Analysis of Off-axis Integral Floating System Using Concave Mirror

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An off-axis integral floating system using a concave mirror is analyzed to resolve the image distortion incurred by the off-axis optical arrangement. The concave mirror can be adopted as the floating device to improve the optical efficiency. The image distortion due to the tilting axis of the concave mirror needs to be analyzed precisely to generate the pre-distortion image. In this paper, we calculate the image deformation in the off-axis structure of the concave mirror using the geometrical optics. Using the calculation results, the compensated elemental image can be generated for the pre-distortion integrated image, which can be projected to the floating 3D image without image distortion. The basic experiments of the off-axis integral floating are presented to prove and verify the proposal.

Keywords : Integral floating, Off-axis structure, Pre-distortion, Integral imaging

OCIS codes : (100.6890) Three-dimensional image processing; (110.2990) Image formation theory

I. INTRODUCTION

Integral imaging is one of the emerging auto-stereoscopic three-dimensional (3D) display techniques, which reconstructs a 3D image to integrate the elemental image through a lens array or pin-hole array [1-4]. The elemental image is composed of the arrayed 2D images which have the directional information of the 3D object. The integral imaging system can express the full parallax and the various depth 3D images which, however, have the limitations of the expressible depth range and the viewing angle. Recently, many techniques have been researched and reported to enhance the 3D image quality of the integral imaging display [5-11]. Moreover, many integral-imaging-based techniques have also been studied to inherit the merits of the integral imaging.

Integral floating is one of the techniques based on the integral imaging [12-17]. The integral floating employs the additional large aperture floating device in front of the integral imaging system, which can enhance the 3D image quality such as the improved depth range, the reduced seam noises from the lens array, and the increased viewing area. However, the general large aperture device is difficult to

manufacture and often results in the image distortion.

The Fresnel lens, which is composed of the tiny concentric circular prisms that refract the parallel rays to a focal point, is generally used as the floating device in the integral floating system because it induces less aberration compared with the general spherical lens. However, the Fresnel lens cannot be perfectly free of the aberrations [18-19] and because of its circular prism patterns, circular seam noise and optical loss are inevitable, causing the degradation of the image quality [5]. The concave mirror which is optically the same with the convex lens can be used as the floating device to reduce the disadvantages of the Fresnel lens and to improve the floating image quality. However, using the concave mirror as the floating device incurs the problem of the on-axis system configuration, which means the object and the observer are located on the same side of the mirror, because the mirror switches the direction of the optical axis. Therefore, the object space and the image space of the concave mirror are overlapped in the on-axis floating system as shown in Fig. 1 (a). In this case, it is impossible to compose the floating system without a beam splitter. Due to the characteristics of the beam splitter, the optical efficiency of the system should be decreased to less than

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FIG. 1. Integral floating system using concave mirror - (a) Conventional system and (b) proposed system.

50%. Furthermore, the system using the beam splitter should be complicated and bulky [17].

The off-axis integral floating where the optical axis of system is tilted from that of the mirror can be a solution of the problems mentioned above as shown in Fig. 1 (b). The off-axis integral floating system can accomplish the relatively simple system configuration with high optical efficiency without the seam noises from the Fresnel lens patterns. But the off-axis integral floating causes the image deformation which is resulted from the optical aberrations. Therefore, the pre-distortion image is needed to reconstruct a correct 3D floated image. In this paper, we analyze and simulate the image deformation due to the tilted angle between the optical axis of the system and the mirror. Using the simulation data, we can make the pre-distortion image which can be floated as the correct image. For the off-axis integral floating system, the compensated elemental image is generated for the pre-distortion integrated image. The experimental results of the image deformation analysis and the off-axis integral floating are presented to prove the feasibility of the proposal.

II. ANALYSIS OF IMAGE DISTORTION

The monochromatic optical aberrations can be classified into the five cases; the spherical aberration, the stigmatism,

FIG. 2. Diagram of calculation for image distortion point Q.

FIG. 3. (a) Experiment for image distortion of a 35mm cube, (b) experimental result and (c) simulation result.

the coma, the field curvature, and the distortion. The former three cases are the focusing error, which means the rays cannot converge on the focal point or the image point because of the surface curvature and the tilting axis, and can influence the image definition. The other two cases are the error about the image position due to the tilting axis and the chief rays. These are about the image deformation and must be considered in the off-axis image process. We start with the simulation of the image deformation due to the tilting angle using the simple geometrical optics.

Figure 2 shows the simulation for the image distortion. Firstly, we select two rays which start to travel from one source point P of the object and have the sufficiently small variations of the angle from the center ray from the source point and travels toward the center point of the spherical concave mirror. And then we find the intersection point of the two reflection rays which is the image point Q. Applying this process to the other points of the object, we can get the whole reflection image of the object. Fig. 3 shows the comparison of the image of the real object and the simulation results. The pattern of simulation results is coincident with the real image distortion.

Based on the simulation of the image distortion, we can make the process to resolve the image distortion because the process of the image formation is reversible. In other

words, we can find the pre-distortion image which forms the correct image through the concave mirror by performing the reverse process of the simulation for the image distortion, so that the pre-distortion image can compensate the image distortion.

III. PRE-DISTORTION PROCESS

In this chapter, we deal with the pre-distortion process to resolve the image distortion, which means the sequential operations to generate the compensated image. In order to find the pre-distortion image, we assume that an observer's eye sees the point P_o which is one point of the correct 3D image as shown in Fig. 4. Then, we can find the pre-distortion point Q_o which converges to the point P_o after the reflection by the concave mirror. The point Q_o can be calculated through finding the chief ray that expresses the viewing direction and applying the reverse method of the simulation for the image distortion.

The chief ray is determined by the point P_o and the position of the observer's eye. The reflection point C_o can be calculated by solving the equations of the chief ray and the concave mirror. To make the calculation simple, the points P_o and C_o should be projected onto the z - x plane. The distance from the point O to the point P_o , $\overline{OP_o}$, and the angle between $\overline{OP_o}$ and the optical axis of the concave mirror, θ_p , are given as follows;

$$\overline{OP_o} = \sqrt{p_{o1}^2 + p_{o2}^2 + p_{o3}^2}, \quad (1)$$

$$\begin{aligned} \theta_p &= \cos^{-1} \left(\frac{\overline{OP_o} \cdot \overline{C_oO}}{|\overline{OP_o}| |\overline{C_oO}|} \right) \\ &= \cos^{-1} \left(\frac{-(p_{o1}c_{o1} + p_{o2}c_{o2} + p_{o3}c_{o3})}{\sqrt{p_{o1}^2 + p_{o2}^2 + p_{o3}^2} \sqrt{c_{o1}^2 + c_{o2}^2 + c_{o3}^2}} \right). \end{aligned} \quad (2)$$

FIG. 4. Diagram of obtaining pre-distortion image point Q_o .

When the point C_o is projected to the point $C(0, 0, -R)$, where R is radius of the concave mirror. The x , y , and z coordinates of the point P , p_1 , p_2 , and p_3 , are given as;

$$p_1 = \overline{OP_o} \sin \theta_p, \quad (3)$$

$$p_2 = 0, \quad (4)$$

$$p_3 = \overline{OP_o} \cos \theta_p. \quad (5)$$

And then, we should calculate the pre-distortion point Q which is the concave mirror image of the point P . Fig. 5 shows the process for calculating the point Q . We select the two rays very near the chief ray, l_B and l_D , which start from the point P . When the reflection rays of l_B and l_D are defined as l_B' and l_D' , the point Q can be obtained by solving the equations of l_B' and l_D' . The point $Q(q_1, q_2, q_3)$ is given by

$$q_1 = (q_3 - b_3) \tan \varphi_B + b_1, \quad (6)$$

$$q_2 = 0, \quad (7)$$

$$q_3 = \frac{b_3 \tan \varphi_B - d_3 \tan \varphi_D + d_1 - b_1}{\tan \varphi_B - \tan \varphi_D}. \quad (8)$$

Because the point Q is the pre-distortion point just on the z - x plane, we should project the point Q onto the original image plane, and then the real pre-distortion point Q_o can be derived. This projection process can be performed by the three linear equations derived by the location relationships among the points on the original image plane and z - x plane. Because the point Q_o is on the original image plane, the first equation of the simultaneous equations is derived by finding the plane equation of the original image plane

FIG. 5. Diagram of calculation process in z - x plane for pre-distortion image point Q .

$$\begin{aligned}
\vec{N} &= \vec{OP}_0 \times \vec{OC}_0 = n_1\vec{x} + n_2\vec{y} + n_3\vec{z} \\
n_1 &= p_{02}c_{03} - p_{03}c_{02}, \\
n_2 &= p_{03}c_{01} - p_{01}c_{03}, \\
n_3 &= p_{01}c_{02} - p_{02}c_{01}, \\
n_1q_{01} + n_2q_{02} + n_3q_{03} &= 0,
\end{aligned} \tag{9}$$

where the vector $\vec{N}$ is a normal vector of the original image plane. The other two equations can be derived by two scalar production;

$$\begin{aligned}
\vec{OC}_0 \cdot \vec{OQ}_0 &= \vec{OC} \cdot \vec{OQ}, \\
c_{01}q_{01} + c_{02}q_{02} + c_{03}q_{03} &= -q_3R,
\end{aligned} \tag{10}$$

$$\begin{aligned}
\text{and } \vec{OP}_0 \cdot \vec{OQ}_0 &= \vec{OP} \cdot \vec{OQ}, \\
p_{01}q_{01} + p_{02}q_{02} + p_{03}q_{03} &= p_1q_1 + p_2q_2 + p_3q_3.
\end{aligned} \tag{11}$$

Finally, the pre-distortion point Q_0 can be obtained by solving the three equations, (9), (10), and (11) as follows;

$$D = n_1(c_{02}p_{03} - c_{03}p_{02}) + n_2(c_{03}p_{01} - c_{01}p_{03}) + n_3(c_{01}p_{02} - c_{02}p_{01}),$$

$$q_{01} = [q_3R(n_2p_{03} - n_3p_{02}) + (p_1q_1 + p_3q_3)(n_2c_{03} - n_3c_{02})] / D, \tag{12}$$

$$q_{02} = [q_3R(n_3p_{01} - n_1p_{03}) + (p_1q_1 + p_3q_3)(n_3c_{01} - n_1c_{03})] / D, \tag{13}$$

$$q_{03} = [q_3R(n_1p_{02} - n_2p_{01}) + (p_1q_1 + p_3q_3)(n_1c_{02} - n_2c_{01})] / D. \tag{14}$$

As mentioned above, the whole pre-distortion image for the correct 3D image can be found by adopting this method to the other points of the correct 3D image. The pre-distortion image can be used to generate the compensated elemental image which can be reconstructed to the pre-distortion integrated image.

The obtained pre-distortion image is for only one view of the observer's eye. If the viewing direction of observer is moved, the chief ray should be changed. Therefore, the pre-distortion image should be generated according to the viewing direction of the observer.

IV. RECONSTRUCTION PROCESS

Using the pre-distortion process, we can reconstruct the correct 3D image without the image distortion using the off-axis integral floating system with the concave mirror. The sequential operations of the reconstruction process are as follows; (1) the 3D image to be displayed is assumed and defined as the certain coordinates. (2) The pre-distortion

image coordinates are calculated by the pre-distortion process as described in chapter 3. (3) The compensated elemental image is generated using the pre-distortion image coordinates in consideration of the system specification of the integral imaging system to be used. (4) The pre-distortion integrated image is reconstructed by the defined integral imaging system. (4) Finally, the concave mirror is set up in front of the integral imaging system with the proper location and angle to project the correct 3D floating image.

Through the process, we can achieve the off-axis integral floating system using the concave mirror with the high efficiency and the simple structure and no distortion.

V. EXPERIMENT

In order to prove the compensation effect of the proposed method, the 15° off-axis integral floating system is set up. The 3D image to be displayed is composed of three characters, 'K', 'H', and 'U' whose heights are 35 mm. As shown in Fig. 6 (a), the character 'H' is located on 300 mm along the z-axis and 15° tilted from the axis. The character 'K'

FIG. 6. (a) Experimental condition and (b) front scene of computer-generated 3D image.

and 'U' are located on 30 mm nearer and farther than the character 'H' along the optical axis, respectively. Fig. 6 (b) shows the front scene of the computer-generated three characters used as a 3D object.

Table 1 describes the specifications of the experimental setup such as the lens array, the display device, and the concave mirror. The pickup process of the compensated elemental image is performed by the computational simulation tools. Fig. 7 (a) shows the elemental images of the non-compensated image while Fig. 7 (b) shows the compensated

TABLE 1. Specifications of experimental setup

Lens array	Number of array($h \times v$)	15×15
	Size	5 mm
	Focal length	10 mm
Display device	Diagonal size	2.5 inch
	Resolution	960×640
Concave mirror	Aperture	190 mm
	Radius of curvature	300 mm
	Focal length	150 mm

(a)

(b)

FIG. 7. Elemental images of (a) non-compensated image and its partially magnified image and (b) pre-distortion image and its partially magnified image when off-axis and viewing angle are 15° .

elemental imaging for the pre-distortion integrated image when both angles of the off-axis and the view are the same as 15° .

Figure 8 shows the implemented off-axis integral floating system. And Fig. 9 shows the pre-distortion image reconstructed by the integral imaging system which is part of the off-axis integral floating system. Fig. 10 and 11 represent the experimental results. The images of experimental results can be taken by camera located on the positions of the observer's eyes indicated in Fig. 6 (a). As shown in Fig. 11, the non-compensated 3D image and the compensated 3D image are compared with each other when the angles of the off-axis and of the view are the same as 15° . The character 'K' becomes larger than the other characters due to the lens effect of the concave mirror. Reversely, the character 'U' becomes smaller due to the same reason for

FIG. 8. Experimental setup of off-axis integral floating system.

FIG. 9. Pre-distortion 3D image reconstructed by integral imaging system with elemental image of Fig. 7 (b).

FIG. 10. (a) Non-compensated 3D image and (b) compensated 3D image for 15° of off-axis angle and viewing angle.

FIG. 11. Simulation images for angle of view in (a) 12.5°, (b) 15°, and (c) 17.5° and experimental results for angle of view in (d) 12.5°, (e) 15°, and (f) 17.5°.

the character 'K'. The characters in the non-compensated 3D image are wider as the characters get farther from the z-axis. The characters in the compensated 3D image have mostly uniform sizes compared with the non-compensated 3D image.

Figure 11 shows the results of the compensated 3D images in three different views, 12.5°, 15°, and 17.5°. Fig. 11 (a), (b), and (c) are the simulation results and Fig. 11 (d), (e), and (f) are the experimental results in each view. The simulation results and the experimental results are coincident with each other.

VI. CONCLUSION

The image distortion of the off-axis integral floating system using the concave mirror is analyzed to resolve. In the integral floating system using the concave mirror, the off-axis structure is inevitable, which results in the image distortion. So, we analyze the image distortion and generate the pre-distortion image which compensates the image deformation caused by the tilted angle through the pre-distortion process. And then, we can realize the distortion-free off-axis integral floating system with the high efficiency and the simple structure. The experimental results for one-view are presented in only one each elemental image. However, the free-viewing direction system can be realized by using the head-tracking method or the directionally selective pick-up method for the integral imaging system.

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