.09614	0
5	1.09614	0	1.09614	0
6	1.09614	0	1.09614	0

*Reference $k_{eff} = 1.09614$ from the whole core transport calculation.

Fig. 12. Relative Error of the Power Distribution for 7-to-2 Group Condensation

Table 7. Comparison of Local/Global Iteration and Conventional Method

Method	Group Condensation			
	7-group to 2-group		7-group to 1-group	
	Local/global iteration (2 nd)	Conventional method	Local/global iteration (2 nd)	Conventional method
k_{eff}	1.09603	1.10494	1.09602	1.10583
Relative error(pcm)	-10	803	-11	884
Relative error(pcm) rRMSE of power density in UO2 assembly (%)	0.03%	1.42%	0.04%	7.48%
rRMSE of power density in MOX assembly (%)	0.09%	5.99%	0.10%	21.14%

4. MONTE CARLO DEPLETION WITH CRITICAL SPECTRUM

4.1 Albedo Eigenvalue Problem

In conventional deterministic lattice physics/depletion codes, e.g., CASMO [27] or HELIOS [28], a leakage-corrected critical spectrum is used for the depletion of burnable nuclides and the buildup of fission products. The critical spectrum is obtained by B_1 (linearly anisotropic scattering) buckling search based on multi-group, homogenized medium asymptotic spectrum (under space-energy separability assumption). In addition, the deterministic lattice physics codes have limitations in treating complex geometry and continuous-energy.

The Monte Carlo method could overcome these limitations, but the Monte Carlo depletion codes currently in use such as the MONTEBURNS code [29] do not have such a feature of the leakage-corrected critical spectrum [30]. Instead they use no-leakage critical spectrum. It is noted though that in Ref. [31] a single-assembly is homogenized by Monte Carlo calculation with net-current zero boundary condition and then B_1 buckling search is performed to provide depletion spectrum.

In [32-34], the authors introduced an approach to Monte Carlo depletion with leakage-corrected critical spectrum, in which the streaming (leakage) term of the transport equation is treated as an eigenvalue term and transformed into a specialized albedo boundary condition problem.

Now, if Boltzmann transport equation is integrated over angular and spatial variables in a volume, the first term (leakage term) becomes:

$$\begin{aligned} \int_V \nabla \cdot \vec{\Omega} \phi(\vec{r}, \vec{\Omega}, E) d\vec{\Omega} dV \\ = \int_{\Gamma} \vec{n} \cdot \int_{4\pi} d\vec{\Omega} \vec{\Omega} \phi(\vec{r}, \vec{\Omega}, E) d\Gamma \\ = \int_{\Gamma} [J^+(\vec{r}, E) - J^-(\vec{r}, E)] d\Gamma \\ = (1 - \alpha) \int_{\Gamma} J^+(\vec{r}, E) d\Gamma. \end{aligned} \quad (4.1)$$

Thus, we note that a k_{eff} -eigenvalue problem can be changed to a $(1-\alpha)$ -eigenvalue problem for an assembly to be depleted. Eq. (4.1) also indicates that α should be unity in critical ($k_{eff}=1$) lattice problem, while $\alpha < 1$ in supercritical ($k_{eff} > 1$) lattice problem to achieve criticality ($k_{eff}=1$) and vice versa. By varying albedo α , we can make in-leakage or out-leakage and obtain critical ($k_{eff}=1$) spectrum. This is depicted in Fig. 13.

In our approach, the MCNP5 code is wrapped around by the secant method. Thus, the MCNP5 code is modified to solve the following albedo boundary balance equation for eigenvalue $k_{eff,i}$ with given albedo α_i by adjusting the reflected Monte Carlo particle weight by α_i :

Fig. 13. Monte Carlo Reflects a Neutron with Weight α at a Surface

$$\begin{aligned} \vec{\Omega} \cdot \phi(\vec{r}, \vec{\Omega}, E) + \sigma_t(\vec{r}, E) \phi(\vec{r}, \vec{\Omega}, E) \\ = \int d\vec{\Omega}' \int dE' \sigma_s(\vec{r}, E' \rightarrow E, \vec{\Omega}', \vec{\Omega}) \phi(\vec{r}, \vec{\Omega}', E') \\ + \frac{1}{k_{eff,i}} \frac{\chi(E)}{4\pi} \int d\vec{\Omega}' \int dE' \nu \sigma_f(\vec{r}, E') \phi(\vec{r}, \vec{\Omega}', E'), \end{aligned} \quad (4.2a)$$

$$\begin{aligned} \phi(\vec{r}, \vec{\Omega}', E) = \alpha_i \phi(\vec{r}, \vec{\Omega}, E), \text{ for } \vec{n} \cdot \vec{\Omega}' \\ = -\vec{n} \cdot \vec{\Omega} < 0, \vec{r} \in \Gamma, \end{aligned} \quad (4.2b)$$

where i is the iteration index of the secant method. Then, α is updated until k_{eff} becomes 1.0 as follows:

$$\alpha_{i+1} = \alpha_i - \frac{k_{eff,i}(\alpha_{i-1} - \alpha_i)}{k_{eff,i-1} - k_{eff,i}}, \quad (4.3)$$

where $\alpha_1=1.0$ and $\alpha_2=0.5$ are used for the initial values.

4.2 Test Results

The critical whole core (based on the SMART reactor [35]) shown in Fig. 14 is considered [36]. In the reference Monte Carlo whole-core calculation, assembly-wise neutron spectra are obtained for C1 and C2 assemblies to assess the albedo search method and the B_1 buckling search method. Other calculational conditions are shown in Table 8.

The neutron spectra of C1 assembly are shown in Fig. 15 while their differences against reference assembly spectrum are shown in Fig. 16. Corresponding results for C2 assembly are shown in Figs. 17 and 18.

The spectra seen by a critical pin (P1 pin) and peripheral pin (P2 pin) in Fig. 19 are shown in Figs. 20 through 23.

Compared to the B_1 search method, the leakage-corrected albedo search method shows better results in critical spectrum for C1 assembly because there is no space-energy separation. However, in C2 assembly (peripheral

Fig. 14. A Whole-core Reference Model

Fig. 15. Neutron Spectra of C1 Assembly

Fig. 16. Differences in Neutron Spectra of C1 Assembly

Fig. 17. Neutron Spectra of C2 Assembly

Table 8. Computational Condition for Monte Carlo Depletion

	B_1	Leakage-corrected albedo	Reference whole core
Cross-section library	ENDF/B-VI.6 continuous-energy library		
Histories per generation	50,000	50,000	250,000
Inactive generations	100	1200*	100
Active generations	600	600	600

* including albedo-search procedure

Fig. 18. Differences in Neutron Spectra of C2 Assembly

assembly whose two interfaces are surrounded by reflector), both methods show poor results due to the lack of dissimilar boundary information. However, in the leakage-corrected

albedo search method, incorporation of the dissimilar boundary information may be possible in an additional albedo search.

Fig. 19. C1 Assembly Model and pin Locations

Fig.22. Neutron Spectra of P2 pin with Normalization in Assembly Power

Fig.20. Neutron Spectra of P1 Pin with Normalization in Assembly Power

Fig.23 Differences in Neutron Spectra of P2 pin with Normalization in Assembly Power

Fig.21. Differences in Neutron Spectra of P1 pin with Normalization in assembly power

In the C1 assembly, we observe significant differences in the spectra seen by the pins depending on their locations. So it is expected that their depletion characteristics will be also significantly different.

5. CONCLUDING REMARKS

This review article discussed three outstanding problem areas in nuclear reactor analysis currently practiced in design, with proposed approaches to resolve them.

The outstanding problems considered important are i) the multi-group approximations traditionally used in the transport equations, ii) the isolated single-assembly calculation for homogenization and few-group condensation, and iii) the lack of critical spectrum in Monte Carlo depletion.

The angle-dependent total cross section in the few-group transport equation, that results from equivalent group condensation, renders the group structure (number and boundaries) of the few groups unimportant. In other words, the number of few groups can be small as long as the group condensation is done *equivalently* (resulting in angle-dependent total cross section). This has been borne out in the fast reactor core-reflector interface problem.

The half-node overlapping local/global (OLG) iteration framework proposed in this article could play a key role in removing the limitations both of i) and ii) above. The OLG iteration framework would provide whole-core transport solutions with reasonable amount of computing resources. It does not require reconstruction process to provide detailed heterogeneous flux distributions; it requires only editing of the local problem solutions. Moreover, local problems are naturally suited for parallel computation. For a slowly converging problem, the local problem computations could be confined, in a later stage of local/global iterations, only on nodes that are slow in convergence. The work in [9] can be slightly modified such that the local Monte Carlo problems are embedded in a global problem of a few-group transport model based on the p-CMFD methodology, and this will be reported elsewhere [37].

With regard to iii), the approach of albedo eigenvalue problem enables Monte Carlo depletion with leakage-corrected critical spectrum based on neutron transport first principles (without relying on ad-hoc assumptions such as those involved in the B_1 buckling search). The preliminary results reported here are encouraging and further refinements would be worthwhile to pursue.

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