Reliability of Strain Estimation on Triangular Network and A Case Study; Deformation of Korea due to 2011 Tohoku Earthquake observed by GPS

Sung-Ho Na¹, Tae Woong Chung^{2*}, Byung-Kyu Choi³, and Sung-Moon Yoo³

¹Korea Space Geodetic Observation Center, Sejong 339-821, Korea (on leave from Ajou University)

²Sejong University, Seoul 143-747, Korea

³Korea Astronomy and Space Science Institute, Daejeon 305-348, Korea

삼각망에서 변형률산출의 신뢰도와 적용례; GPS로 관측된 2011 토호쿠지진에 의한 한반도 변형

나성호¹ · 정태웅^{2*} · 최병규³ · 유성문³

¹한국우주측지관측센터 (아주대 파견)

²세종대학교 에너지자원공학과

³한국천문연구원

Abstract: A stable procedure is presented to attain most probable and unbiased estimate of principal strain, rotation, and dilatation for 2-dimensional geodetic data on triangular network. The proper network size should be chosen carefully, because the errors of these estimates of strain tensor and other associated observables grow inversely proportional to the area of station triangle. As a case study, the deformation observables for the GPS-monitored co-seismic displacement in Korea due to the 2011 Tohoku-Oki earthquake were attained accordingly.

Keywords: Deformation, Unbiased Estimate, Network Size, Stable Procedure, GPS, 2011 Tohoku Earthquake

요 약: 삼각망에서 얻은 2차원 변위자료로부터 주축변형, 회전, 팽창등을 산출하는 안정된 계산순서를 제시하였으며, 이들 추산치의 오차가 삼각망의 면적간에 반비례하므로 자료망의 크기를 택함에 유의하여야함을 지적하였다. 이 순서를, 2011년 토호쿠지진에 의해 발생한 한반도지역의 변위의 GPS관측자료에 적용하여 안정된 추산치들을 얻었다.

주요어: 변형, 최적추정치, 자료망의 크기, 안정된 계산순서, GPS, 2011년 토호쿠지진

Introduction

Most probable and unbiased estimation by imposing the condition of least sum of square error was first developed by Gauss. This concept of least square technique has been widely used in branches of science and engineering dealing with precise measurements. One of the disciplines, which demand frequent application of ‘least square estimation’ or

‘least square adjustment’, is geodetic measurement and analysis. There were studies of both theory and application based on different approaches to derive principal strain from displacement data.

In this report, we summarize a direct and slightly different calculation procedure to attain the most probable and unbiased estimates of strain tensor, principal strain, and rotation for given triangular network measurement. Unlike conventional procedure, in which strain tensor and rotation angle are simultaneously determined (Hamdy 2004; Pietrantonio and Riguzzi, 2004), we use two step procedures – first calculate deformation tensor and then calculate other observables accordingly.

Often one may prefer smaller-spacing network to attain strain pattern in the given area for better resolution. However, we assure that the network triangle size must be taken properly with care. This is due to abrupt increase of uncer-

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*Corresponding author

E-mail: chungtw@sejong.ac.kr

Address: Department of Energy & Mineral Resources, Sejong University, Seoul 143-747, Korea

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tainty in those estimates with decrease of triangle size. Therefore proper size network should be chosen with error assessment. We applied our procedure to the co-seismic GPS displacement in South Korea due to the 2011 Tohoku-Oki Earthquake (M=9.0) and attained the estimate of associated strain field and related.

Gauss-Markov Observation Equation: Brief Review

Method of observation equation for linear model is briefly summarized below. More description on the least square estimation technique is given in Appendix. There are standard materials about the Gauss-Markov observation equation (see for example, Schaffrin (1999), Uotila (1986), Mikhail and Ackermann (1976)). Write the linear model of observation and parameters as

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix} \quad (1)$$

where $y_1, y_2, y_3, \dots, y_n$ are observations with errors as $e_1, e_2, e_3, \dots, e_n$, and $x_1, x_2, \dots, x_m$ are the parameters to be determined ($n > m$). This model can be expressed as $y_i = \sum_j a_{ij}x_j + e_i$ or $Y = AX + \varepsilon$. The variance-covariance matrix for observed quantities is denoted as

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_n^2 \end{bmatrix}$$

where $\sigma_i^2 = \text{var}(y_i)$ and $\sigma_{ij} = \text{cov}(y_i, y_j)$. Forward and inverse relations between Σ_y and Σ_x are given as $\Sigma_y = A\Sigma_x A^T$ and $\Sigma_x = (A^T A)^{-1} A^T \Sigma_y A (A^T A)^{-1}$. By minimizing the weighted sum of square error $S = \varepsilon^T \Sigma^{-1} \varepsilon$, the best unbiased estimate of X is found as

$$\hat{X} = (A^T \Sigma^{-1} A)^{-1} (A^T \Sigma^{-1} Y). \quad (2)$$

Strain Estimation from Displacement of Triangular Array

There have been various studies about both methodology

and application of the strain estimation from displacement data (Pietrantonio and Riguzzi, 2004 and references cited therein, Hamdy, 2004). Both Pietrantonio & Riguzzi (2004) and Hamdy (2004) expressed the strain tensor and rotation angle in one observation equation so that they would be determined simultaneously. However, we suggest another procedure, which is composed of two step calculations. Given the 2-dim displacement vector measurements at three stations (Fig. 1), we firstly acquire the most probable and unbiased estimate of strain tensor by using the least square technique, and then, estimate principal strain, rotation angle as following step. In Fig. 1, three 2-dim displacement vectors denoted as (u_i, v_i) are represented as arrows, while the locations of three stations are denoted as (x_i, y_i) for $i = 1, 2$ and 3. The center of the triangle is denoted as (x_c, y_c) , and the relative locations of each station with respect to the center are denoted as $(\Delta x_i, \Delta y_i)$.

Write the displacement field as (u, v) , then with neglecting the second and higher order terms we can express three sets of (u_i, v_i) as follows (Love, 1927).

$$u_i = \frac{\partial u}{\partial x} \Delta x_i + \frac{\partial u}{\partial y} \Delta y_i \quad \text{and} \quad v_i = \frac{\partial v}{\partial x} \Delta x_i + \frac{\partial v}{\partial y} \Delta y_i \quad (i = 1, 2 \text{ and } 3)$$

We rewrite these relations as the observation equation as the following.

$$\begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{bmatrix} = \begin{bmatrix} \Delta x_1 & \Delta y_1 & 0 & 0 \\ 0 & 0 & \Delta x_1 & \Delta y_1 \\ \Delta x_2 & \Delta y_2 & 0 & 0 \\ 0 & 0 & \Delta x_2 & \Delta y_2 \\ \Delta x_3 & \Delta y_3 & 0 & 0 \\ 0 & 0 & \Delta x_3 & \Delta y_3 \end{bmatrix} \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial y} \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{bmatrix} \quad (3)$$

Fig. 1. Two dimensional displacement vectors measured at three stations.

The most probable and unbiased estimate of the four parameters; $\partial u/\partial x$, $\partial u/\partial y$, $\partial v/\partial x$, and $\partial v/\partial y$ can readily be acquired as least square error solution. The four parameters compose deformation tensor. Given the variance-covariance Σ_y for observation, the variance-covariance of these parameters Σ_x can be acquired by using the relation; $\Sigma_x = (A^T A)^{-1} A^T \Sigma_y A (A^T A)^{-1}$, while Eq. (3) is expressed as $\hat{Y} = A \hat{X}$.

The strain tensor and principal strain are acquired accordingly, once the four deformation rates are known. The strain tensor $E_{ij} = 1/2(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$ can be written as

$$\begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) \\ \frac{1}{2}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) & \frac{\partial v}{\partial y} \end{bmatrix} \quad (4)$$

The deformation tensor D_{ij} can be separated into symmetric part – strain tensor E_{ij} and antisymmetric part – rotation tensor O_{ij} , i.e., $D_{ij} = E_{ij} + O_{ij}$, and matrix representation of this relation can be written as follows.

$$\begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) \\ \frac{1}{2}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) & \frac{\partial v}{\partial y} \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{2}(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}) \\ \frac{1}{2}(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}) & 0 \end{bmatrix}$$

The nonvanishing component of the rotation tensor; O_{12} corresponds to the rotational angle in counterclockwise sense (small rotation of the whole region involved). Dilatation, which is the expansion rate, can be expressed as $\nabla \cdot \bar{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$.

The definition of principal strain is that the strain is parallel to displacement along the principal direction, which can be written as

$$\begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \lambda \begin{bmatrix} x \\ y \end{bmatrix} \quad (5)$$

Both the direction and magnitude of the principal strain are acquired from this relation. First magnitude of principal strain λ (often called eigenvalue) is found from the condition of nontrivial solution for (x, y) , which is

$$\begin{vmatrix} E_{11} - \lambda & E_{12} \\ E_{21} & E_{22} - \lambda \end{vmatrix} = 0$$

or can be written as

$$\lambda^2 - (E_{11} + E_{22})\lambda + E_{11}E_{22} - E_{12}^2 = 0$$

Two values of the principal strain are acquired as

$$\lambda = \frac{E_{11} + E_{22}}{2} \pm \frac{1}{2} \sqrt{(E_{11} - E_{22})^2 + 4E_{12}^2} \quad (6)$$

Two perpendicular directions of principal strain can be found by inserting the value of λ to Eq. (5) and then solving for (x, y) . The direction of principal strain can be stably identified as the eigenvector of Eq. (5) by using a FORTRAN built-in function 'ATAN2.' This direction may simply be acquired by a formula; $\theta = 1/2 \tan^{-1}(2E_{12}/E_{11} - E_{22})$, which is mathematically equivalent (Hamdy, 2004). The two step procedure of this study has been found to yield at least one more effective digits for observables. The robustness came from isolation of rotation angle from the first step of estimation and also from stability of direct estimation of rotation angle through considering sine and cosine of argument.

Coseismic Deformation of South Korea due to the 2011 Tohoku-Oki Earthquake

In March 11 of the year 2011, an extremely large and devastating earthquake occurred in Japan. Its magnitude was known to be 9.0, and therefore its energy roughly amounts to 2.0×10^{18} joule or explosion of TNT 480 mega ton (Gutenberg and Richter, 1956). Coseismic displacements of a few meters were observed in Japan near the focus. In Fig. 2, the coseismic displacement of Korea measured by 73 GPS stations are illustrated (Baek, 2012), values of the two components and others are also given in Table 1 for convenient use in future. All the stations were found to have undergone Eastward displacement. Largest displacement of 5.38 cm was found at DOKD station, while smallest displacement of 1.05 cm was found at MARA station.

Fig. 2. GPS coseismic displacement of Korea due to the 2011 Tohoku earthquake.

Table 1. GPS measurement data for the co-seismic displacement in Korea due to the 2011 Tohoku-Oki earthquake. Each longitude and latitude of GPS station given in [deg] of East and North, and two components of the displacement vector and their standard deviations given in [cm] (after Baek, 2012).

	Station	Longitude	Latitude	U	$\sigma(u)$	V	$\sigma(v)$
01	BHAO	128.976	36.164	2.73	0.13	0.41	0.16
02	DAEJ	127.374	36.399	2.16	0.13	0.26	0.13
03	JEJU	126.462	33.288	1.13	0.13	0.29	0.16
04	MKPO	126.381	34.817	1.52	0.11	0.27	0.14
05	MLYN	128.744	35.491	2.35	0.12	0.39	0.15
06	SBAO	128.457	36.934	2.84	0.16	0.13	0.18
07	SKCH	128.565	38.251	3.31	0.20	-0.01	0.16
08	SKMA	126.918	37.494	2.35	0.11	0.12	0.13
09	KOHG	127.519	34.454	1.53	0.14	0.35	0.13
10	ANHN	126.135	36.674	1.65	0.17	0.21	0.15
11	CCHN	127.715	37.994	2.78	0.14	0.12	0.18
12	DANG	126.603	34.098	1.28	0.18	0.32	0.16
13	DOKD	131.870	37.239	5.37	0.11	0.39	0.13
14	EOCH	125.968	36.125	1.77	0.16	0.24	0.14
15	GASA	126.043	34.461	1.25	0.16	0.26	0.12
16	GEOM	127.322	34.008	1.52	0.13	0.30	0.16
17	HGDO	125.204	34.711	1.23	0.19	0.17	0.15
18	HOMI	129.567	36.078	2.87	0.14	0.44	0.15
19	JEOJ	128.399	38.552	3.03	0.17	-0.13	0.17
20	JUKB	129.429	37.058	3.19	0.17	0.25	0.15
21	JUMN	128.834	37.898	3.27	0.15	0.03	0.14
22	MARA	126.269	33.117	1.01	0.12	0.28	0.14
23	MLDO	126.315	35.858	1.74	0.15	0.20	0.13
24	MOOJ	127.583	35.903	2.07	0.14	0.31	0.14
25	PYCH	128.488	37.350	3.01	0.19	0.11	0.17
26	SEOI	128.738	34.788	1.86	0.15	0.48	0.15
27	SOCH	124.729	37.760	1.66	0.12	0.11	0.14
28	SOHE	125.099	34.095	1.14	0.17	0.27	0.16
29	SORI	127.801	34.412	1.50	0.18	0.36	0.14
30	SUWN	127.054	37.276	2.33	0.13	0.18	0.15
31	ULLE	130.798	37.518	4.66	0.18	0.19	0.16
32	YND0	129.071	35.062	2.22	0.12	0.63	0.11
33	BOEN	127.730	36.488	2.38	0.12	0.30	0.13
34	CHCN	127.711	37.869	2.51	0.38	0.12	0.22
35	CHEN	127.155	36.878	2.04	0.16	0.09	0.17
36	CHJU	126.530	33.514	1.16	0.14	0.48	0.16
37	CHLW	127.415	38.164	2.51	0.11	0.02	0.16
38	CHNG	128.478	35.533	2.26	0.09	0.48	0.11
39	CHSG	129.056	36.436	2.77	0.15	0.43	0.13
40	CHWN	128.693	35.236	2.00	0.15	0.58	0.16
41	CHYG	126.801	36.459	2.01	0.12	0.29	0.16
42	CNJU	127.461	36.627	2.23	0.12	0.32	0.15
43	DOND	127.061	37.902	2.41	0.11	0.11	0.14
44	GOCH	127.943	35.668	2.24	0.17	0.44	0.17
45	GSAN	127.787	36.816	2.55	0.12	0.22	0.16
46	HADG	127.709	35.162	1.83	0.20	0.40	0.16
47	HONC	128.194	37.709	2.87	0.15	0.21	0.13
48	INCH	126.686	37.420	2.12	0.14	0.16	0.17
49	INJE	128.171	38.069	3.05	0.22	-0.09	0.31
50	JAHG	126.900	34.675	1.61	0.14	0.55	0.17
51	JINJ	128.050	35.173	1.94	0.10	0.41	0.13

Table 1. Continued.

	Station	Longitude	Latitude	U	$\sigma(u)$	V	$\sigma(v)$
52	JUNG	126.974	35.623	1.85	0.14	0.39	0.12
53	JUNJ	127.135	35.843	1.90	0.11	0.33	0.14
54	KANR	128.868	37.771	3.31	0.16	0.10	0.15
55	KUNW	128.574	36.233	2.77	0.19	0.37	0.15
56	KWNJ	126.910	35.178	1.94	0.16	0.26	0.16
57	MUJU	127.661	36.003	2.11	0.11	0.39	0.13
58	NAMW	127.396	35.423	1.90	0.12	0.40	0.13
59	NONS	127.099	36.186	2.04	0.11	0.37	0.14
60	PAJU	126.738	37.746	2.22	0.11	0.09	0.13
61	SEOS	126.494	36.776	1.97	0.10	0.23	0.11
62	SNJU	128.144	36.379	2.35	0.14	0.33	0.14
63	SONC	127.486	34.957	1.72	0.13	0.43	0.14
64	SOUL	127.080	37.630	2.34	0.12	0.07	0.13
65	TABK	128.976	37.161	3.09	0.29	0.24	0.18
66	TEGN	128.802	35.906	2.48	0.14	0.42	0.14
67	WNJU	127.947	37.337	2.60	0.11	0.23	0.12
68	WOLS	129.416	35.504	2.58	0.14	0.58	0.14
69	WULJ	129.413	36.992	3.24	0.13	0.29	0.12
70	YANP	127.506	37.454	2.70	0.12	0.12	0.13
71	YECH	128.446	36.651	2.52	0.12	0.33	0.14
72	YONK	126.516	35.279	1.52	0.16	0.18	0.15
73	YOWL	128.462	37.183	2.85	0.11	0.11	0.13

To see the most representative amount of principal strain, least square solution for the largest triangle was acquired; +64.42 and -6.25 nanostrain along the directions of 13.89 and -76.11 degrees with respect to the East (Fig. 3). The value of dilatation for this single largest triangle was found as $\Delta = E_{11} + E_{22} = 58.2$ nanostrain. Therefore, South Korea, as a whole, was slightly elongated in roughly the East-West direction and was very slightly compressed in North-South

Fig. 3. Principal strain estimated for coseismic displacements due to the Tohoku earthquake at the largest triangular GPS station array in Korea.

Fig. 4. Principal strain, dilatation, and rotation estimated for coseismic displacements due to the Tohoku earthquake at 8 large triangle GPS station network in Korea; (a) principal strain, (b) dilatation, and (c) rotation.

direction due to the earthquake. The value of rotation was found as $O = -1.23 \times 10^{-8}$ (rad), so that South Korea was

slightly rotated in clockwise direction.

The principal strain, dilatation, and rotation for 8 triangle network were estimated accordingly and are illustrated (Fig. 4). Their values are given in Table 2(b). A little detail of estimates for this network is given as the following. Since the epicenter of the earthquake is located at far East from Korea, not only the displacement but also the strain was large in the Eastern part of Korea. Both the displacement and strain varied almost inversely proportional to the distance from the epicenter. The value of elongation principal strain E_{11} was largest at the Northeastern region and smallest at the Southwestern region, and varied from 84.7 to 21.4 nanostrain (from East to West). Likewise, the value of compression principal strain E_{22} was largest at the Northeastern region and smallest at the Southwestern region, and varied from -21.1 to -7.1 nanostrain (from East to West). Rotation angles estimated for each triangular arrays were all small and counterclockwise, and varied from 2.33×10^{-9} to 6.52×10^{-10} (unit: radian). Rotation was largest at the East lower region and smallest at Kyunggi-Choongnam region.

As the size of each triangular array become smaller, the acquired principal strain better show localized pattern in the whole principal strain. The finer the network becomes, the more details of the localized pattern could be known. However, care should be taken in choosing the triangular network size and configuration, because calculation instability also grows larger for smaller network. To see this, we go back to the observation equation (Eq. 3). The elements of A matrix of Eq. (3) (in the form of $Y = AX + \varepsilon$) are Δx_i and Δy_i , which are proportional to the size of each triangular array in the network. Evidently the variance-covariance matrix of deformation tensor estimate will become larger for smaller network, according to the law of error propagation. One may expect the variance be inversely proportional to the square of the dimension, i.e., to the area.

The variance-covariance matrix of the deformation tensor $\Sigma_x = (A^T A)^{-1} A^T \Sigma_y A (A^T A)^{-1}$ were calculated for the three cases of 1, 8 and 59 triangle networks, and their r.m.s values were found as 0.077, 0.91 and 18 nanostrain respectively. Therefore the least square solution for the 59 triangle network is prone to contain large amount of error (same order of strain estimate itself), while the estimate for 1 or 8 triangle network is either quite stable or acceptable. In Table 2(a) ~ (c), r.m.s. values of error in the estimate of deformation tensor for each triangles are given together with principal strain, dilatation and rotation for the three kinds of triangular net-

Table 2. (a)-(c) Triangular network configurations and the estimates of principal strain, dilatation, rotation, and standard deviation of deformation tensor (a-c: three kinds networks of 1, 8, and 59 triangles each). Principal strain, dilatation, standard deviation are dimensionless and given in nanoscale. Negative values of strain refer to compression. Principal direction is denoted by counterclockwise angle from the East (in parenthesis). The value of rotation is given as counterclockwise angle in radian. Only some parts of content are shown for Table 2(c).

(a) Estimated Parameters (1 triangle)

triangle	stations	2 principal strain (angle)		dilatation	rotation	s.t.d.
1	27-22-13	64.4 (13.9°)	-6.3 (-76.1°)	58.1	-1.23E-8	0.077

(b) Estimated Parameters (8 triangles)

triangle	stations	2 principal strain (angle)		dilatation	rotation	s.t.d.
1	27-14-43	36.0 (10.4°)	-8.3 (-79.6°)	27.7	-7.32E-9	0.61
2	14-62-43	30.8 (17.2°)	-1.3 (-72.8°)	17.3	-6.52E-9	0.72
3	43-62-07	59.5 (11.8°)	-1.9 (-78.2°)	40.7	-2.01E-8	1.00
4	07-62-13	84.7 (10.6°)	-2.1 (-79.4°)	63.6	-1.23E-8	0.52
5	14-28-03	21.4 (26.7°)	-7.3 (-63.3°)	14.1	-1.10E-8	1.55
6	14-03-62	32.4 (23.3°)	-7.1 (-66.7°)	25.3	-9.62E-9	0.61
7	03-26-62	26.2 (35.5°)	-2.0 (-54.5°)	6.0	-8.94E-9	1.09
8	62-26-13	83.8 (16.6°)	-1.5 (-73.4°)	68.4	-2.33E-8	0.66

(c) Estimated Parameters (59 triangles)

triangle	stations	2 principal strain (angle)		dilatation	rotation	s.t.d.
1	19-49-07	96.6 (178.5°)	-20.6 (-91.5°)	76.0	3.74E-8	34.3
2	37-49-12	81.6 (179.1°)	-3.9 (-90.9°)	77.7	-1.54E-8	5.4
3	37-34-49	95.1 (9.7°)	-54.2 (-80.3°)	40.9	-4.88E-8	27.0
4	37-43-34	19.1 (8.2°)	-31.7 (-81.8°)	-12.6	-7.42E-9	17.1
5	43-30-34	19.1 (12.0°)	-11.4 (-78.0°)	7.7	-5.13E-9	7.0
6	43-60-30	60.9 (9.6°)	-12.2 (-80.4°)	48.7	9.98E-10	15.9
7	34-30-67	34.0 (2.6°)	-16.0 (-87.4°)	18.0	5.31E-9	9.3
8	34-67-47	92.3 (8.7°)	-15.6 (-81.3°)	76.7	-8.33E-10	24.5
9	34-47-49	105.7 (6.3°)	-77.7 (-83.7°)	28.0	-3.03E-8	23.3
10	60-61-30	60.1 (6.5°)	-15.1 (-83.5°)	45.0	-2.73E-9	7.9
11	49-21-07	65.3 (26.4°)	-12.6 (-63.6°)	52.7	-9.99E-9	28.6
12	49-47-21	55.7 (9.5°)	-78.8 (-80.5°)	-23.1	-2.57E-8	21.8
13	47-25-21	68.4 (169.7°)	1.3 (-100.3°)	69.7	-2.07E-8	11.5
14	47-67-25	83.7 (178.3°)	8.5 (-91.7°)	92.2	-2.26E-8	11.2
15	21-25-65	39.4 (20.1°)	-31.1 (-69.9°)	8.4	-4.03E-9	12.3
-	-	-	-	-	-	-
-	-	-	-	-	-	-
-	-	-	-	-	-	-
57	10-05-68	61.5 (46.9°)	16.9 (-43.1°)	78.4	-8.19E-9	27.5
58	05-32-68	56.3 (25.3°)	-49.9 (-64.7°)	6.4	-8.91E-9	7.6
59	05-26-32	93.6 (30.5°)	-48.7 (-59.5°)	44.9	-3.52E-11	16.9

works.

In Fig. 5(a)~(c), the principal strain, dilatation, and rotation estimated for the 59 triangular networks are illustrated. The value of principal strain varies more or less in the same range, however, the principal direction show large fluctuation at regions. This complicated feature possibly reflects locally

different direction of principal strain, but also possibly was misled by erroneous estimation due to too small size of each network triangles. Likewise estimated dilatation and rotation show various features over the all part of the Korean peninsula, but not all of these features can be taken as real. We found the same type erroneous estimation (worse than ours

Fig. 5. Principal strain, dilatation, and rotation estimated for coseismic displacements due to the Tohoku earthquake at 59 triangle GPS station network in Korea; (a) principal strain, (b) dilatation, and (c) rotation.

for the 59 triangular networks) in a pre-published report, and such has been one of our motives to write this article.

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Appendix:

Least Square Estimation: More Description

(This appendix is included by the first author wishing it be helpful as tutorial. Basic discipline is its content in statistics, geodesy, and surveying, but not in most other branches of science and technology.)

Prior to describe the general least square error estimation with Gauss-Markov model, two simple examples for least square error estimation are considered. The first one is 'simple arithmetic mean'. Suppose we are to measure the weight of a coin using a scale. In case we make just one measurement and acquire m_1 as a result, then it would be the value we can take. If we measure twice and acquire m_1 and m_2 , then normally we choose the value as $m_0 = (m_1 + m_2)/2$, which is simple arithmetic mean. If we write the each differences of the two measurements from the mean value m as $e_1 = m_1 - m$ and $e_2 = m_2 - m$. Write the sum of the squares of two errors as $S = e_1^2 + e_2^2$, we find the sum S becomes minimum for $m = (m_1 + m_2)/2 = m_0$. Extension to multiple measurements is obvious, and we similarly find the corresponding arithmetic mean $m = (m_1 + m_2 + m_3 + \dots + m_n)/n$ as least square error estimate of the coin's weight.

As second example, consider a set of two dimensional data; $(x_1, y_1), (x_2, y_1), (x_3, y_3), \dots, (x_n, y_n)$. Assuming an elongated distribution of the data in x - y plane, we seek a line; $y = ax + b$, which somehow well fits the distributed data on the plane (Fig. 2). Write the difference between each data point and the fit line in y -direction as e_i ; that is

$$e_i = y_i - (ax_i + b).$$

Then we can define the sum of square error as

$$S = e_1^2 + e_2^2 + e_3^2 + \dots + e_n^2 = \sum_i e_i^2 = \sum_i (y_i - ax_i - b)^2.$$

To determine the two coefficients a and b , we impose conditions as $\partial S / \partial a = 0$ and $\partial S / \partial b = 0$, which are written as follows.

$$\frac{\partial}{\partial a} \sum_i (y_i - ax_i - b)^2 = -2 \sum_i x_i (y_i - ax_i - b) = 0,$$

$$\frac{\partial}{\partial b} \sum_i (y_i - ax_i - b)^2 = -2 \sum_i (y_i - ax_i - b) = 0.$$

After rearranging terms, we can rewrite the two conditions in matrix form as

$$\begin{bmatrix} \sum_i x_i^2 & \sum_i x_i \\ \sum_i x_i & n \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum_i x_i y_i \\ \sum_i y_i \end{bmatrix}.$$

Invert this relation and then we find values of a and b of desired fit line.

Method of observation equation for linear model can be summarized as below. Suppose we have redundancy in observation, *i.e.*, more observations than parameters to be determined. Write the linear model as

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix}$$

where $y_1, y_2, y_3, \dots, y_n$ are observations with each errors as $e_1, e_2, e_3, \dots, e_n$, and $x_1, x_2, \dots, x_m$ are the parameters to be determined ($n > m$). This relation can be equivalently rewritten as $y_i = \sum_j a_{ij} x_j + e_i$ or $Y = AX + \varepsilon$. Since the errors are random ones, then their expectation values are zeroes, and then we have $\hat{Y} = A\hat{X}$, where the hats denote expectation values. We may proceed to minimize sum of the squares of errors, however we would introduce concept of proper weight to observations in the minimizing condition. The variance-covariance matrix for observed quantities is denoted as

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \dots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \dots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \dots & \sigma_n^2 \end{bmatrix}$$

where $\sigma_i^2 = \text{var}(y_i)$ and $\sigma_{ij} = \text{cov}(y_i, y_j)$. The law of error propagation is stated as; $\Sigma_y = A \Sigma_x A^T$ when two column vectors X and Y are related by $Y = AX$. To calculate Σ_y from Σ_x can be readily done using their relation; $\Sigma_y = A \Sigma_x A^T$. To do the reverse we can equate as follows; $\Sigma_x = (A^T A)^{-1} A^T \Sigma_y A (A^T A)^{-1}$.

Write the weighted sum of square errors as $S = \varepsilon^T \Sigma^{-1} \varepsilon$. As the next step, we minimize S and identify $\hat{X}$ as the corresponding values of parameters of X . The sum $S = \varepsilon^T \Sigma^{-1} \varepsilon$ can be explicitly shown as

$$\begin{aligned} S &= (Y - AX)^T \Sigma^{-1} (Y - AX) \\ &= Y^T \Sigma^{-1} Y - Y^T \Sigma^{-1} AX - X^T A^T \Sigma^{-1} Y + X^T A^T \Sigma^{-1} AX \\ &= Y^T \Sigma^{-1} Y - 2Y^T \Sigma^{-1} AX + X^T A^T \Sigma^{-1} AX \end{aligned}$$

where the last step comes due to $Y^T \Sigma^{-1} AX$ is just a number, and so $X^T A^T \Sigma^{-1} Y = Y^T \Sigma^{-1} AX$.

Take the partial derivative of the sum S with respect to the parameters X , then claim it to vanish for $\hat{X}$ as follows.

$$\frac{\partial S}{\partial X} = 2A^T \Sigma^{-1} AX - 2A^T \Sigma^{-1} Y \Rightarrow 0 \quad \text{for } X = \hat{X}$$

$$A^T \Sigma^{-1} A \hat{X} - A^T \Sigma^{-1} Y = 0$$

i.e. $\hat{X} = (A^T \Sigma^{-1} A)^{-1} (A^T \Sigma^{-1} Y)$. For those, who are not familiar with matrix manipulation, alternative notation showing indexes explicitly is recommended ($e_i = y_i - \sum_j a_{ij} x_j$, $S = \sum_i \sum_j e_i \sigma_{ij} e_j$, equate $\partial S / \partial x_k$ and then claim it to be zero).

We can deduce the same result as above for the two examples by using Gauss-Markov formulation. Suppose the weight of a coin is determined by the n-times repeated measurements. Then observation equation can be written as

$$\begin{bmatrix} m_1 \\ m_2 \\ \cdot \\ \cdot \\ m_n \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \cdot \\ \cdot \\ 1 \end{bmatrix} m + \begin{bmatrix} e_1 \\ e_2 \\ \cdot \\ \cdot \\ e_n \end{bmatrix}$$

The least square estimate of the weight is given as $\hat{m} = (A^T \Sigma^{-1} A)^{-1} (A^T \Sigma^{-1} Y)$. The simple arithmetic mean is readily found as $\hat{m} = (m_1 + m_2 + \dots + m_n)/n$ by assuming equal and uncorrelated measurements. Likewise, linear regression of two dimensional data can be done by using observation equation as follows. To draw a line ($y = ax + b$) for a given set of two dimensional data; $(x_1, y_1), (x_2, y_2), (x_3, y_3), \dots, (x_n, y_n)$, we can write observation equation as

$$\begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \cdot & \cdot \\ \cdot & \cdot \\ x_n & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \cdot \\ \cdot \\ e_n \end{bmatrix}$$

Then estimate of $\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix}$ as $(A^T \Sigma^{-1} A)^{-1} (A^T \Sigma^{-1} Y)$ by assuming equal and uncorrelated data leads to the same result.