Design of a Mega-pixel Grade Catadioptric Panoramic Lens with the Rectilinear Projection Scheme

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A dedicated catadioptric panoramic lens with the rectilinear projection scheme is designed. The resolution is high enough to provide separate VGA quality images for the respective four directions and the image distortion is less than 0.5%.

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I. INTRODUCTION

A catadioptric panoramic lens is a useful device for monitoring the full 360° surroundings using a single

camera [1-8]. As schematically shown in Fig. 1, a catadioptric panoramic lens is composed of a catoptric (i.e., mirror) lens and a dioptric (refractive) lens. The rotational symmetry axis (the z-axis in the figure) of the panoramic mirror coincides with the optical axis of the camera. We will designate the ray before the reflection at the panoramic mirror as the incident ray and the ray after the reflection at the mirror as the reflected ray. The angle δ of the *incident ray* is measured as an altitude angle and the angle θ of the *reflected ray* is measured as a zenith angle.

The shape of the mirror and the distance between the panoramic mirror and the refractive lens is such that an incident ray from an object point P having an altitude angle δ is reflected at a point M on the panoramic mirror and passes through the nodal point N of the refractive lens with a predetermined zenith angle θ . The nodal point is the location of the pinhole in the pinhole camera model. Because of the geometrical construction of the catadioptric panoramic lens, scenes from full 360° surroundings are captured within an annular region in the image sensor I as schematically shown in Fig. 2. This raw panoramic image can be unwarped using a simple polar-to-rectangular transformation as is shown in Fig. 3a. Figure 3b shows details of the unwarped image. The image quality of the unwarped panoramic image can be comparable to that of the raw panoramic image when a bi-linear interpolation algorithm is used for the unwarping.

Referring to the Fig. 1, an exemplary object point P is at a horizontal distance D and a vertical height

FIG. 1. Schematic diagram illustrating the operational principle of a catadioptric panoramic lens.

H from the nodal point N. Then for a vertically extended object to appear distortion-free in the unwarped panoramic image, the pixel distance ξ should be a linear function of the object point height H. In our previous study, we have derived a semi-analytical formula for the panoramic mirror profile that yields a practically distortion-free panoramic image given as;

$$r(\theta) = r(\theta_i) \exp \left[\int_{\theta_i}^{\theta} \frac{\sin \theta' + \cot \phi \cos \theta'}{\cos \theta' - \cot \phi \sin \theta'} d\theta' \right], \quad (1)$$

where $r(\theta)$ is the radial distance from the nodal point N to the point of reflection M, θ_i is the lower bound

of the indefinite integral, and ϕ is the zenith angle of the tangent plane T to the mirror at M [8,9]. Then, the law of specular reflections states that the inclination angle ϕ is the arithmetic average of the zenith angles of the incident and the reflected rays as given by

$$\phi = \frac{\theta + \left(\frac{\pi}{2} - \delta \right)}{2}. \quad (2)$$

If the angular range of the incident ray is between δ_1 and δ_2 (i.e., $\delta_1 \leq \delta \leq \delta_2$), and the corresponding range of the reflected ray is between θ_1 and θ_2 (i.e., $\theta_1 \leq \theta \leq \theta_2$), then the incidence angle δ in the rectilinear projection scheme is given as a function of the reflection angle θ as;

$$\delta(\theta) = \tan^{-1} \left[\frac{(\tan \delta_2 - \tan \delta_1)}{(\tan \theta_2 - \tan \theta_1)} (\tan \theta - \tan \theta_1) + \tan \delta_1 \right]. \quad (3)$$

Although not limited to, the lower bound θ_i of the indefinite integral in Eq. (1) is usually chosen between θ_1 and θ_2 . The location of the mirror point M can also be given in a cylindrical coordinate as a set of coordinate pairs (ρ, z) , where z is the height measured along the symmetry axis and ρ is the axial radius (i.e., horizontal distance measured perpendicular to the rotational symmetry axis). The variables in the two coordinate systems are related to each other by simple geometrical relations given as $z(\theta) = r(\theta) \cos \theta$ and $\rho(\theta) = r(\theta) \sin \theta$. When the maximum mirror radius $\rho_{\max} \equiv \rho_2$ is small compared to the object distance D - which is true for the usual settings of panoramic image acquisition -, then the image distortion is negligible and all the objects in the unwarped panoramic image will have correct proportions.

In our previous study, we have designed and fabricated a panoramic mirror with the rectilinear projection scheme [9,10]. Unlike other panoramic mirrors based on hyperbolic mirror profiles, our panoramic mirror has negligible distortion and the vertical field of view (VFOV) can be arbitrarily chosen. This catoptric lens could be used with a digital camera to capture a panoramic photograph of a scenic place or could be used with a video camera for security/surveillance or web-based virtual touring. However, the hybrid lens composed of a panoramic mirror and a dioptric lens is bulky, heavy, and mechanically unprotected. The purpose of the present article is to report the result of a further study conducted with a view of developing a compact and standalone catadioptric panoramic lens with a mega-pixel grade resolution.

II. IMAGE SENSOR REQUIREMENTS

In a multiple-viewpoint panoramic imaging systems, two or more cameras are simultaneously employed, and

FIG. 2. Illustrative raw panoramic image. The horizontal line represents a reference line for the unwarping.

(a)

(b)

FIG. 3. (a) Unwarped panoramic image obtained by applying a simple trigonometric transformation to the raw panoramic image in Fig. 2, (b) Details of Fig. 3a.

all the individual images are seamlessly joined together to yield a panoramic image. When two cameras are used, then each camera must be equipped with a fisheye lens having a FOV larger than 180° . Since such a fisheye lens is overly expensive, a rather common multiple-viewpoint panoramic imaging system is composed of four cameras and each camera is equipped with a wide-angle lens with a FOV larger than 90° .

These multiple-viewpoint panoramic imaging systems have two drawbacks. First of all, all the horizontal directions are not equally treated. Let's assume that the four cameras are pointing to the North, South, East and West, individually. Then the resulting panoramic image is more or less a scene of a right-rectangular room where each wall has an image of the chosen direction. In other words, it is scene of the North joined by scenes of the East and the West on their respective directions, and etc. Therefore if a region of interest (ROI) lies at the North-East direction, for example, then the view of the ROI is actually a half of Northern view and a half of Eastern view. Therefore the hybrid view in the Northeast direction cannot be as natural as that in the North or the East direction. If the object is near the panoramic imaging system, then the problem aggravates and the Northeast view can be unnatural in that a split image is obtained. In other words, the image of an object lying in the Northeast direction can be similar to the image of an object seen through a broken looking glass.

Secondly, although a miniature size multiple-viewpoint camera is readily available, it is difficult to expect that the size of the imaging system can be further reduced dramatically. Furthermore, it takes time and resources to join four separate images into one panoramic image. Hardware image processing can expedite the process, but there are always applications where such an image processing cannot match up in speed, such as mobile robots, telematics and military applications.

Despite the above-mentioned demerits of the multiple-viewpoint panoramic imaging system, it had one distinctive advantage over the catadioptric panoramic imaging system. It is the image size problem. The most common security camera has a VGA resolution. In a VGA-grade camera, the image sensor is 640 pixels wide and 480 pixels high, totaling 307,200 pixels. Therefore if four different VGA-grade cameras were employed in multiple-viewpoint panoramic imaging system, the resulting panoramic image can be as big as 1.2 million pixels, and object – such as the face of an intruder – can be recognized fairly well. On the other hand, until now, the resolution of the catadioptric panoramic imaging system has been rather low, and often it was difficult to recognize a face in the panoramic image.

Figure 4 shows a schematic diagram of an image sensor, which can yield a panoramic image comparable to that of the multiple-viewpoint panoramic imaging

FIG. 4. Schematic diagram illustrating a square image sensor having just enough pixels to provide four separate VGA-grade images for the respective four directions.

system. In a catadioptric panoramic imaging system, a raw panoramic image is obtained within an annular region. Because of the annular shape of the raw panoramic image, the region near the inner radius is lower in horizontal resolution than the region near the outer radius. Therefore, if the unwarped panoramic image must match up with that of the multiple-viewpoint panoramic imaging system having four separate VGA-grade cameras, then the perimeter of the inner circle must be four times the width W of a VGA-grade image. Therefore $2\pi r$ must equal $4W$, where r is the radius of the inner circle. As a result, the radius of the inner circle must be given as $r = \frac{4W}{2\pi}$ pixels long. Furthermore, the width of the annular region must not be smaller than the height H of a VGA-grade image. Therefore when a square image sensor is employed, then one side of the image sensor should be as wide as $2(r + H) = 2(408 + 480) = 1776$ pixels, and the square image sensor must have at least 3.15 mega-pixels. Such a high-resolution image sensor is difficult to find, slow in speed, and high in price. Considering the fact that a square image sensor is rather a rarity, and width: height ratio of most of the image sensors are 4: 3, the pixel number must be multiplied by $4/3$ in order to obtain comparable panoramic image using standard 4: 3 ratio image sensor. In this case, the image sensor should be 4.2 mega-pixels at the least.

III. DEVICE OUTLINE

From the considerations in the preceding section, it is apparent that we need an image sensor that has

more than 4 mega pixels. One camera model from Lumenera corporation (LW625c) has 6.6 mega-pixel color CMOS sensor. Therefore we decided to develop a catadioptric panoramic lens that matches this camera model. This non-standard 2/3-inch CMOS sensor is 3000 pixels wide, and 2200 pixels high. As schematically shown in Fig. 5, the sensor is 10.5 mm wide and 7.8 mm high, and each pixel is square in shape with one side measuring $3.5 \mu\text{m}$. Because a complete ring must reside within the active area of the image sensor, we divided the image sensor active area such that the outer radius of the annular region is 3.9 mm, and the inner radius is 1.95 mm. In other words, the inner perimeter will be half the size of the outer perimeter.

The panoramic lens will be nearly standard in field of view (FOV), and the vertical field of view of the panoramic lens will be similar to the 46° FOV of an average human. Allowing some safety margin, the VFOV is set as 60° , and the outer and the inner perimeter will correspond to the 30° above and below the horizon,

FIG. 5. Image sensor layout for the panoramic lens design.

FIG. 6. Required field of view of the refractive lens determined by the image sensor size and the focal length of the refractive lens.

respectively.

Figure 6 illustrates the relation between the image sensor size and the required field of view of the refractive lens (not the panoramic lens). The distance between the rear nodal point of the refractive lens and the image sensor is close to the nominal focal length of the refractive lens. Comparing with Fig. 5, we could induce that the field of view θ_v of the refractive lens is given as

$$\theta_v = \tan^{-1}\left(\frac{H}{2f}\right). \quad (4)$$

To have this angle within a reasonable range, the focal length of the refractive lens has been tentatively chosen as 8.0 mm, and θ_v is given as 25.989° . As shown in Fig. 7, the minimum distance between the front nodal point of the refractive lens and the mirror is chosen as 40 mm. The larger this number becomes, the easier it is to design a satisfactory refractive lens, but the mirror also becomes larger, and the overall lens might end up too large to keep its value as a commercial product. This number has been chosen as a compromise

FIG. 7. Optical system layout.

FIG. 8. The altitude angle δ of the incident rays as a function of the zenith angle θ of the reflected rays.

between these two requirements.

Figure 8 shows the altitude angle of the incident rays as a function of the zenith angle of the reflected rays. This graph is obtained using Eq. (3) for the chosen design parameters. From the Fig. 1, we can see that a reflected ray having a zenith angle θ will hit a pixel

FIG. 9. The altitude angle δ of the incident ray and their desired image height ξ .

FIG. 10. The designed panoramic mirror profile.

FIG. 11. The mirror profile and the trajectories of the principal rays.

in the image sensor that is x mm away from the center as given by

$$\xi = f \tan \theta. \quad (5)$$

Using Eq. (3) and Eq. (5), we can assign each pixel with a unique altitude angle of the incident ray. Fig. 9 shows this relation. From the graph, we can see that the image center correspond to -60° , the inner radius ($\rho = 1.95$ mm) and the outer radius ($\rho = 3.9$ mm) correspond to -30° and $+30^\circ$, respectively. Finally, Fig. 10 shows the mirror profile obtained using Eqs. (1-3). Plotted in the graph is the relative height of the mirror measured from the bottom of the mirror (i.e., $h(\rho) = z(\rho) - \min(z(\rho))$). The mirror is approximately 55mm in diameter. Fig. 11 shows the traces of the principal rays. Since this mirror follows the rectilinear projection scheme, if the horizontal intervals between adjacent reflected rays are uniform, then vertical intervals between adjacent incident rays are also uniform.

IV. LENS DESIGN

To design a dedicated refractive lens, the panoramic mirror profile should be expressed in a form that can be recognized by lens design software such as Code V or Zemax. From the mirror profile shown in Fig. 10, it must be clear that this mirror profile cannot be written in the standard even aspheric lens formula. A more general form of the even aspheric lens formula is the odd aspheric lens formula, where odd powers of ρ are allowed as is given in Eq. (6).

$$h(\rho) = \frac{\frac{\rho}{R^2}}{1 + \sqrt{1 - (1+k)\left(\frac{\rho}{R}\right)^2}} + \sum_{i=1}^{\infty} C_i \rho^i. \quad (6)$$

The symbols R and k stand for the radius of curvature at the vertex and the conic constant, respectively. By setting the radius of curvature equal to infinity, this odd aspheric lens formula is reduced to a polynomial in ρ . To use this form of the aspherical lens profile, the mirror profile is fitted to an 8th-order polynomial in ρ as given in Eq. (7).

$$h(\rho) = \sum_{i=1}^8 C_i \rho^i. \quad (7)$$

This polynomial should pass through the origin, and therefore the constant term (i.e., C_0) has been omitted. The best set of coefficients is obtained using the generalized linear least squares algorithm. Table 1 shows the fitting coefficients and Fig. 12 shows the residual error between the actual mirror profile and the fitted profile given by Eq. (7). The maximum axial

radius $\rho_{\max}$ of the panoramic mirror is given as $r(\theta_2)\sin\theta_2$ and the diameter of the panoramic mirror is given as $2\rho_{\max} \cong 54.97$ mm. As can be seen, the error is less than $1\ \mu\text{m}$ over the whole active area of the mirror surface. Since the mirror is relatively far away from the refractive lens, this error shouldn't cause any problem.

The dedicated refractive lens has been designed using standard procedure [11]. Figure 13 shows the optical layout of the optimized catadioptric panoramic lens.

TABLE 1. Fitting coefficients of the panoramic mirror profile to an 8th order polynomial in ρ .

$\rho_{\max}$	27.48557
C_1	2.691686e-001
C_2	-5.331967e-005
C_3	7.397843e-004
C_4	-7.496730e-005
C_5	7.656002e-006
C_6	-3.472886e-007
C_7	7.045602e-009
C_8	-5.344482e-011

TABLE 2. Optimized lens prescriptions.

Surface no.	Surface type	Comment	Radius	Thickness	Index	Abbe number	Glass	Semi-Diameter
Obj	Std.		1.0e10	1.0e10				∞
1	Odd asp.	MIRROR	∞	22.658			MIRROR	29.999
2	Even asp.	Lens A	14.602	7.994	1.4918	57.44	PMMA	14.076
3	Std.		27.625	8.538				11.815
4	Std.	Lens B	10.788	2.606	1.7234	37.96	S-BAH28	7.593
5	Std.		7.397	8.271				6.133
6	Std.	Lens C	-6.703	2.070	1.7015	41.24	S-BAH27	3.986
7	Std.		41.471	4.313				4.015
8	Std.	Lens D	-8.000	4.501	1.6210	35.88	TIM11	4.256
9	Std.		-8.373	19.302				5.111
10	Std.	Lens E	16.311	1.907	1.6259	35.70	PBM1	5.123
11	Std.		12.989	7.001				4.926
stop	Std.	IRIS	∞	2.000				4.150
13	Std.	Lens F	21.069	1.500	1.6210	35.88	PBM11	5.627
14	Std.	Lens G	14.864	4.814	1.6400	60.08	S-BSM81	5.674
15	Std.		-56.177	2.301				5.761
16	Std.	Lens H	-203.658	2.390	1.6180	63.33	S-PHM52	5.718
17	Std.	Lens I	-17.961	2.533	1.6727	32.10	PBM25	5.715
18	Std.		-56.622	2.787				5.734
19	Std.	Lens J	29.428	3.499	1.4875	70.24	S-FSL5	5.584
20	Std.		-650.896	3.896				5.328
21	Std.	Lens K	-16.366	2.600	1.6200	36.26	PBM2	4.898
22	Std.		-80.886	31.496				4.950
IMA	Std.	CMOS	∞					3.904

Table 2 shows the lens prescriptions for this design. It was difficult to design a satisfactory lens with spherical lenses only. Therefore we added one aspherical lens (lens A) just below the mirror. Also, we allowed the mirror coefficients to vary at the last stage of the lens

FIG. 12. The residual error between the designed mirror profile and its best fit to an 8th order polynomial in ρ .

design. Table 3 summarizes the modified fitting coefficients for the optimized mirror. The lens A just below the mirror is made of PMMA, and the front surface is an even aspherical surface, given as;

$$h(\rho) = \frac{\frac{\rho^2}{R}}{1 + \sqrt{1 - (1+k)\left(\frac{\rho}{R}\right)^2}} + A\rho^4 + B\rho^6 + C\rho^8 + D\rho^{10}. \quad (8)$$

In Eq. (8), A, B, C and D are the 4th, 6th, 8th and 10th deformation coefficients. Table 4 shows the set of coefficients for this even aspherical surface. Figure 14 compares the original mirror profile (solid line) with the modified mirror profile (round dot). It can be seen that the overall profile is not significantly different.

As a whole, the catadioptric panoramic lens is com-

FIG. 13. The final optical layout of the optimized catadioptric panoramic lens.

TABLE 3. Modified fitting coefficients of the panoramic mirror profile.

$\rho_{\max}$	29.9994
C_1	-2.8409849e-001
C_2	1.6244699e-004
C_3	-7.4683688e-004
C_4	7.5733631e-005
C_5	-7.6955090e-006
C_6	3.5691724e-007
C_7	-7.4449040e-009
C_8	5.8304787e-011

TABLE 4. Fitting coefficients of the front surface of the PMMA lens to an even aspheric lens formula.

variable	value
R	14.60246
k	-0.3509195
A	-5.7742051e-005
B	9.5188809e-008
C	7.3261880e-011
D	-2.5358094e-012

posed of one aspherical mirror, one aspherical lens and ten spherical lenses. Stop radius is manually set as 4.15 mm and the resulting F# is 5.53. All the spherical lenses are chosen from the Ohara glasses. To allow an adjustable iris, an ample space has been assigned around the iris, and the distance between the last lens element and image sensor is well over 2.0 cm. Therefore, this lens can be made into a C-mount lens without causing any packaging problem.

Separate from the vertical field of view of the panoramic mirror, the completed panoramic lens has its own vertical field of view. The VFOV of the mirror is $\pm 30^\circ$. However, the VFOV of the optimized lens is from -40° to $+20^\circ$.

Figure 15 shows the MTF plot. The pixel of the chosen camera has $3.5 \mu\text{m}$ square pixel, and therefore the Nyquist frequency is $1/(2 \times 3.5 \mu\text{m}) = 143 \text{lp/mm}$. Therefore, the MTF at 150lp/mm has been considered. It can be seen that the MTF is over 0.2 over most of the active area of the image sensor. Figure 16 shows the spot diagram.

Distortion cannot be calculated in the usual way. Figure 17 illustrates the scheme of the distortion calculation. In our rectilinear panoramic mirror, the tangent of the altitude angle δ of the incident ray is proportional to the image height ξ . The range of the altitude angles of the incident rays is from $\delta_1 = -40^\circ$ to $\delta_2 = +20^\circ$, and these rays form an annular panoramic

FIG. 14. The original (solid line) and the modified (round dot) mirror profiles.

FIG. 15. The MTF plot.

FIG. 16. The spot diagram.

FIG. 17. The scheme for determining the distortion of the panoramic lens.

image with an inner radius $\xi_1 = 1.9$ mm and an outer radius $\xi_2 = 3.9$ mm. For every incident ray having an altitude angle δ within the valid range between δ_1 and δ_2 , a unique image height ξ is assigned according to the Eq. (9).

$$\xi(\delta) = \xi_1 + \left(\frac{\xi_2 - \xi_1}{\tan \delta_2 - \tan \delta_1} \right) (\tan \delta - \tan \delta_1). \quad (9)$$

FIG. 18. Image distortion as a function of the altitude angle.

This is used as a target in the merit function. On the other hand, the actual image height x of a ray having an altitude angle δ is given as $x(\delta)$. Then the distortion is calculated as

$$\text{error}(\delta) = \frac{\xi(\delta) - x(\delta)}{\xi(\delta)} \times 100\%. \quad (10)$$

Figure 18 shows the distortion calculated according to Eq. (10). The distortion is less than 0.5%. It is said that if distortion is less than 4%, then it is impossible for an unaided eye to notice an image distortion. Therefore the distortion characteristics are fair [12].

The anomaly in the middle corresponds to the horizon with zero altitude angle. Although not significant, this anomaly reflects the singularity the usual ray tracing algorithm commonly possesses.

V. CONCLUSION

In conclusion, we have designed a dedicated catadioptric panoramic lens with the rectilinear projection scheme. This panoramic lens will provide a raw panoramic image that only needs a simple trigonometric transformation to yield a perspectively normal panoramic image. The panoramic image will be rich in detail as to provide separate VGA quality images for the respective four directions.

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